

# ESE3700: Circuit-Level Modeling, Design and Optimization for Digital Systems

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Lec 4: February 5, 2024  
Regenerative Property





# Today

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- We know how to make our logic functional, but how do we make sure logic is robust?
  - To enable design cascading gates into any (feed forward) graph and still tolerate voltage drops and noise, while maintaining digital abstraction



# Outline

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- Two signal problems → Gate Cascade failure
- Regeneration Solution → Gate Abstraction
  - Transfer Curves
  - Noise Margins
  - Non-linearity



# Two Signal Problems

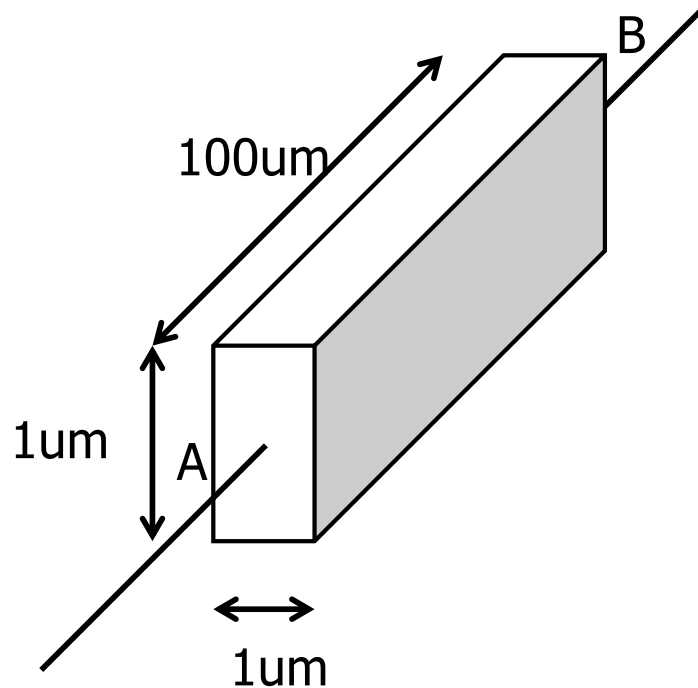
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1. Output does not go to rail  
Stops short of  $V_{dd}$  or Gnd
2. Signals may be perturbed by noise

$$V_x = V_{\text{ideal}} \pm V_{\text{noise}}$$

# Wire Resistance (Preclass 1)

□ Resistance of 100  $\mu\text{m}$  long wire?

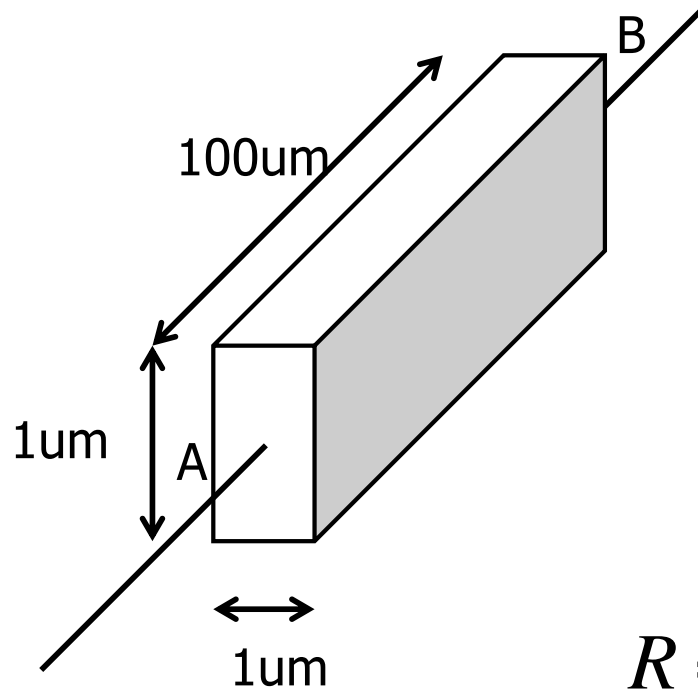


$$\rho = 10^{-7} \Omega \cdot \text{m}$$

$$R = \frac{\rho L}{A}$$

# Wire Resistance

□ 100  $\mu\text{m}$  long wire?



$$R = \frac{\rho L}{A}$$

$$R = \frac{10^{-7} \Omega \cdot m \cdot 100 \mu m}{1 \mu m \cdot 1 \mu m} = 10 \Omega$$



# Wire Resistance

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- ❑ 1 mm long wire?
- ❑ 1 cm long wire?
- ❑ Length of integrated circuit chip side?
  - (we often call an IC chip a “die”)



# Die Sizes

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Chip	#Trans.	Year	Maker	process	mm <sup>2</sup>
GK10 Kepler	7B	2012	NVIDIA	28nm	561
22-core Xeon Broadwell-E5	7B	2016	Intel	14nm	456
GC2 IPU	23.6B	2018	Graphcore	16nm	825
Apple A12X Bionic	10B	2018	Apple	7nm	122
Tegra Xavier SoC	9B	2018	Nvidia	12nm	350
Navi 10	10B	2021	AMD	7nm	251
AMD Instinct MI300A	146B	2023	AMD	5nm	1,017

source: [http://en.wikipedia.org/wiki/Transistor\\_count](http://en.wikipedia.org/wiki/Transistor_count)

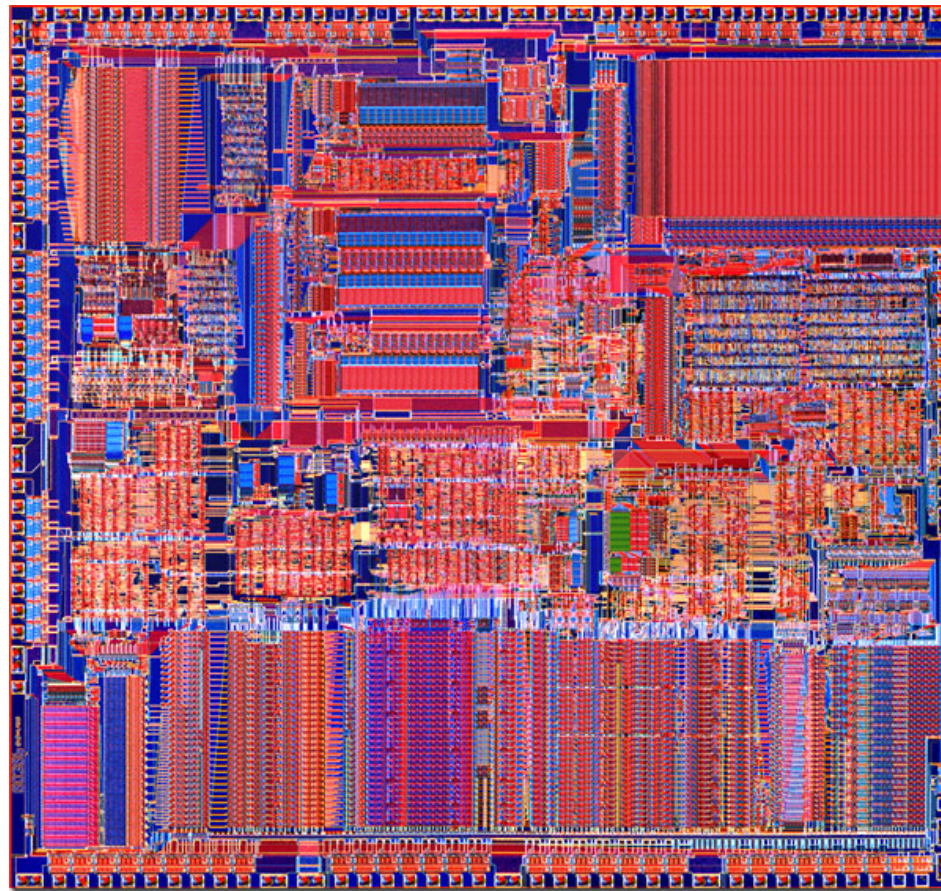




# Implications

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- ❑ What does the circuit really look like for an inverter in the middle of the chip?

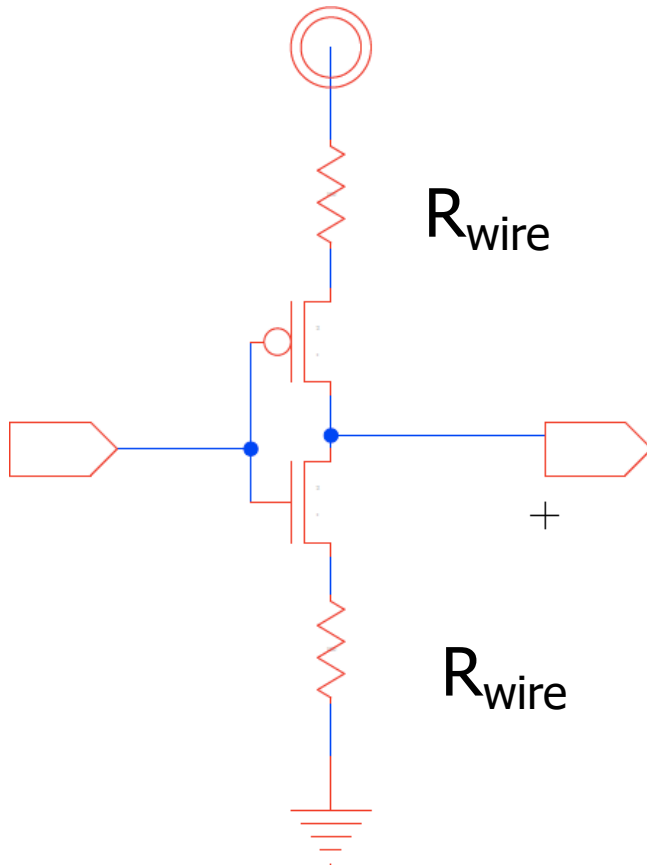




# Implications

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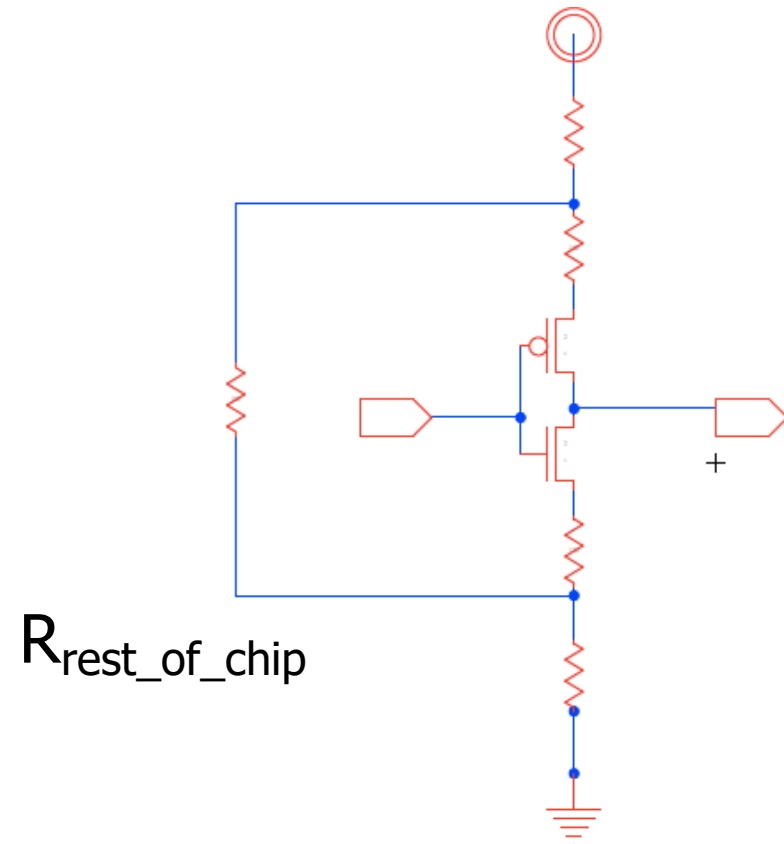
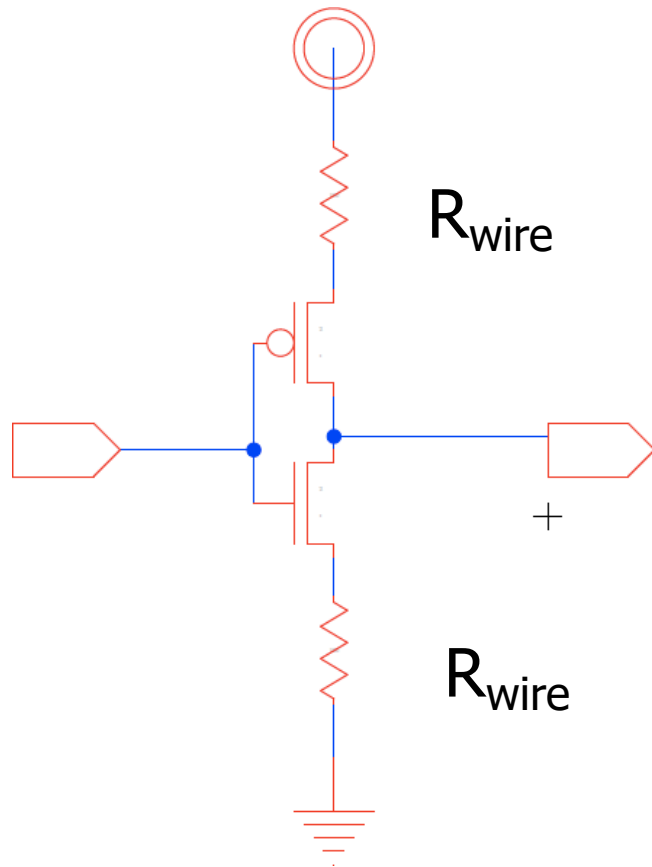
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# Implications

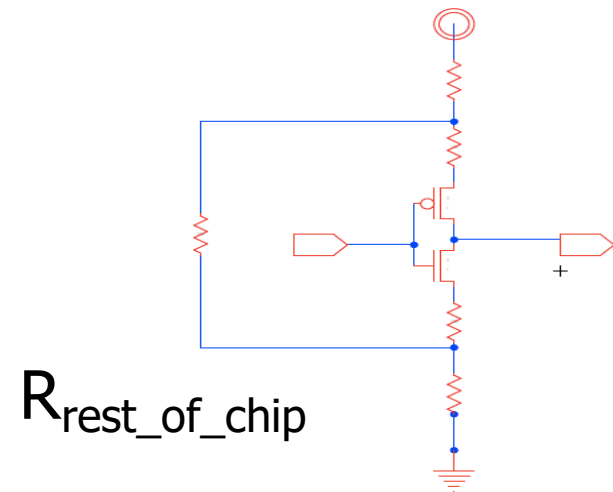
- ❑ What does the circuit really look like for an inverter in the middle of the chip?





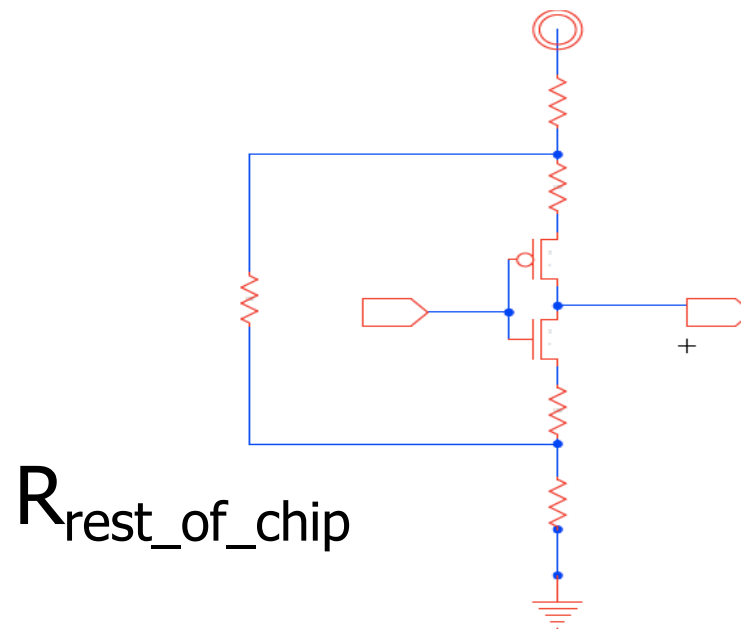
# IR-Drop

- ❑ Since interconnect is resistive and gates pull current off the supply interconnect
  - The  $V_{dd}$  seen by a gate is lower than the supply Voltage by
    - $V_{drop} = I_{supply} \times R_{distributed}$
  - Two gates in different locations
    - See different  $R_{distributed}$
    - Therefore, see different  $V_{drop}$



# Output does not go to Rail

- Due to  $V_{\text{drop}}$ , “rails” for two communicating gates may not match





# Two Signal Problems

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1. Output not go to rail  
Different swing for gates
2. Signals may be perturbed by noise  
Voltage seen at input to a gate may be lower/higher than input voltage



# Noise Sources?

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- ❑ Signal coupling
  - Crosstalk
- ❑ Inductive noise
- ❑ Leakage
- ❑ Ionizing particles (shot noise)



# Signals **will** be degraded

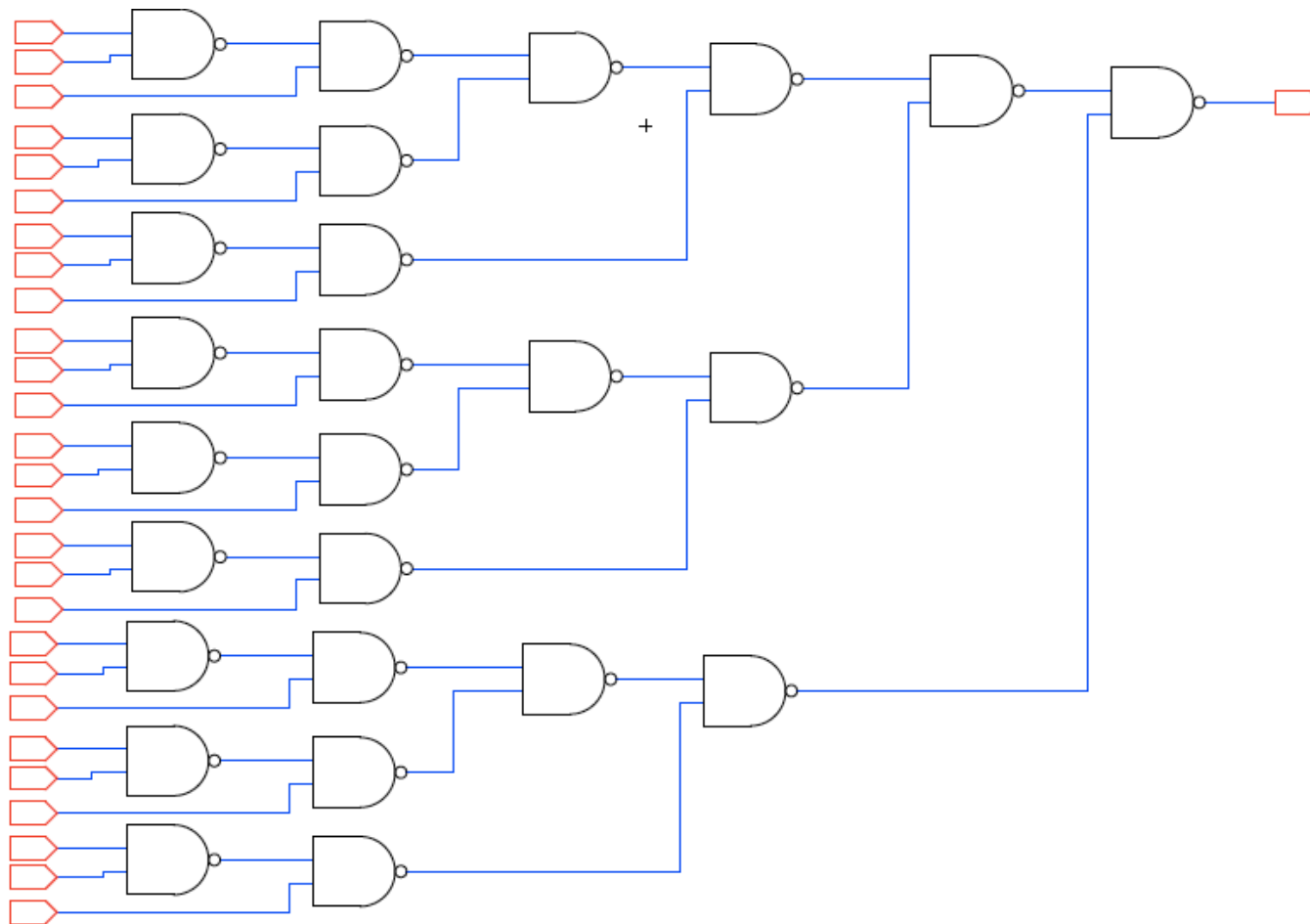
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1. Output not go to rail
    - Is this tolerable?
  2. Signals may be perturbed by noise
    - Voltage seen at input to a gate may be lower/higher than expected input voltage
- What happens to degraded signals?



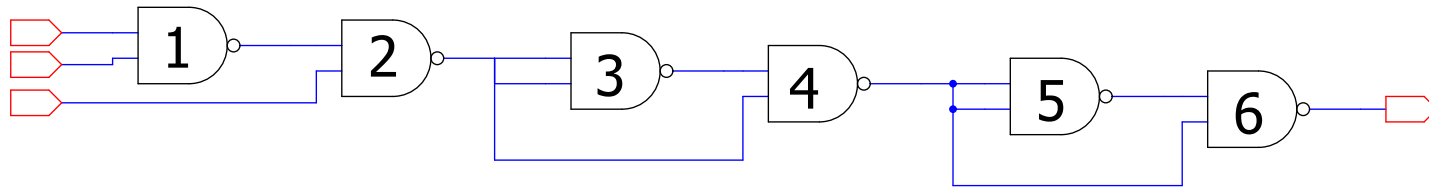
# Gate Cascade Example (Preclass 2)

- What is the output when all inputs are all 1s?



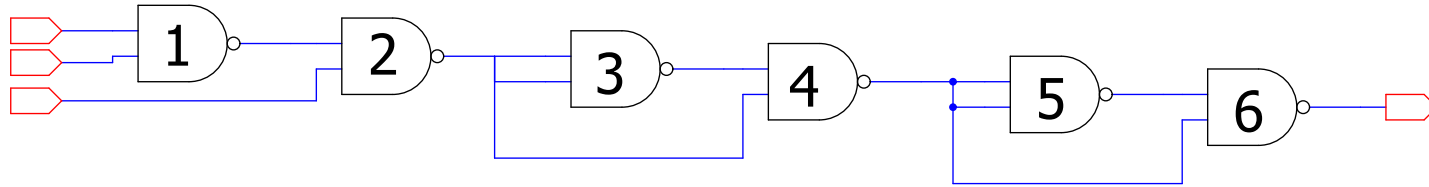
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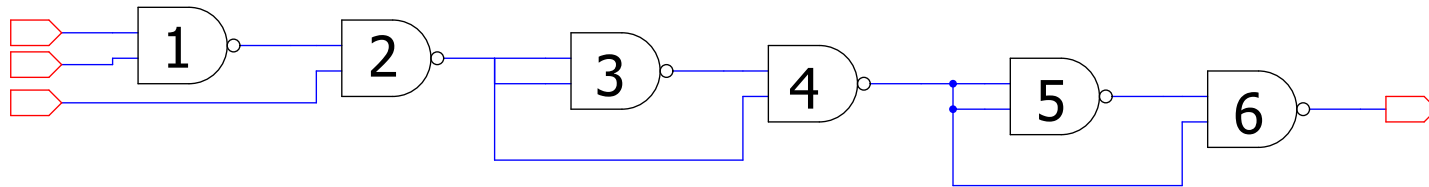
# Gate Cascade Example (Preclass 2)

- What is the output when all inputs are all 1.0 and  $\text{NAND}(A, B) = 1 - A * B$ ?



# Gate Cascade Example (Preclass 2)

- What is the output when all inputs are all 0.95 and  $\text{NAND}(A, B) = 1 - A * B$ ?





# Degradation

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- ❑ Cannot have signal degrade across cascaded gates
- ❑ Want to be able to cascade arbitrary set of gates
  - No limit on number of gates to maintain signal integrity



# Gate Creed

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- Gates should leave the signal “better” than they found it
  - “better” → closer to the rails



# Regeneration Discipline

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- Define legal inputs
  - Gate works if  $V_{in}$  “close enough” to the rail



# Regeneration Discipline

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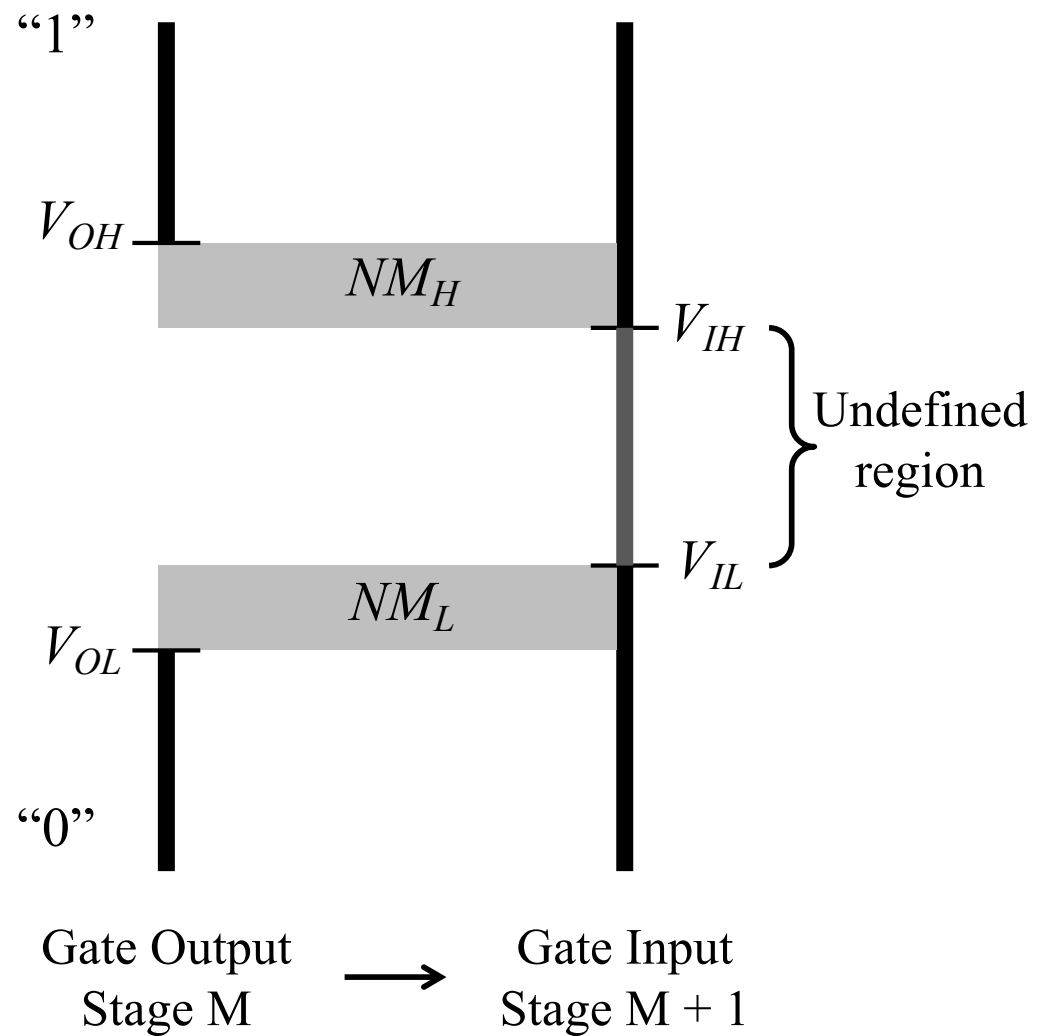
- Define legal inputs
  - Gate works if  $V_{in}$  “close enough” to the rail
  
- Regeneration
  - Gate produces  $V_{out}$  “closer to rail”
    - This tolerates some drop between one gate and next (between out and in)
    - Call this our “Noise Margin”

Regeneration/Restoration/Static Discipline



# Noise Margin

- $V_{OH}$  – output high
- $V_{OL}$  – output low
  
- $V_{IH}$  – input high
- $V_{IL}$  – input low
  
- $NM_H = V_{OH} - V_{IH}$
- $NM_L = V_{IL} - V_{OL}$



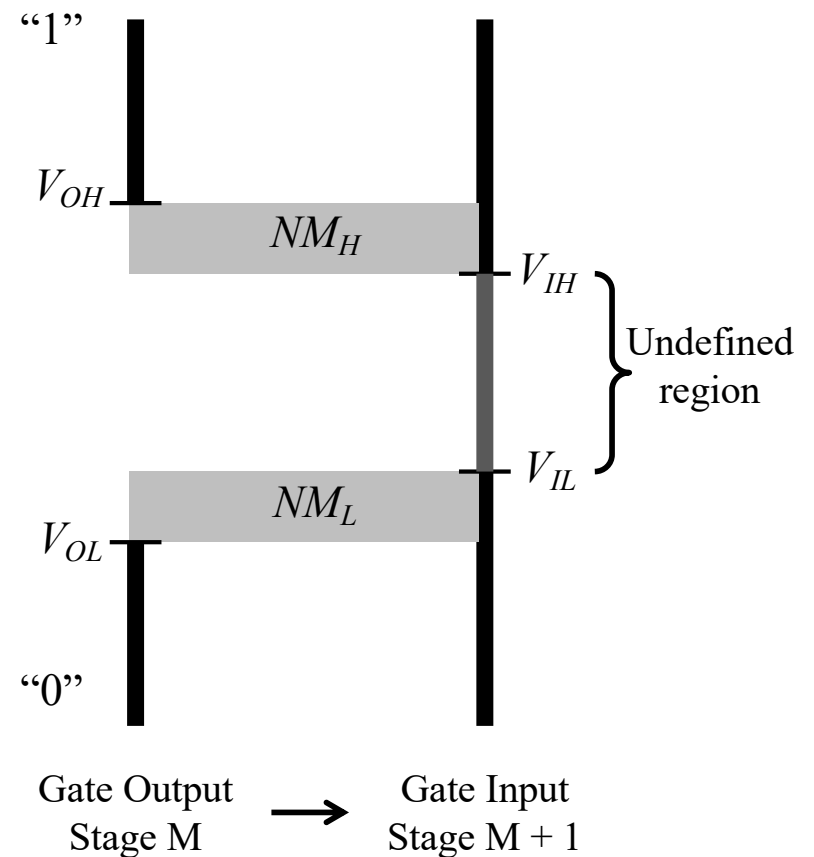
# Regeneration Discipline (getting precise)

## □ Define legal inputs

- Gate works if  $V_{in}$  “close enough” to the rail
- $V_{in} > V_{IH}$  or  $V_{in} < V_{IL}$

## □ Regeneration

- Gate produces  $V_{out}$  “closer to rail”
  - $V_{out} < V_{OL}$  or  $V_{out} > V_{OH}$



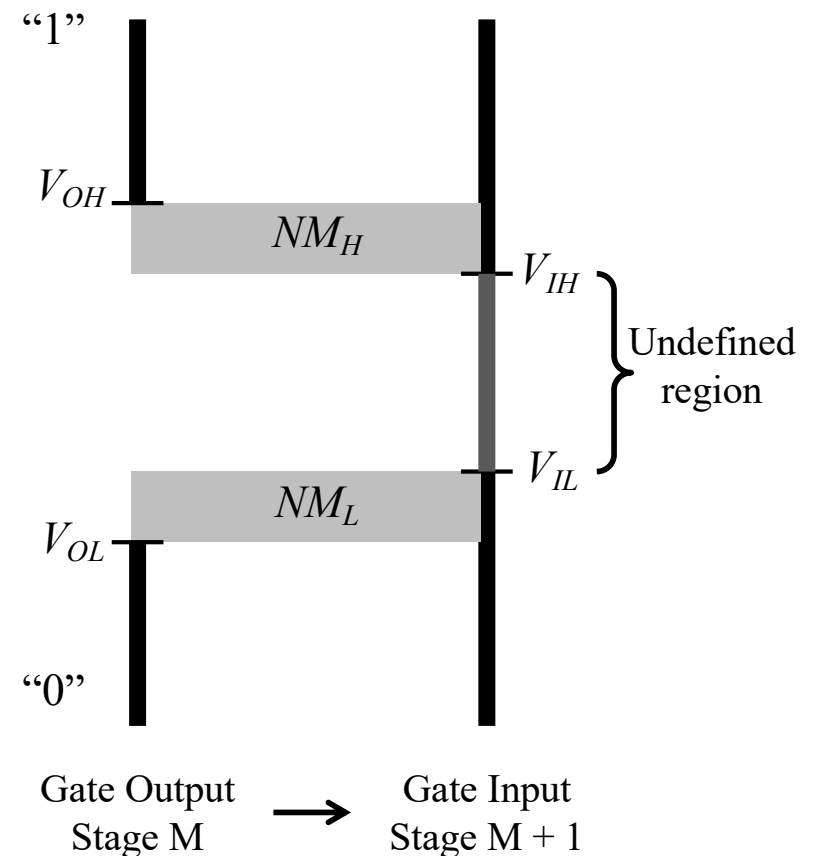
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## □ Regeneration

- Gate produces  $V_{out}$  “closer to rail”
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# Transfer Function

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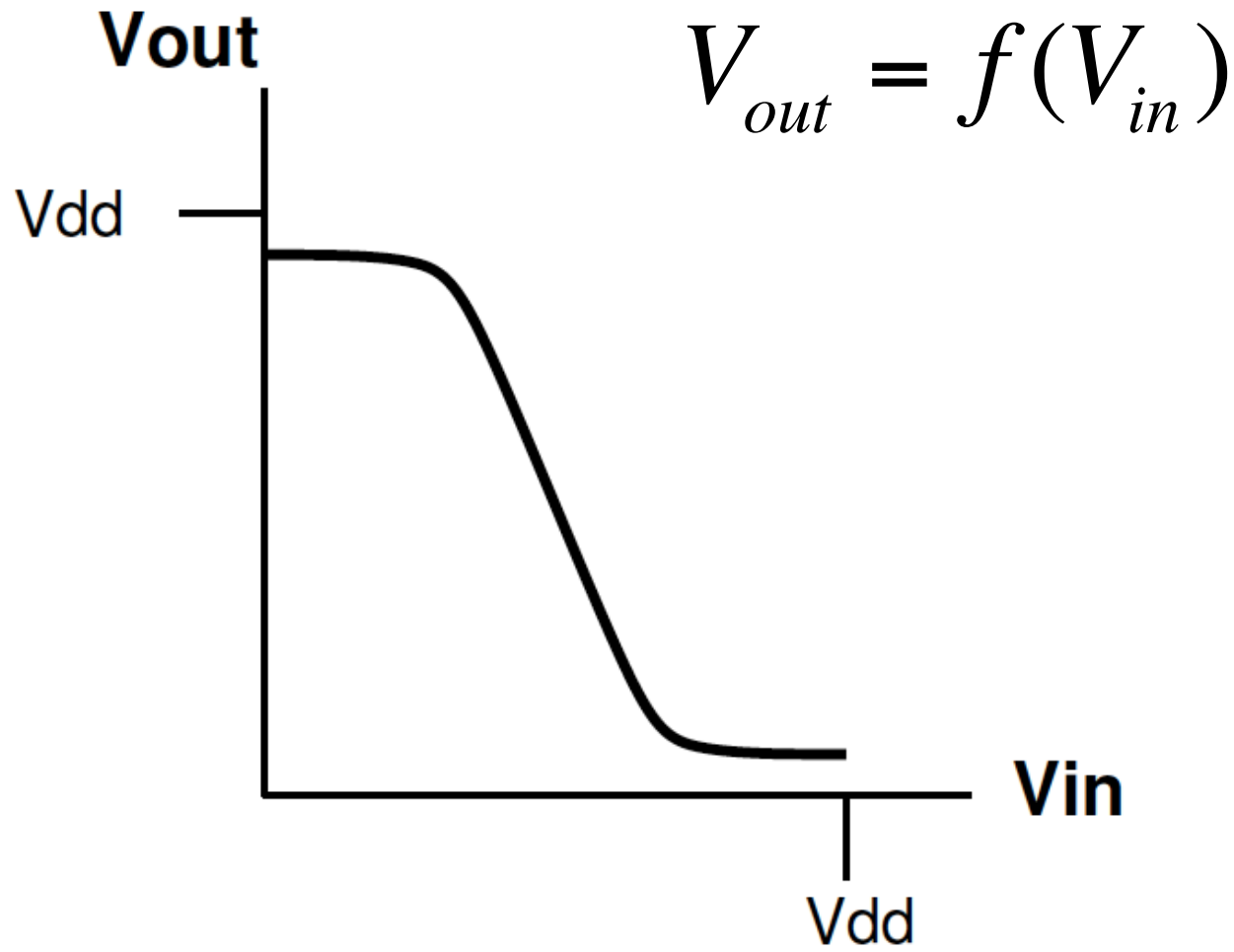
- Describes what the output is given logic gate input
  - $V_{\text{out}} = f(V_{\text{in}})$



# Transfer Function

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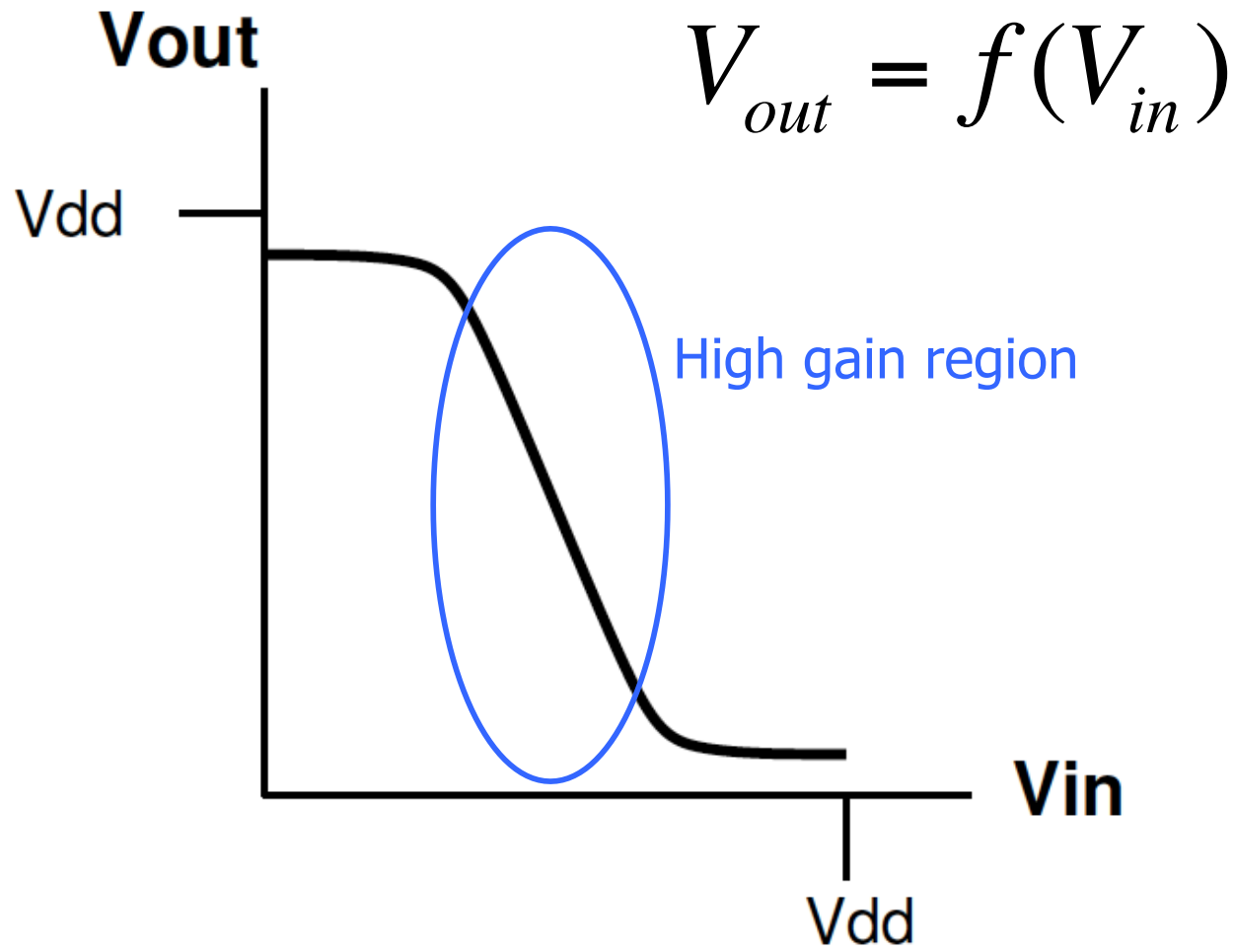
- What gate is this?





# Transfer Function

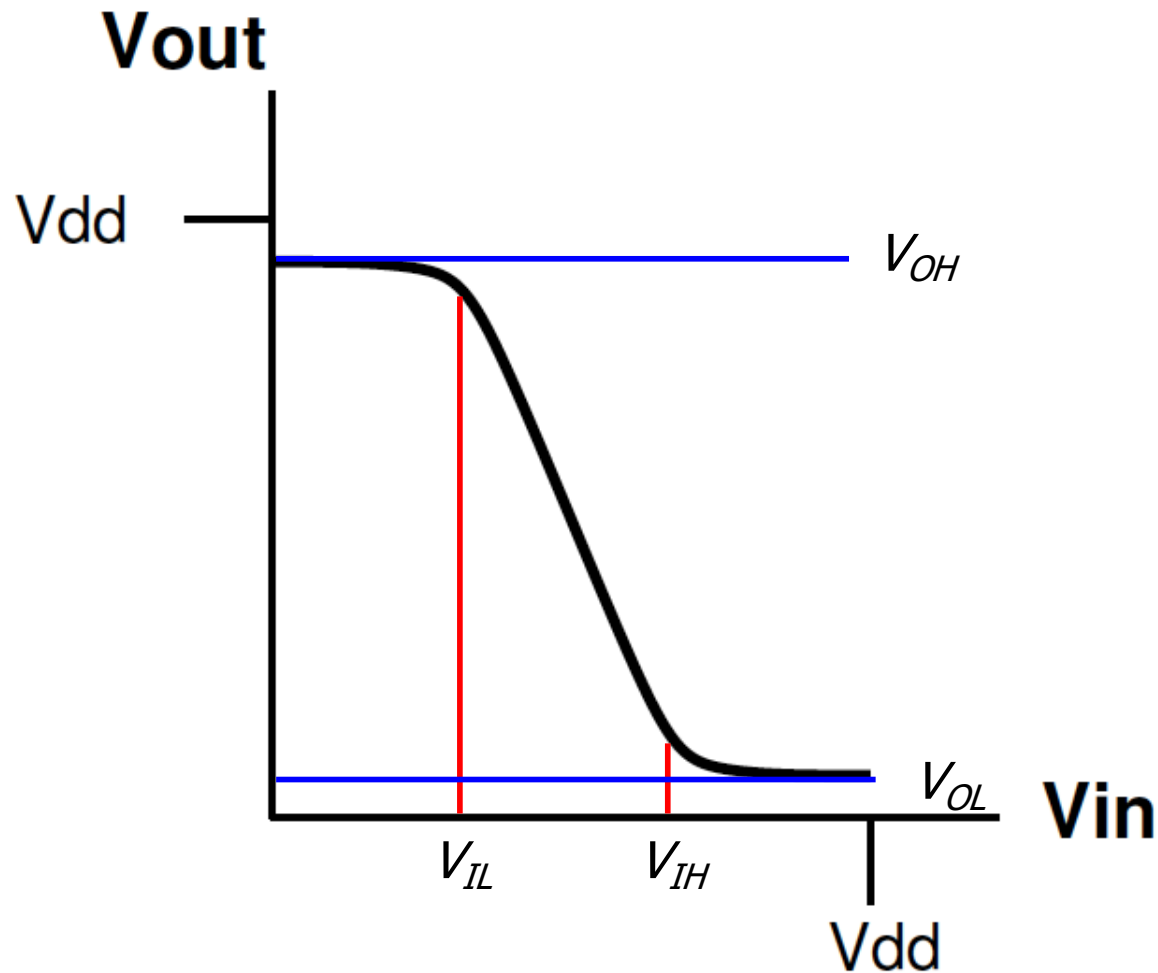
□ What gate is this?





# Regenerating Transfer Function

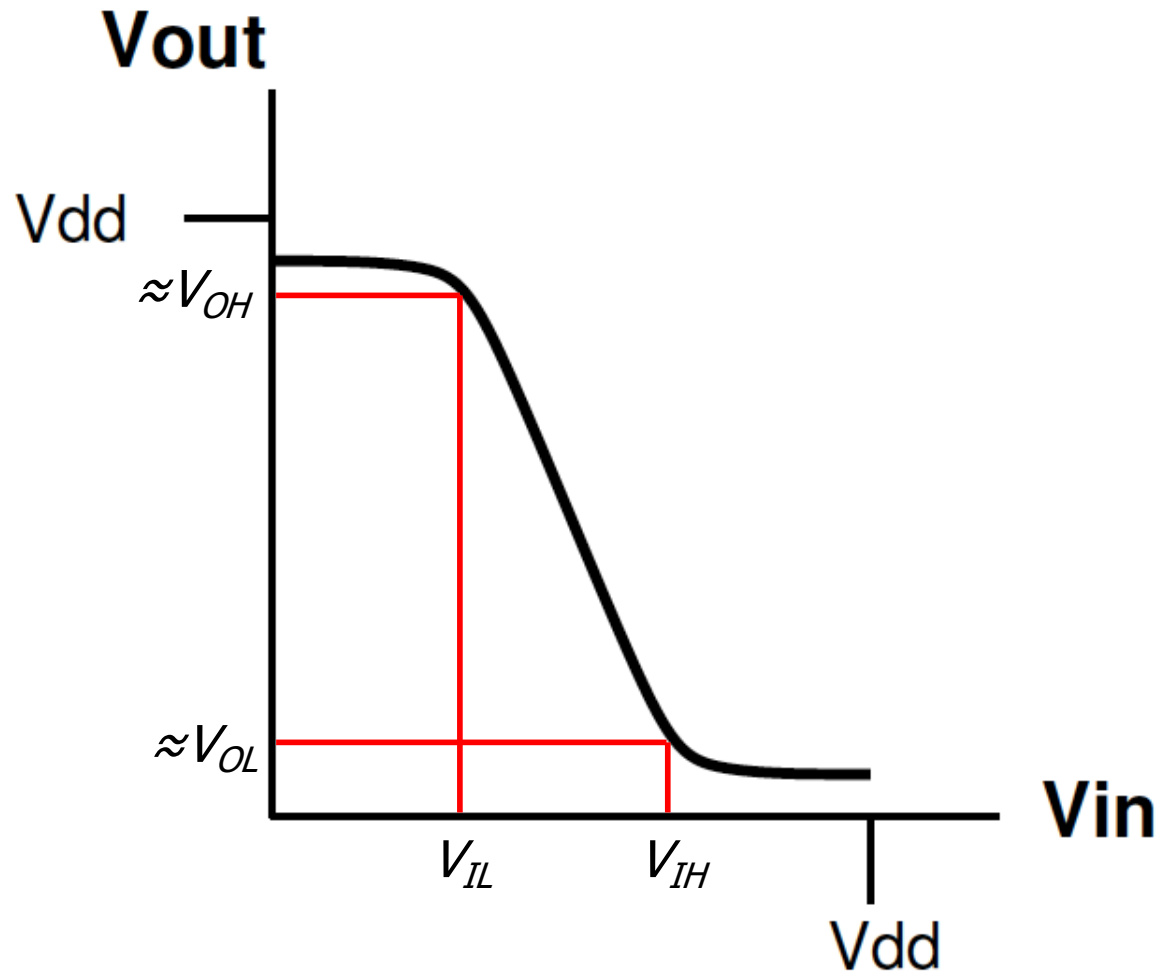
- Use gain (i.e. slope) to define noise margins





# Regenerating Transfer Function

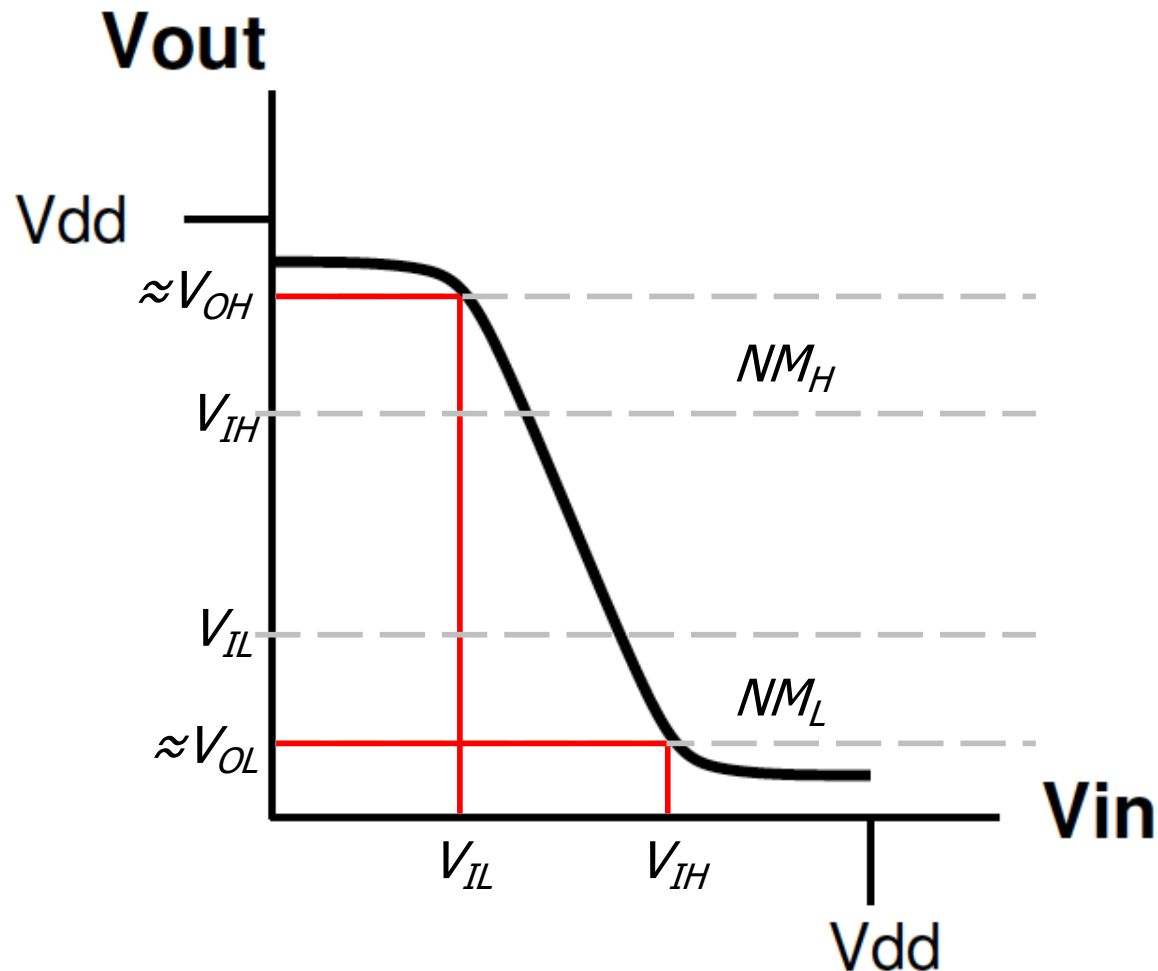
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# Regenerating Transfer Function

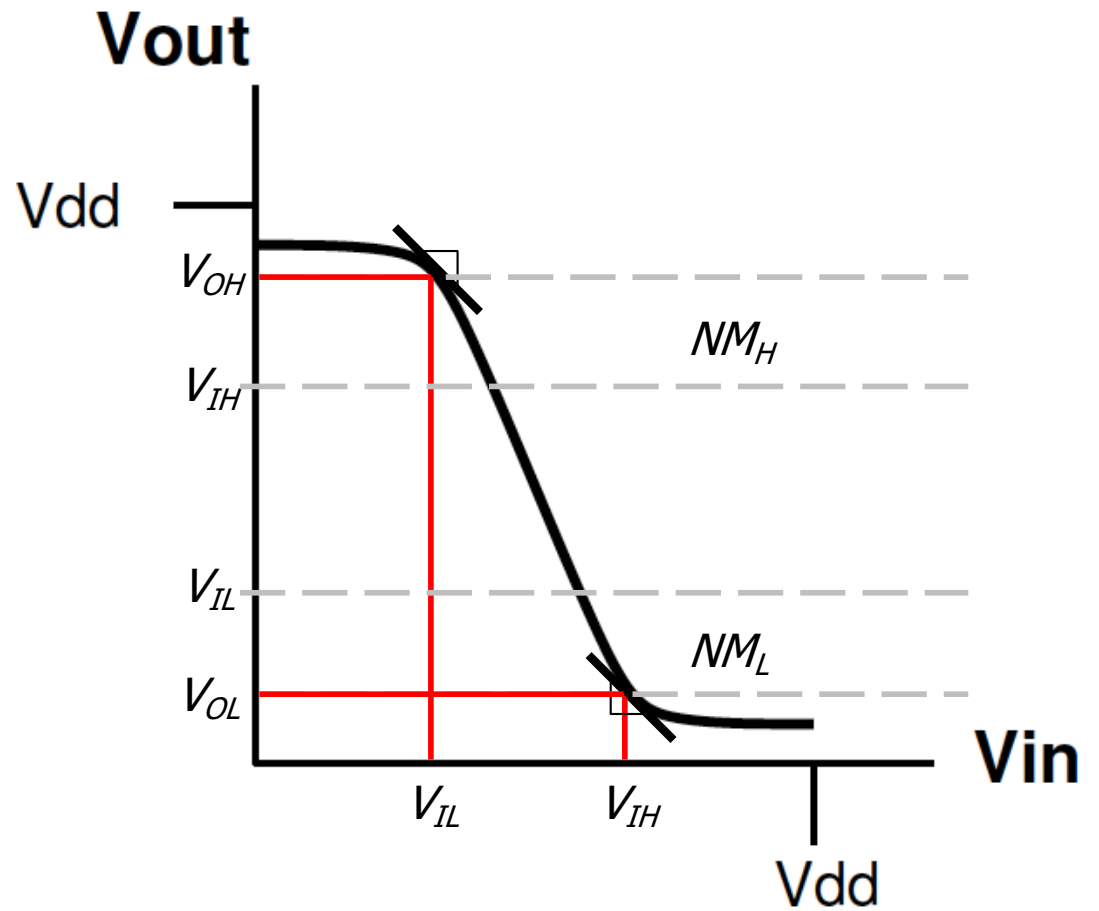
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# Decomposing

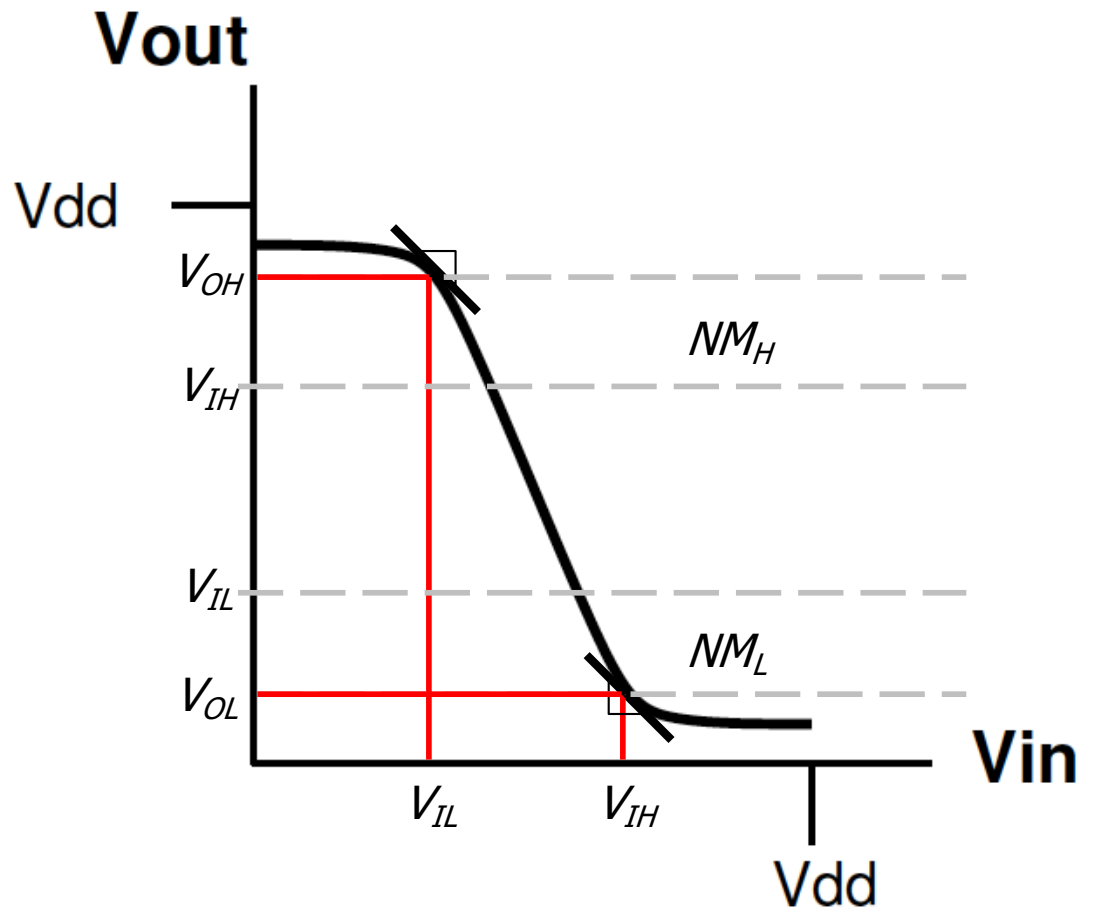
- What is gain?
  - $|\Delta V_{out}/\Delta V_{in}|$





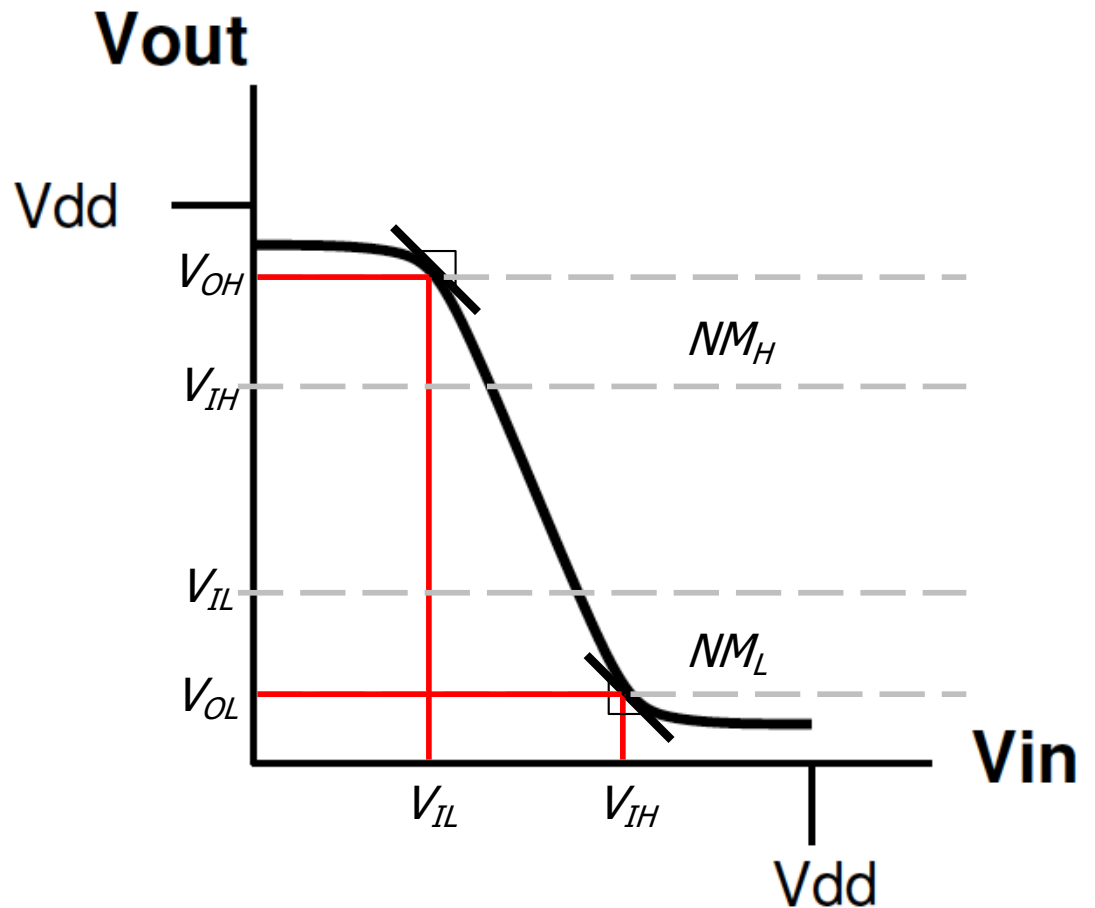
# Decomposing

- What is gain?
  - $|\Delta V_{out}/\Delta V_{in}|$
- Where is there high gain?
  - $|\Delta V_{out}/\Delta V_{in}| > 1$



# Decomposing

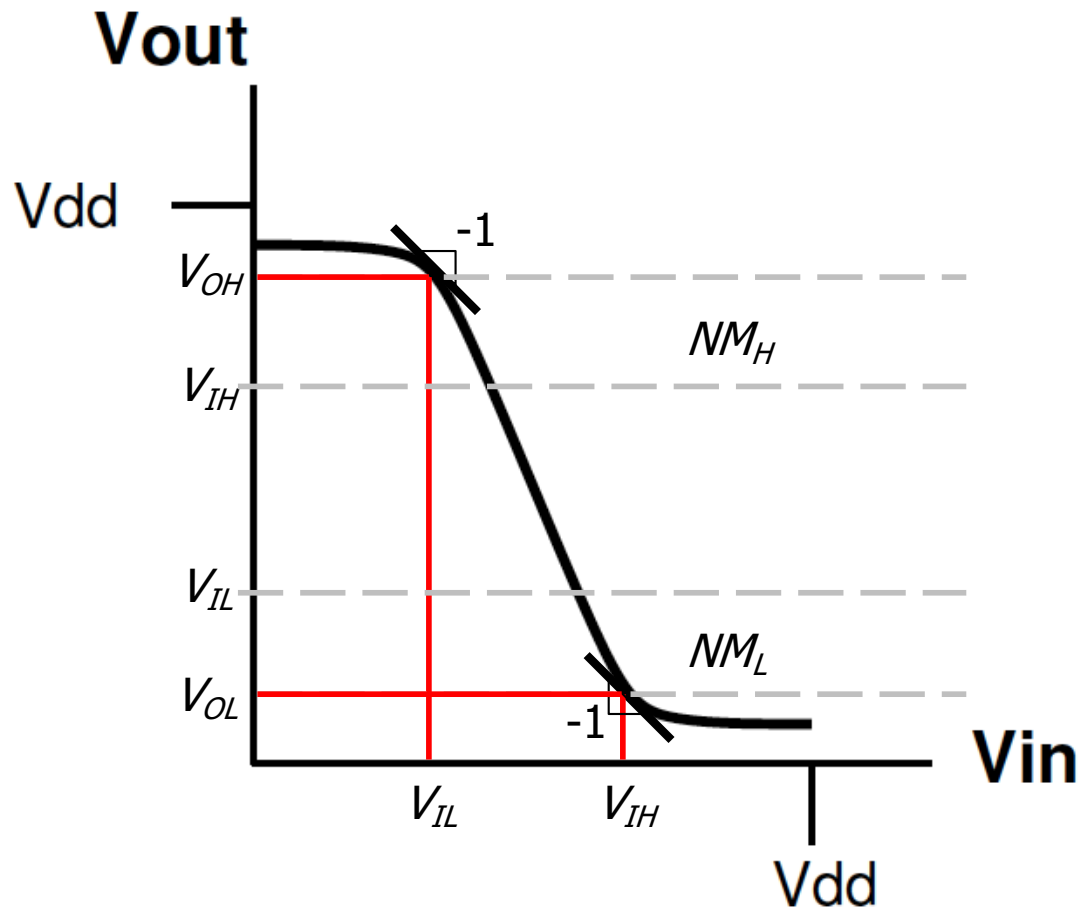
- What is gain?
  - $|\Delta V_{out}/\Delta V_{in}|$
- Where is there high gain?
  - $|\Delta V_{out}/\Delta V_{in}| > 1$
- Where is there low gain?
  - $|\Delta V_{out}/\Delta V_{in}| < 1$



# Decomposing

- What is gain?
  - $|\Delta V_{out}/\Delta V_{in}|$
- Where is there high gain?
  - $|\Delta V_{out}/\Delta V_{in}| > 1$
- Where is there low gain?
  - $|\Delta V_{out}/\Delta V_{in}| < 1$
- Dividing point?

$$\left. \frac{\delta V_{out}}{\delta V_{in}} \right|_{V_{IL}} = \left. \frac{\delta V_{out}}{\delta V_{in}} \right|_{V_{IH}} = -1$$

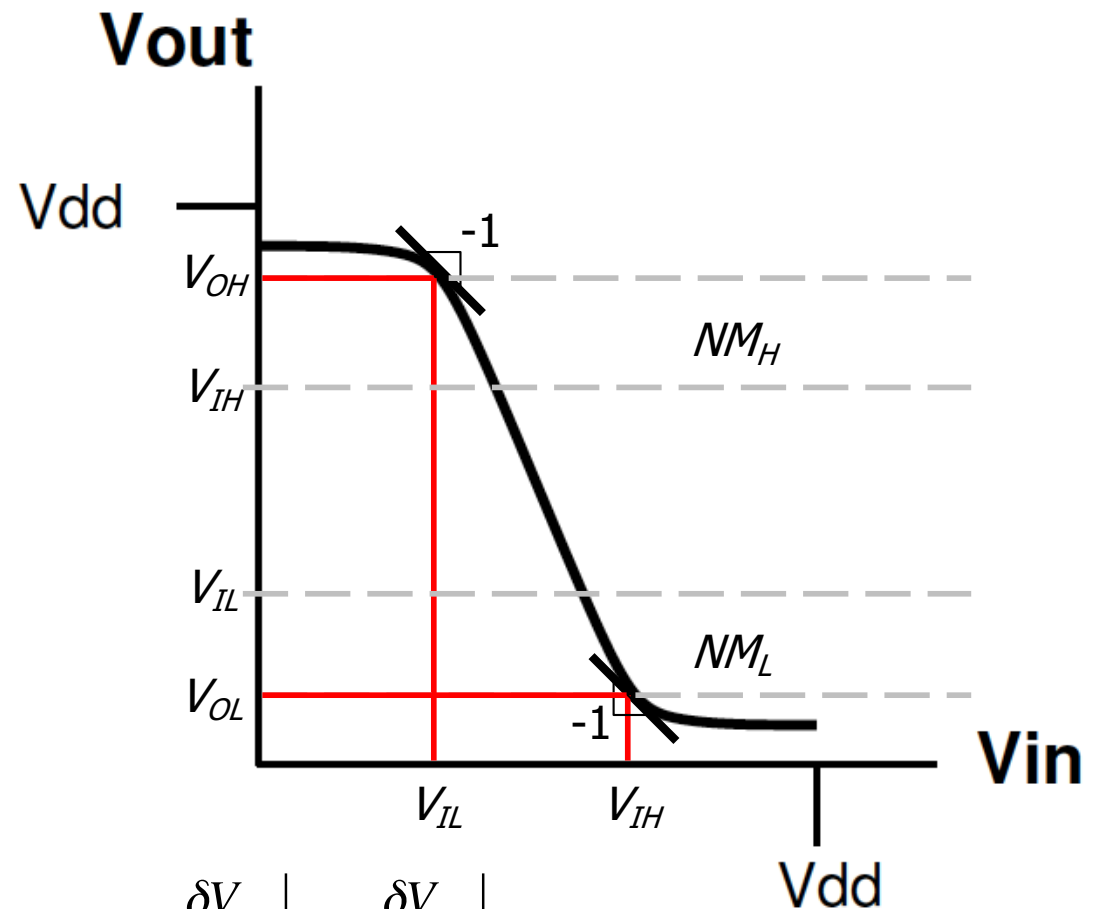


$$V_{OH} = f(V_{IL})$$

$$V_{OL} = f(V_{IH})$$

# Decomposing

- An input closer to rail than  $V_{IL}$ ,  $V_{IH}$  doesn't make much difference on  $V_{out}$ 
  - i.e transfer function is flat for input close to rails
- Defining  $V_{IL}$  lower (or  $V_{IH}$  higher) would reduce NMs and increase our undefined region



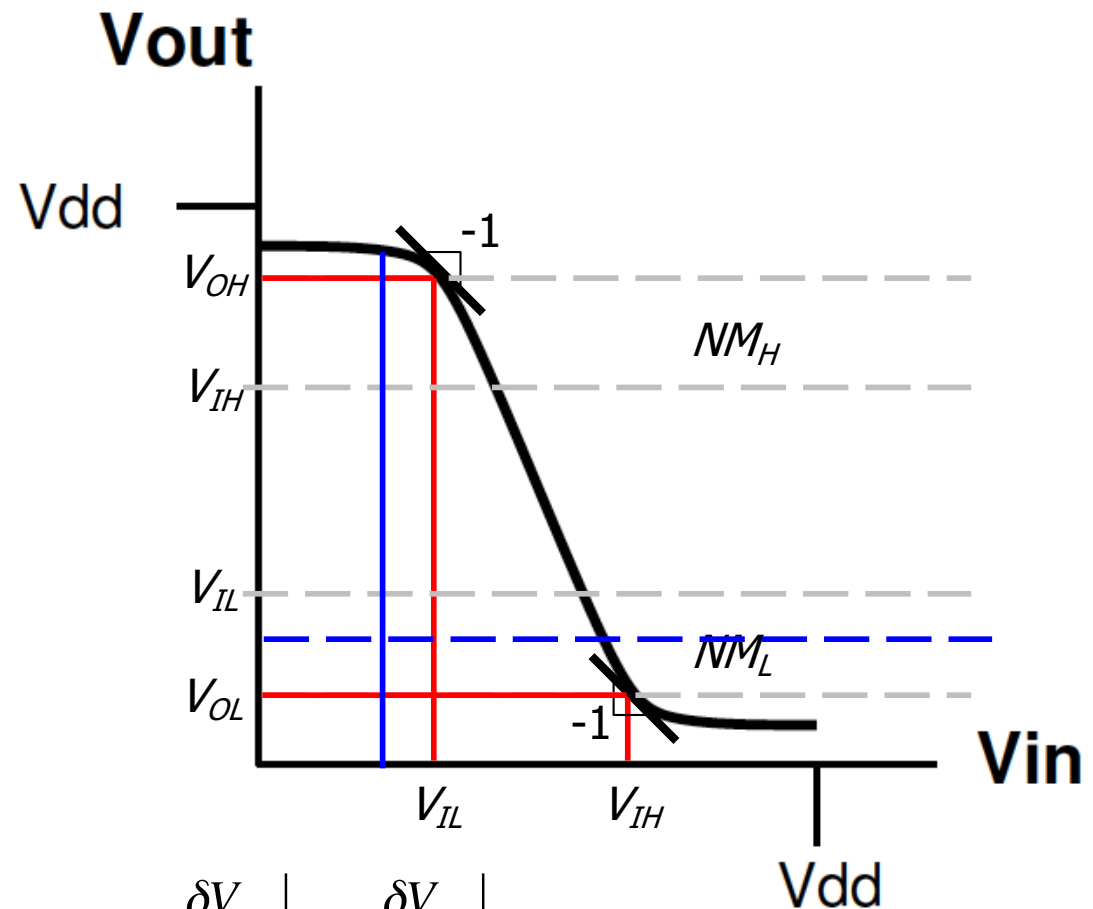
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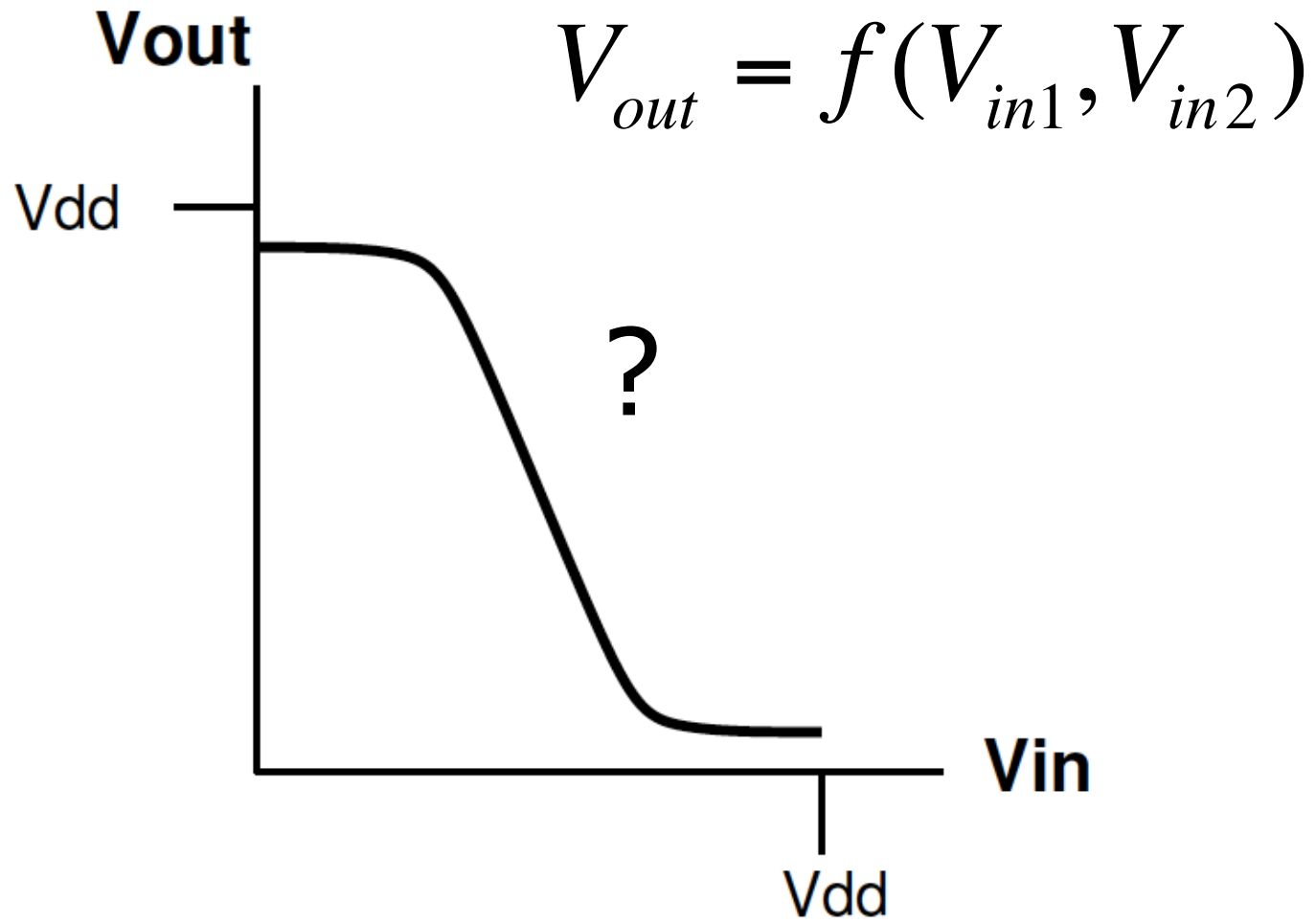


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$$V_{OH} = f(V_{IL})$$

$$V_{OL} = f(V_{IH})$$

# Transfer Function for Multiple Inputs







# Controlling Input

---

- ❑ Consider a nand2 gate
  - If want A to control the output
  - What value should B be?
- ❑ We call B the **non-controlling** input since it does not determine the output



# Controlling Input

---

- ❑ Consider a nand2 gate
  - If want A to control the output
  - What value should B be?
- ❑ We call B the **non-controlling** input since it does not determine the output

A	B	NAND
0	0	1
0	1	1
1	0	1
1	1	0




# Controlling Input

---

- ❑ Consider a nand2 gate
  - If want A to control the output
  - What value should B be?
- ❑ We call B the **non-controlling** input since it does not determine the output
- ❑ What should the non-controlling input value be for a nor2 gate?

A	B	NOR	A	B	NAND
0	0	1	0	0	1
0	1	0	0	1	1
1	0	0	1	0	1
1	1	0	1	1	0



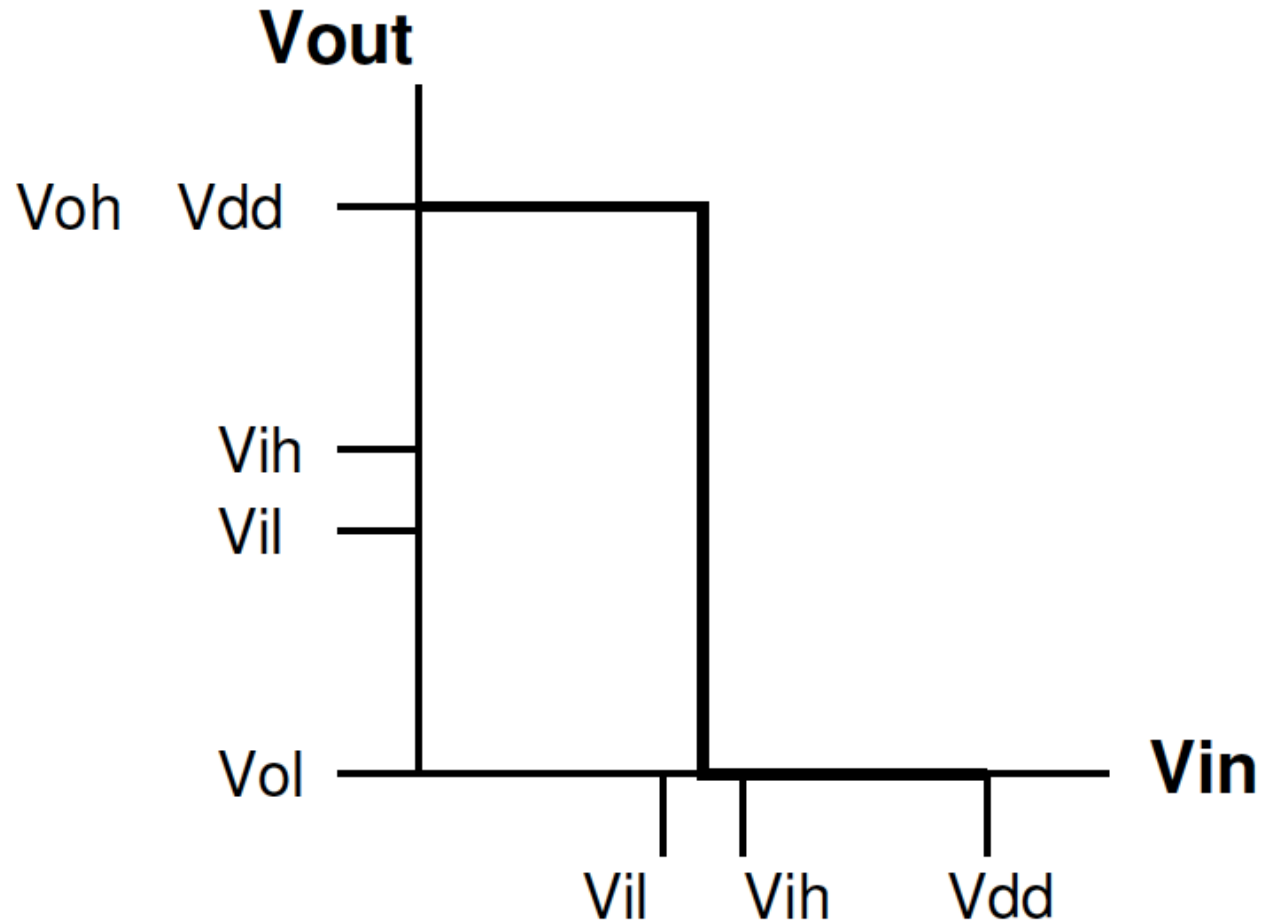
# Controlling Input for Worst Case

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- Consider a nor2/nand2 gate
  - If want A to control the output
  - What value should B be?

# Ideal Transfer Function for Inverter

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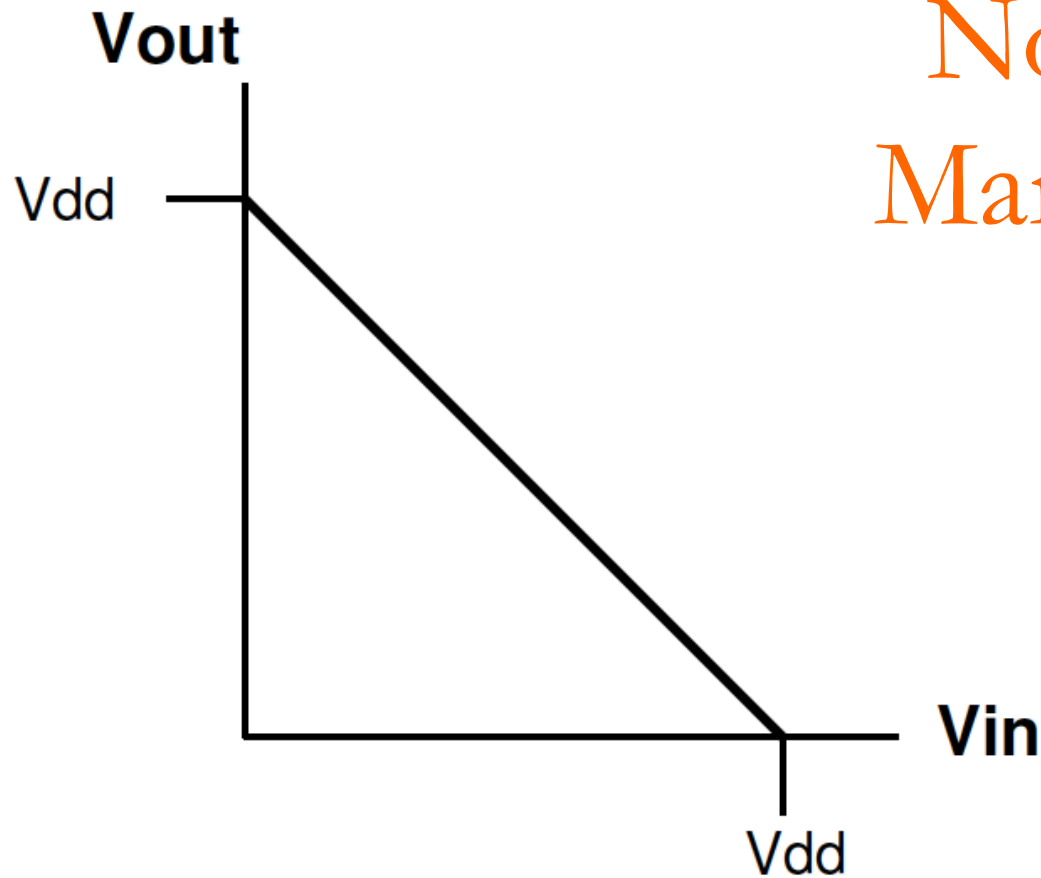




# Linear Transfer Function?

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□  $V_{out} = V_{dd} - V_{in}$



Noise  
Margin?

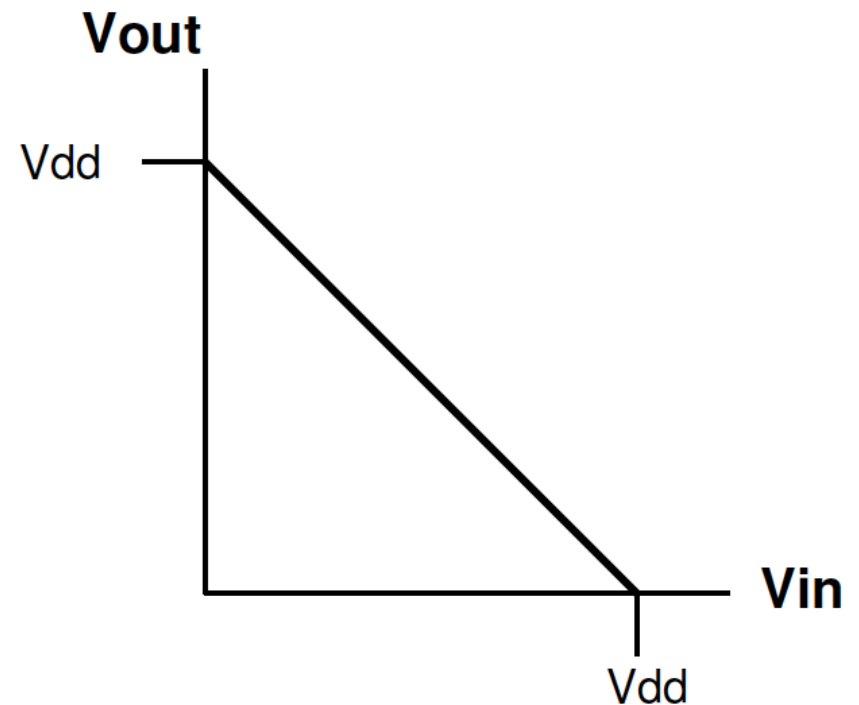
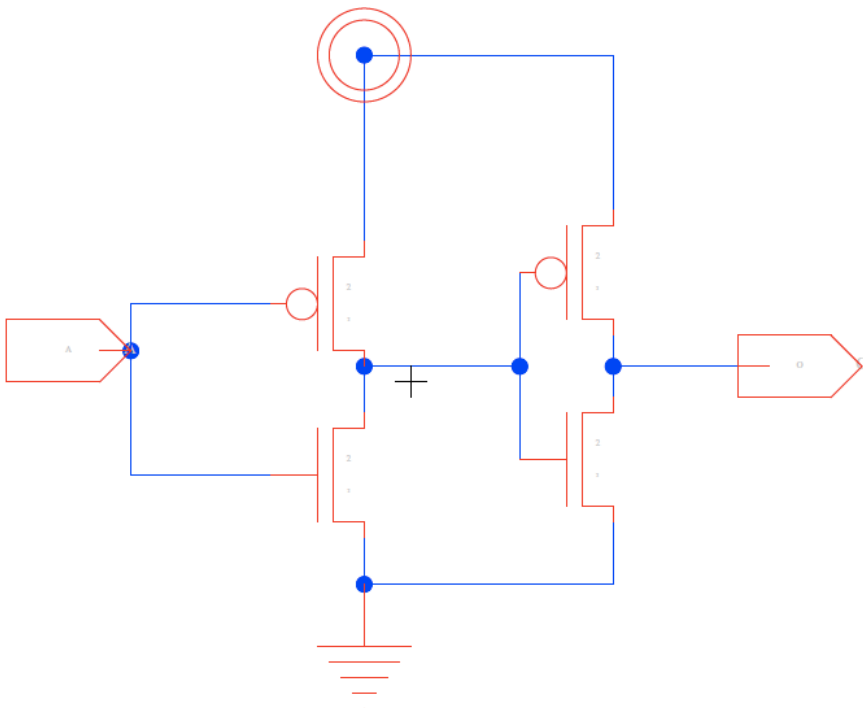
# Linear Transfer Function?

- Consider two in a row (buffer)

- $V_{out1} = V_{dd} - V_{in1}$

- What is transfer function to buffer output  $V_{out2}$ ?

- $V_{out2} = V_{dd} - V_{in2} = V_{dd} - V_{out1} = V_{dd} - (V_{dd} - V_{in1}) = V_{in1}$

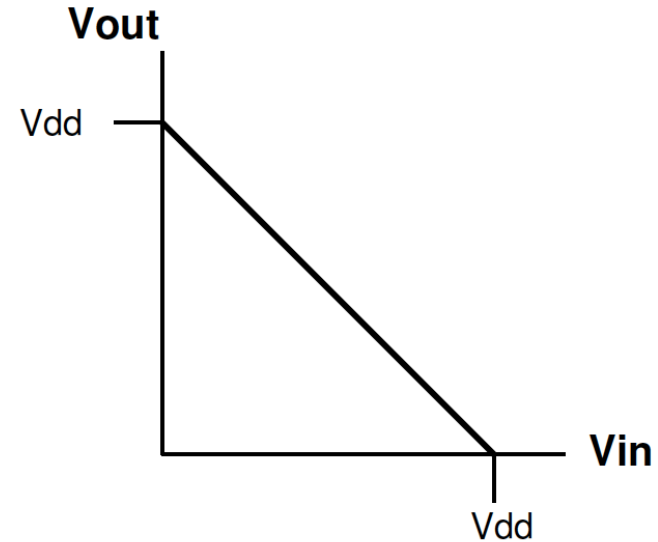


# Linear Transfer Function?

- ❑ For buffer:  $V_{\text{out}2} = V_{\text{in}1}$
- ❑ Consider a chain of buffers
- ❑ What happens if  $V_{\text{in}1}$  drops  $\Delta$  volts between each buffer?

$$A_{i+1} = A_i - \Delta$$

**Conclude:** Linear transfer functions do not provide restoration.







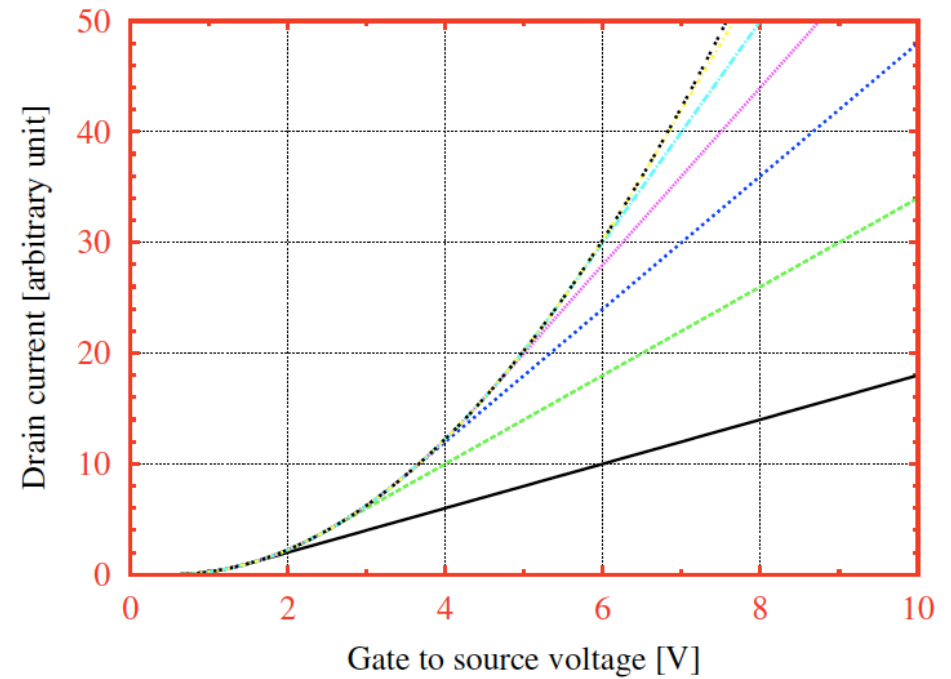
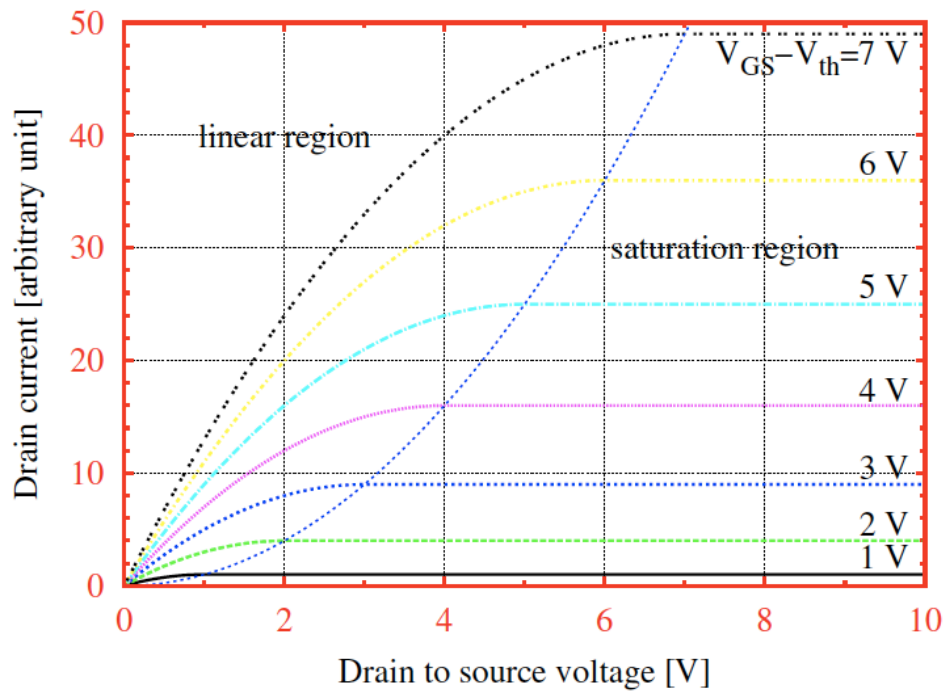
# Non-linearity

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- ❑ **Need** non-linearity in transfer function
- ❑ Could not have built restoring gates with R, L, C circuit
  - R, L, C are all linear elements



# Transistor Non-Linearity





# All Gates

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- If we hope to assemble design from collection of gates,
  - Voltage levels must be consistent and supported across all gates
  - Must adhere to a  $V_{IL}$ ,  $V_{IH}$ ,  $V_{OL}$ ,  $V_{OH}$  that is valid across entire gate set of digital circuit

$$V_{ol} = \mathbf{MAX}_{g \in G} (g.V_{ol})$$

$$V_{il} = \mathbf{MIN}_{g \in G} (g.V_{il})$$

$$V_{oh} = \mathbf{MIN}_{g \in G} (g.V_{oh})$$

$$V_{ih} = \mathbf{MAX}_{g \in G} (g.V_{ih})$$

# Semiconductor Physics

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# Conduction

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- ❑ Metal – conducts
- ❑ Insulator – does not conduct
- ❑ Semiconductor – can act as either

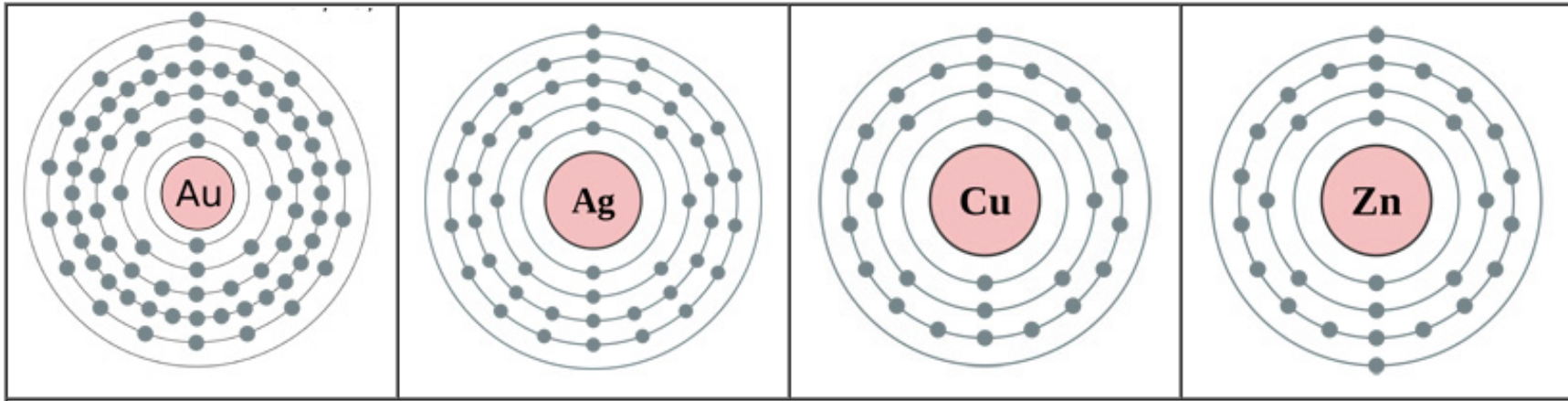
# Why does metal conduct?

Gold

Silver

Copper

Zinc

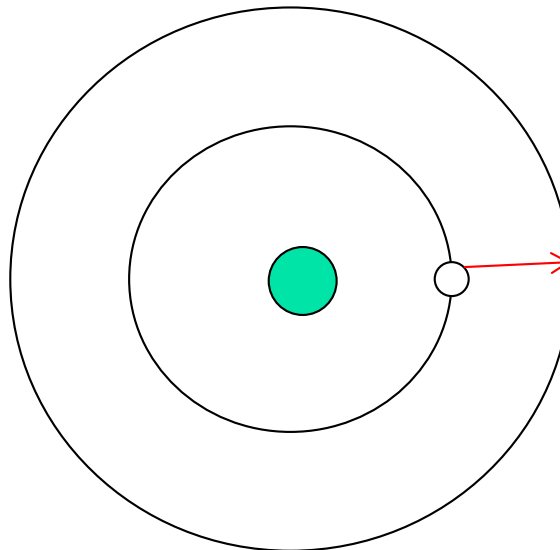




# Atomic States

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- ❑ Quantized Energy Levels (bands)
  - Valence and Conduction Bands
- ❑ Must have enough energy to change level (state)

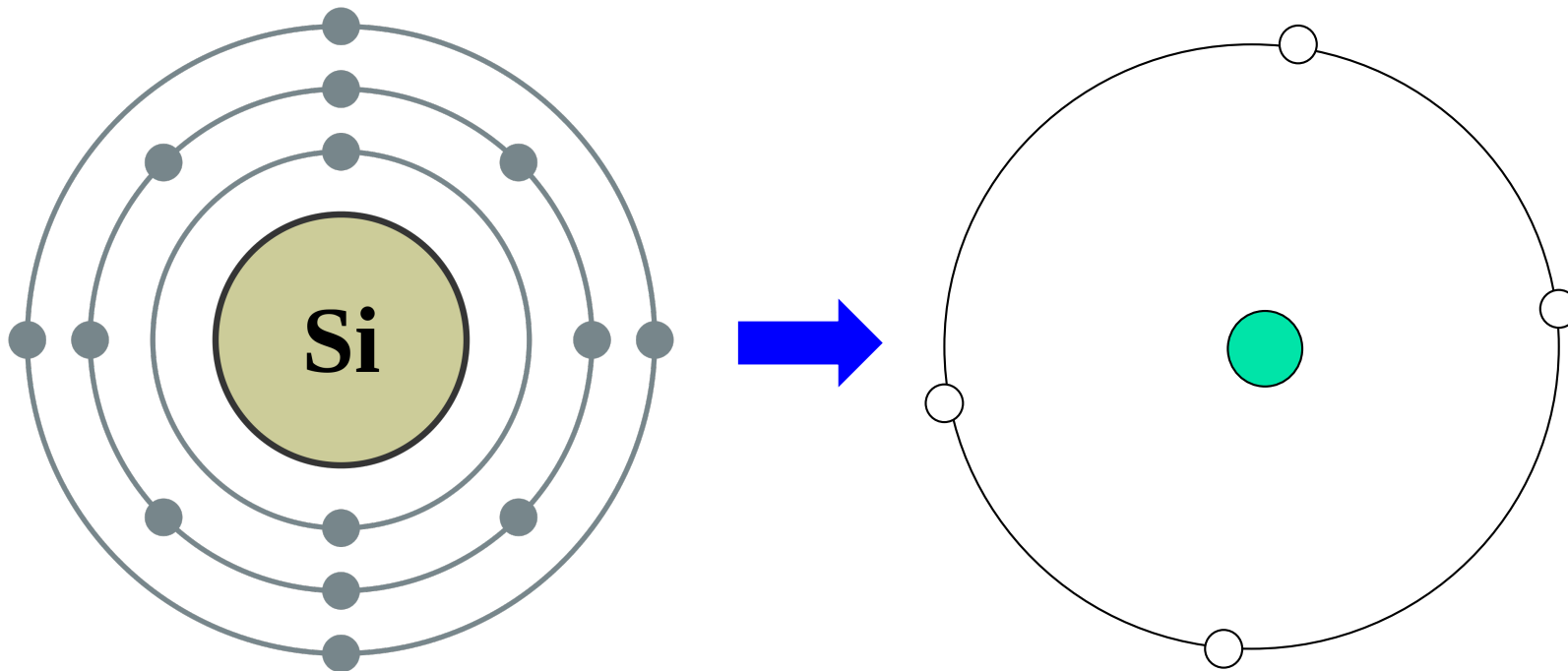




# Silicon Atom

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- 4 valence electrons
  - Inner shells filled
  - Only outer shells contribute to chemical interactions

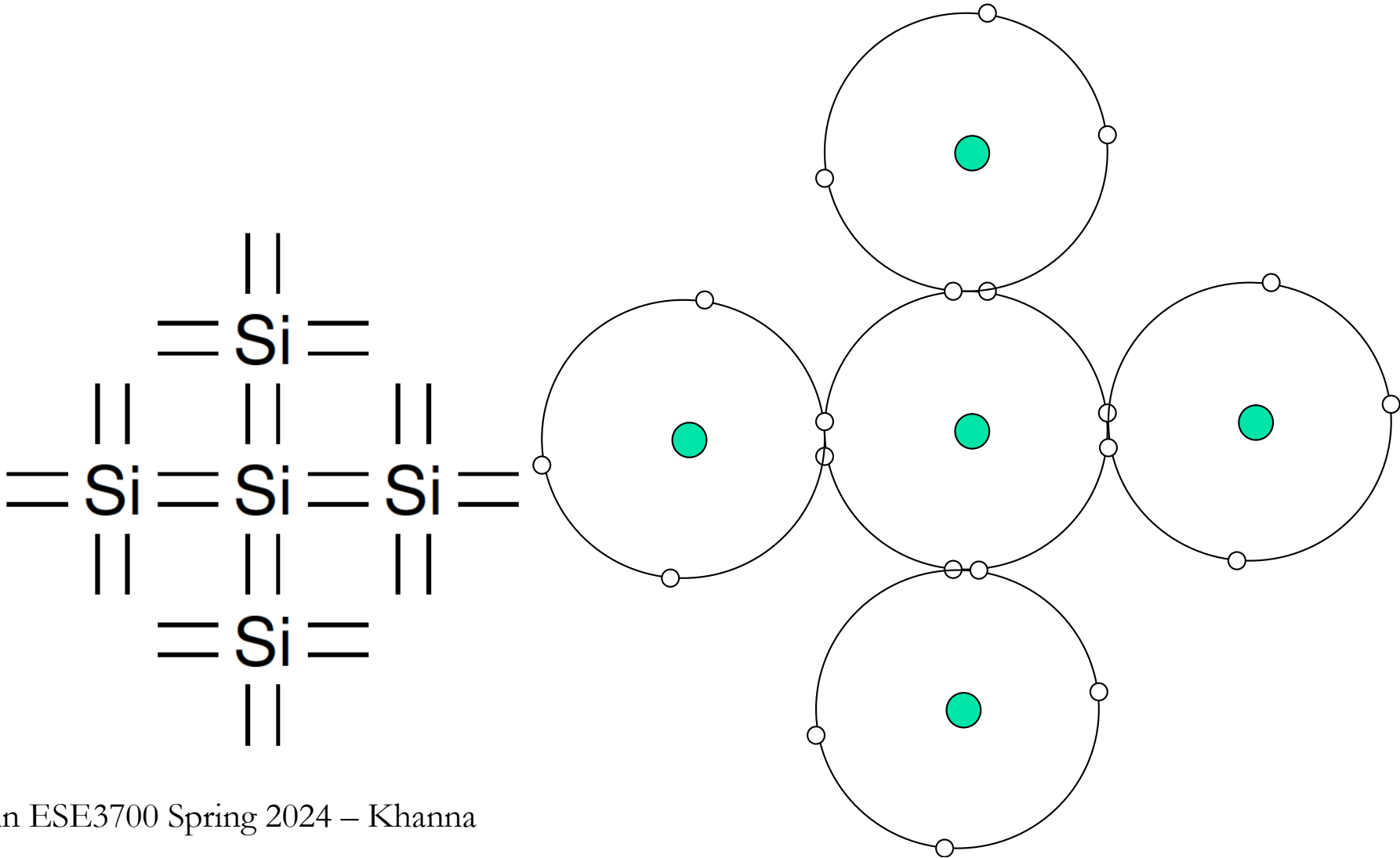






# Silicon-Silicon Bonding

- Can form covalent bonds with 4 other silicon atoms





# Silicon Ingot

1 impurity atom  
per 10 billion  
silicon atoms

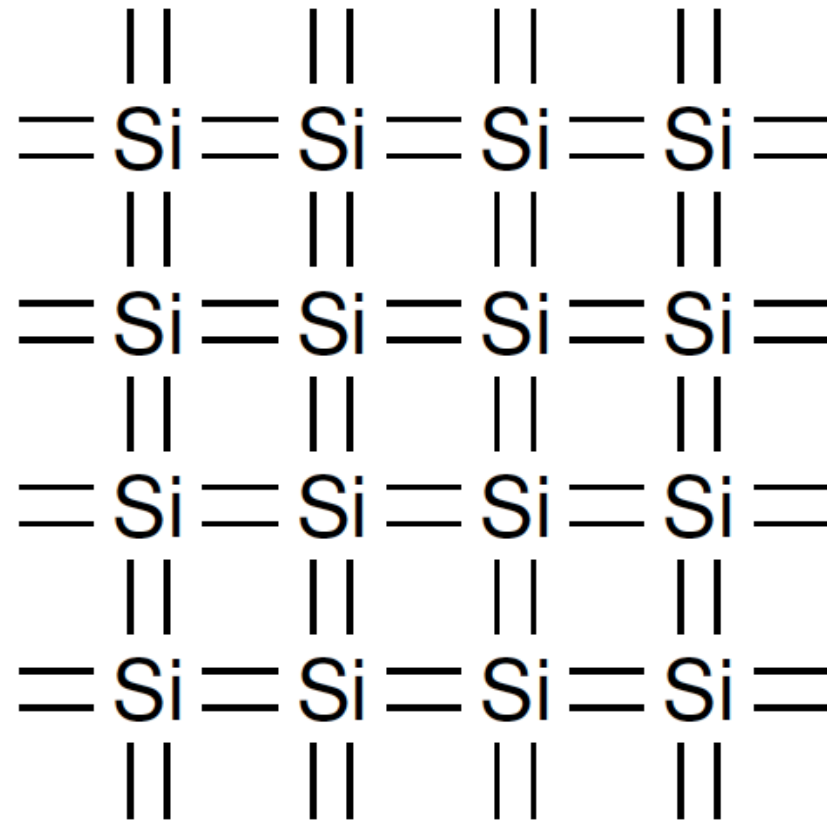




# Silicon Lattice

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- Cartoon two-dimensional view

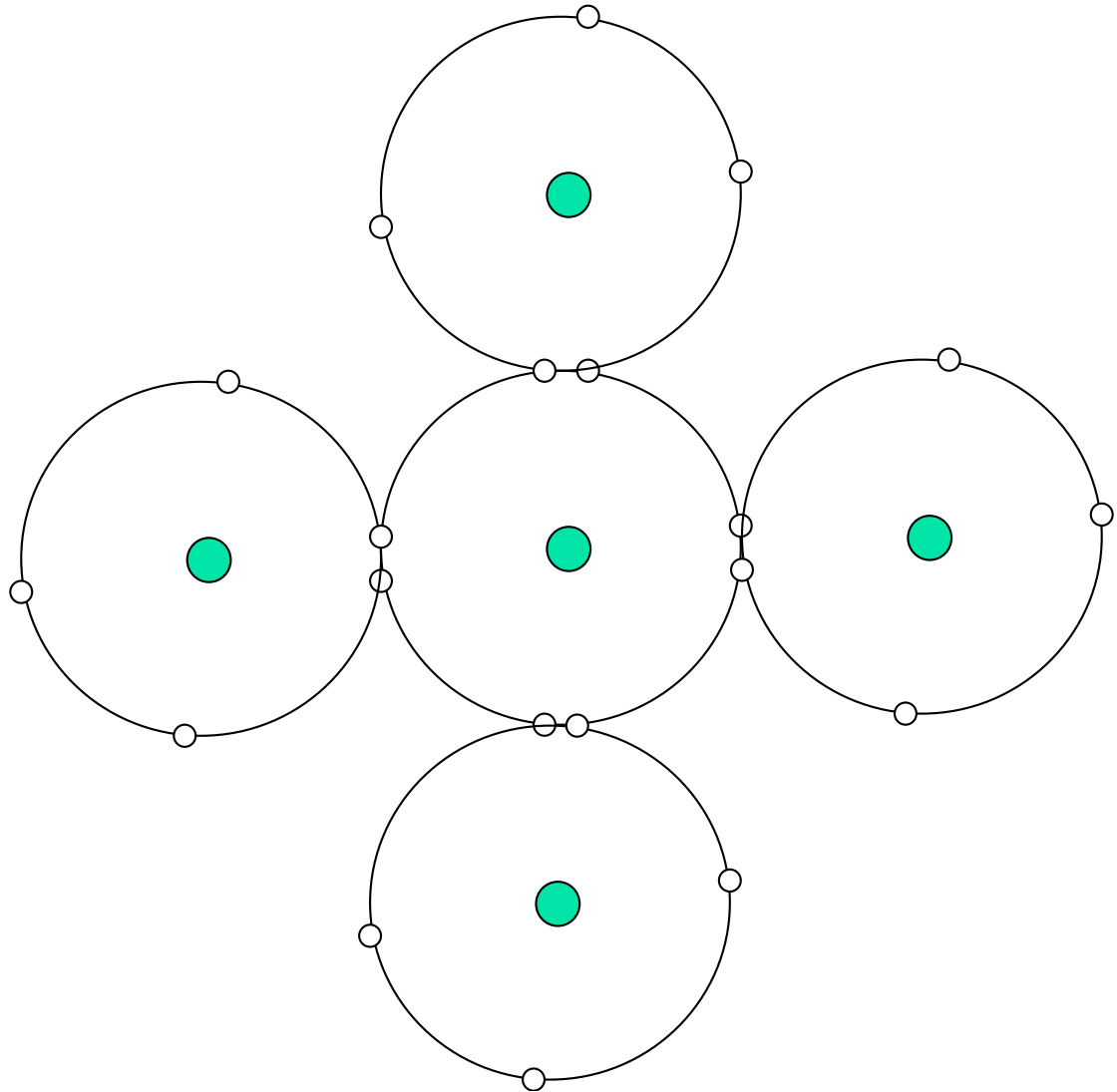




# Energy?

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- What does this say about energy to move electron?





# Energy State View

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Conduction Band— all states empty

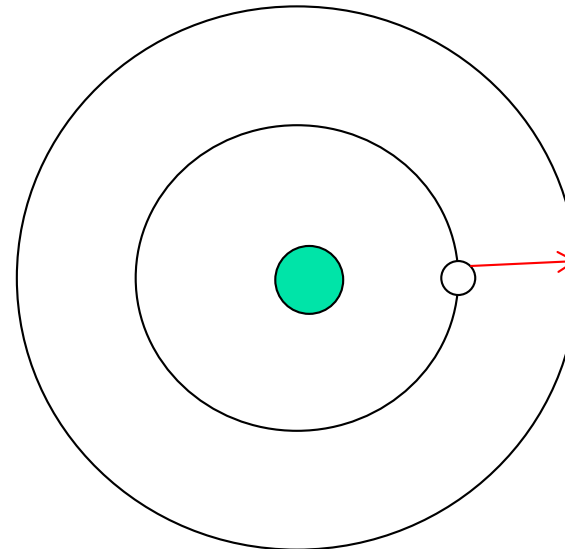
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Energy

Band Gap

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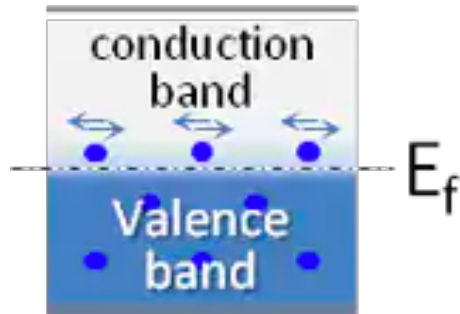
Valance Band – all states filled



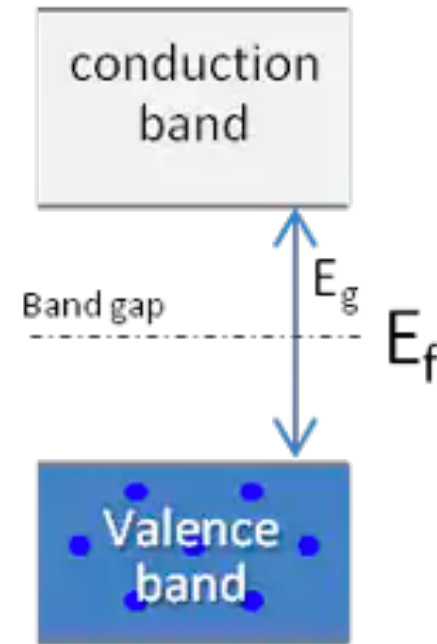


# Band Gap and Conduction

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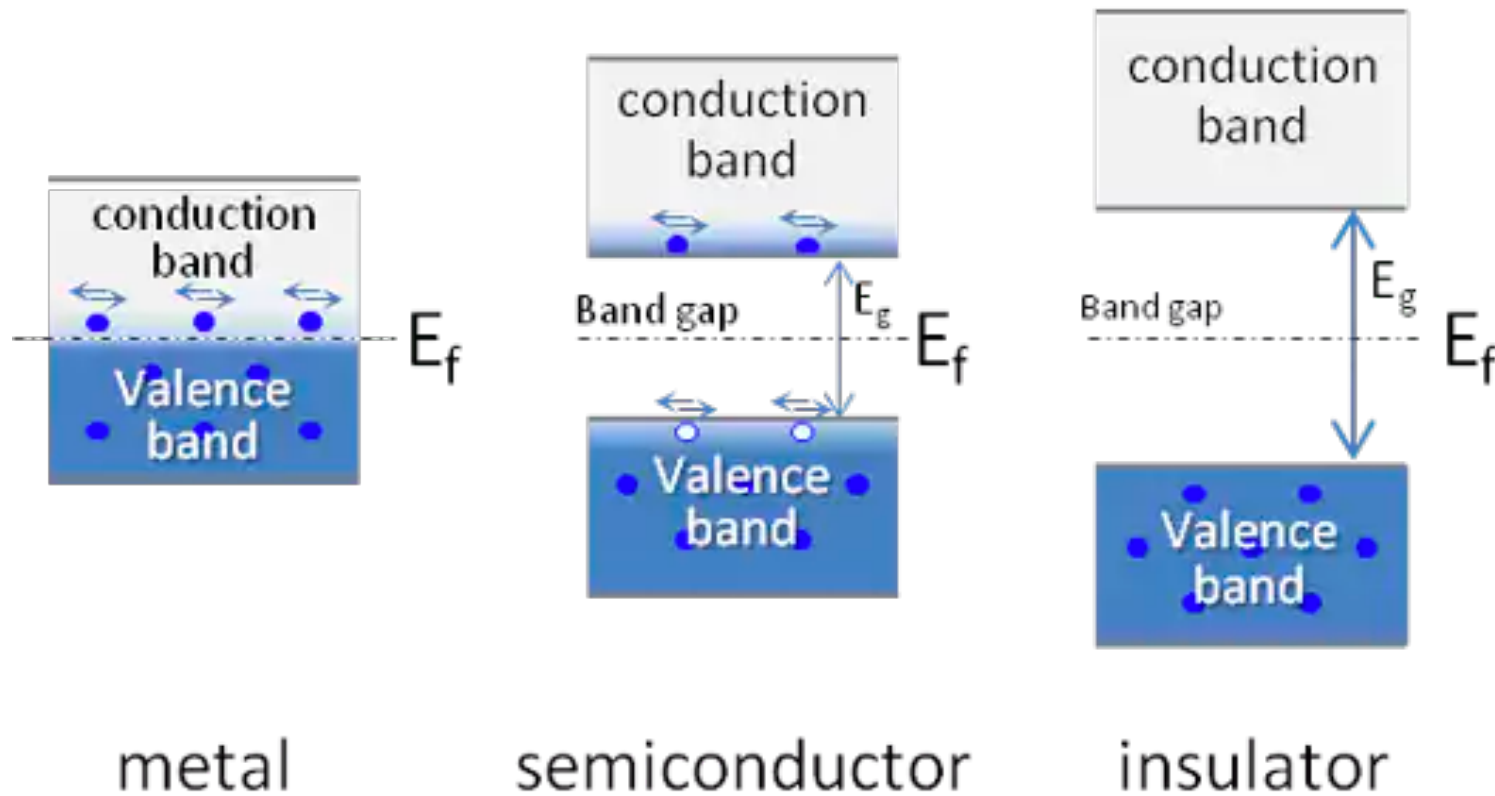


metal



insulator

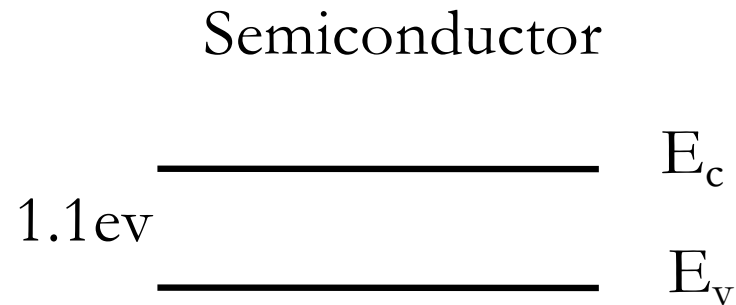
# Band Gap and Conduction





# Band Gap and Conduction

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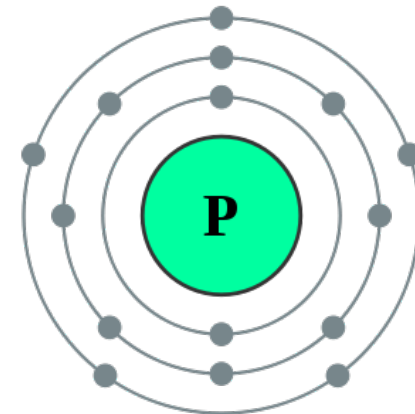
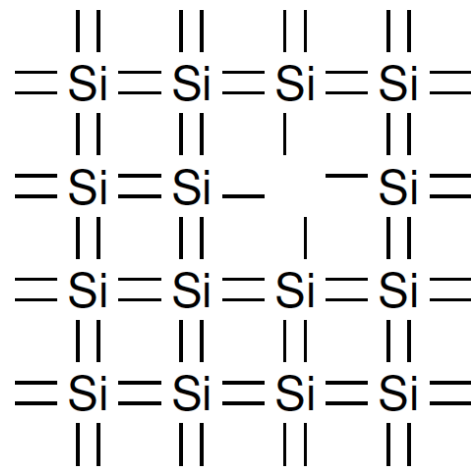
$$1\text{eV} = 160 \text{ zeptojoules } (10^{-21} \text{ J})$$





# Doping

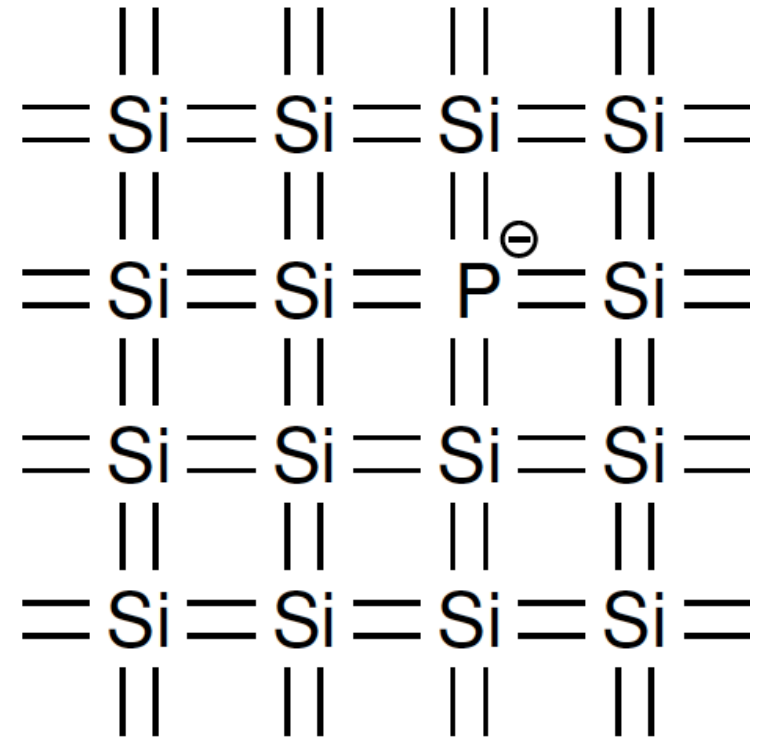
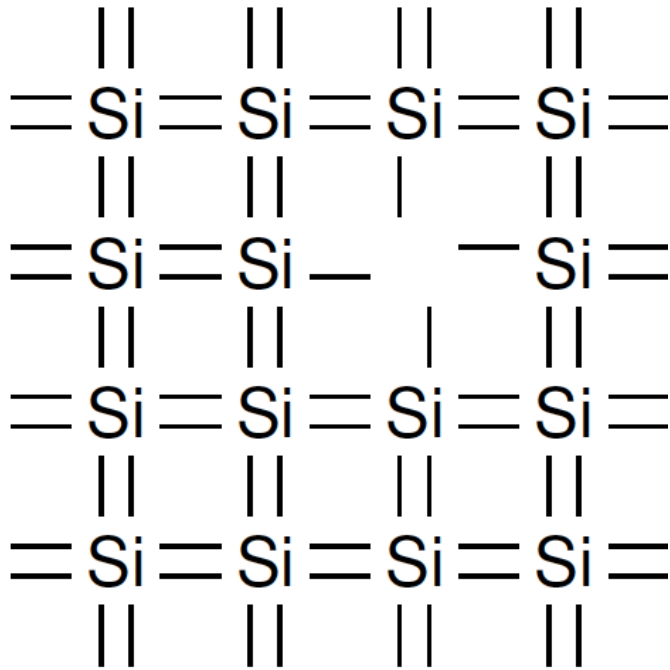
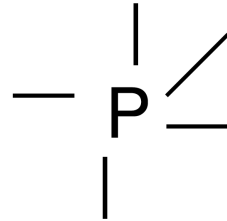
- ❑ Add impurities to Silicon Lattice
  - Replace a Si atom at a lattice site with another
- ❑ Add a Group 15 element
  - *E.g.* P (Phosphorus)
  - How many valence electrons?





# Doping with P

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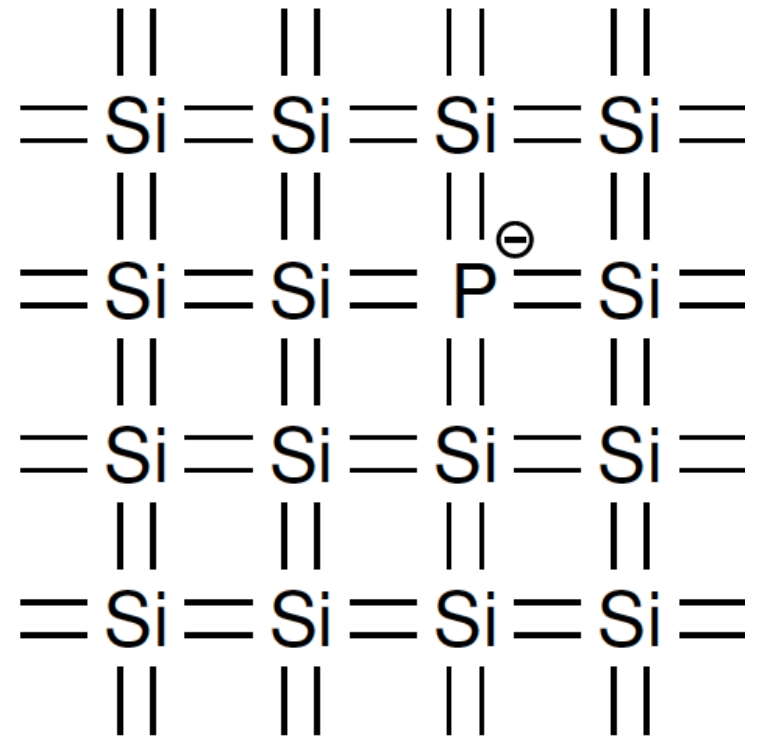




# Doping with P

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- ❑ End up with extra electrons
  - Donor electrons
- ❑ Not tightly bound to atom
  - Low energy to displace
  - Easy for these electrons to move

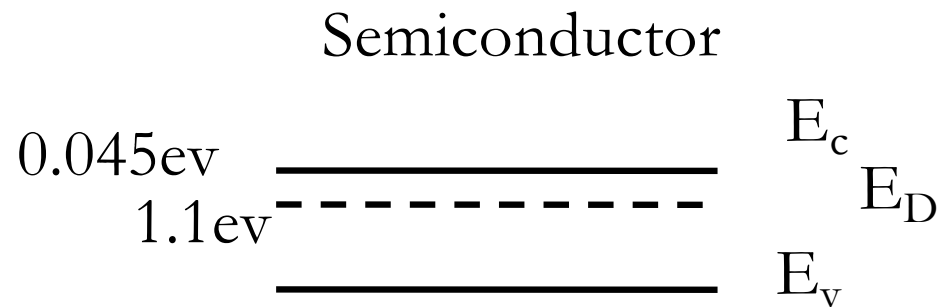




# Doped Band Gaps

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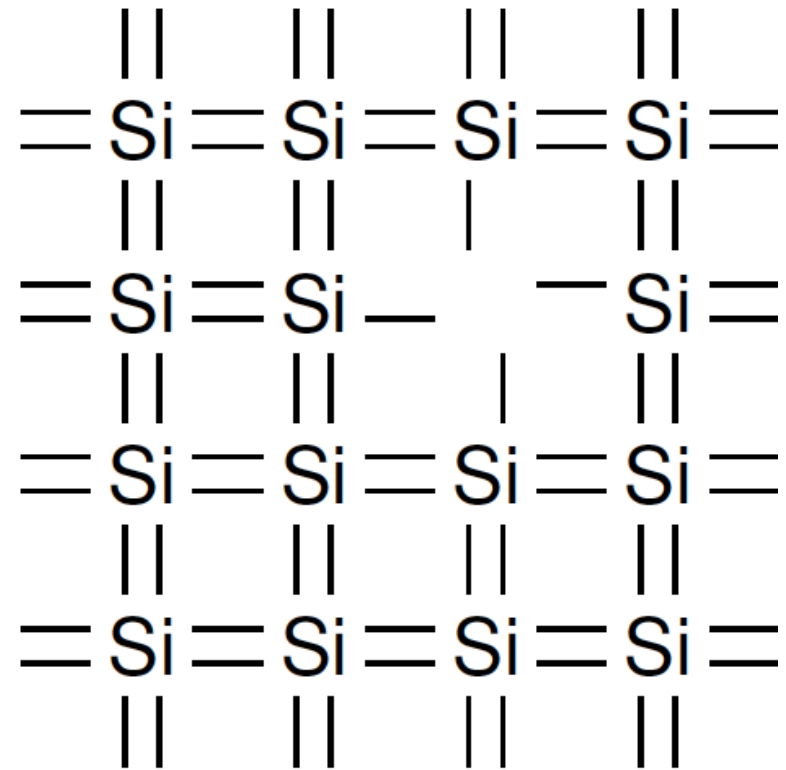
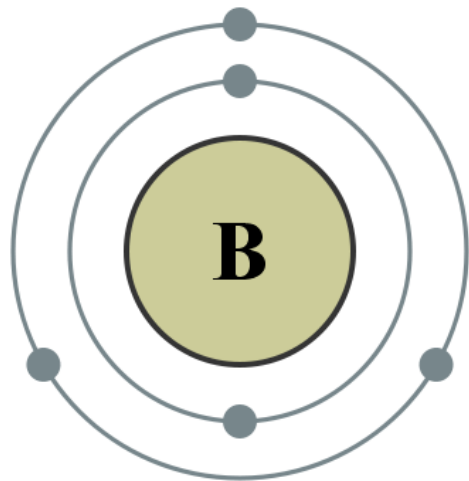
- Addition of donor electrons makes more metallic
  - Easier to conduct





# Doping

- ❑ What happens if we replace Si atoms with group 13 atom instead?
  - E.g. B (Boron)
  - Valance band electrons?

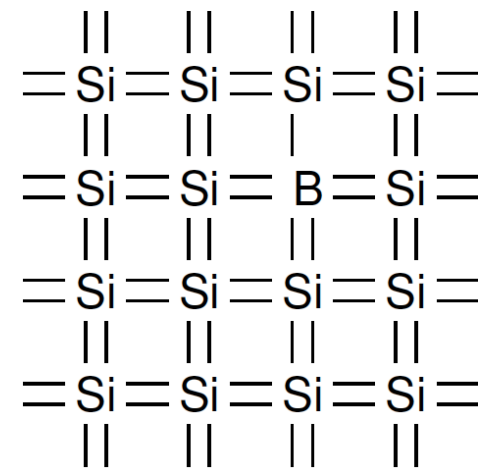




# Doping with B

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- ❑ End up with electron vacancies -- Holes
  - Acceptor electron sites
- ❑ Easy for electrons to shift into these sites
  - Low energy to displace
  - Easy for the electrons to move
    - Movement of an electron best viewed as movement of hole

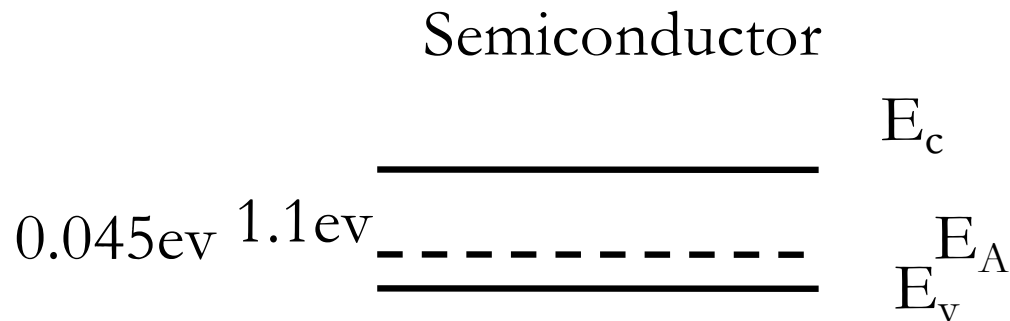




# Doped Band Gaps

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- Addition of acceptor sites makes more metallic
  - Easier to conduct





# MOSFETs

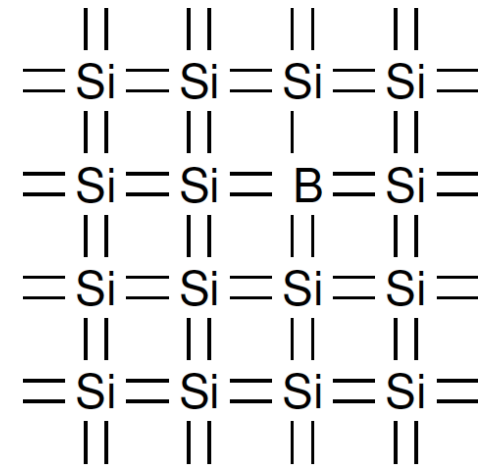
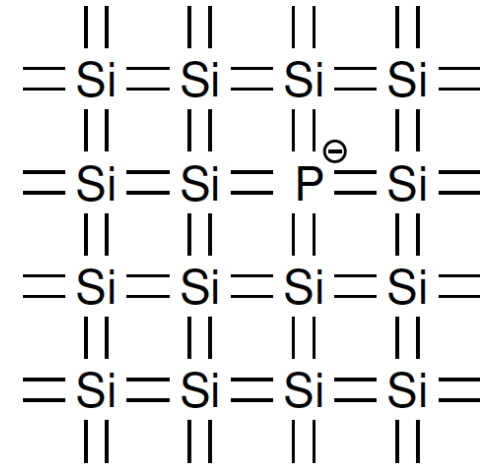
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## □ Donor doping

- Excess electrons
- Negative or N-type material
- NFET

## □ Acceptor doping

- Excess holes
- Positive or P-type material
- PFET





# PN Junction

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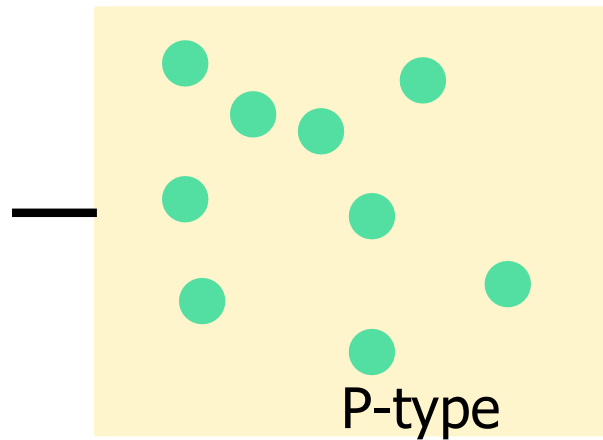




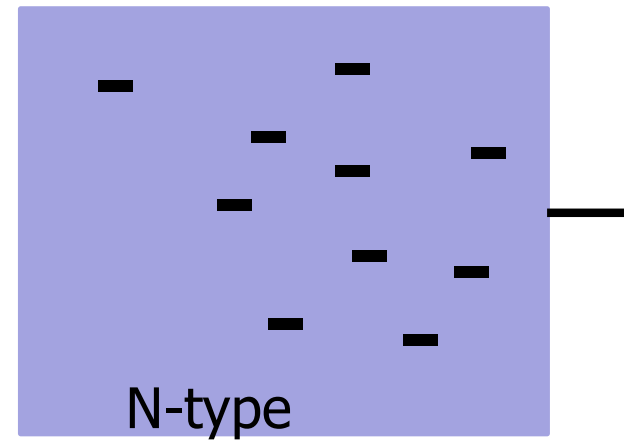
# Doped Silicon

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● = hole  
- = electron



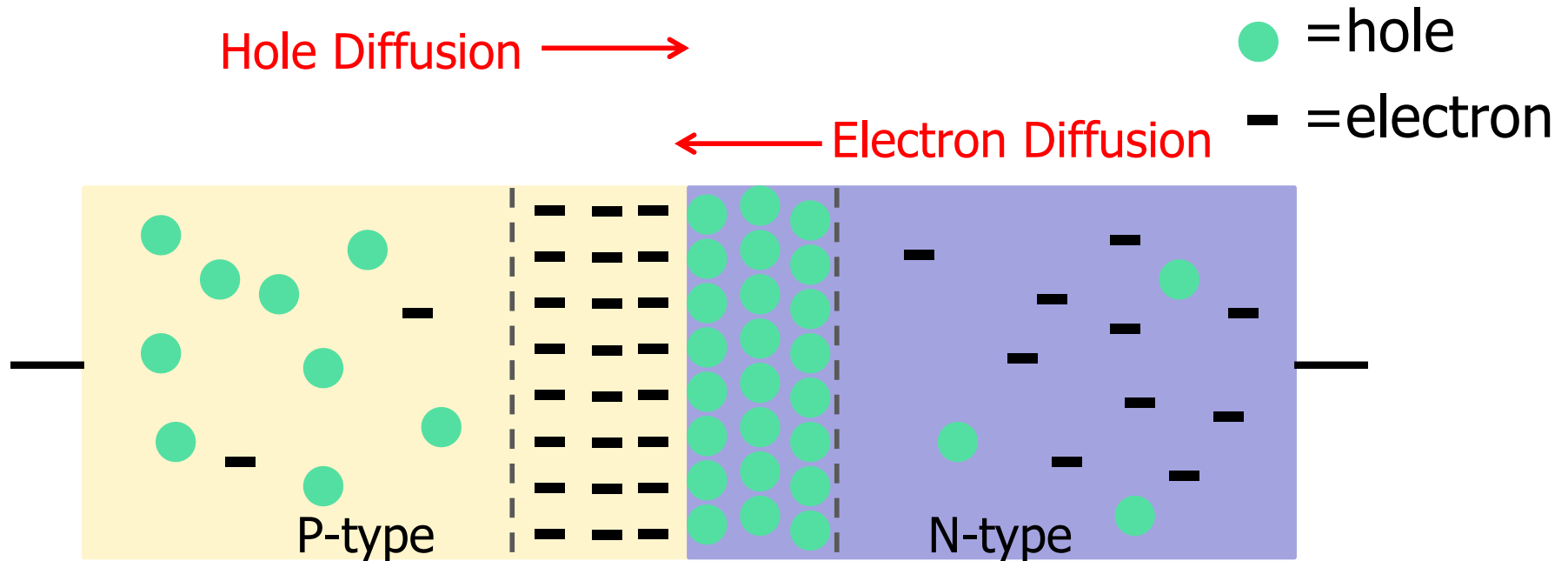
Excess holes



Excess electrons



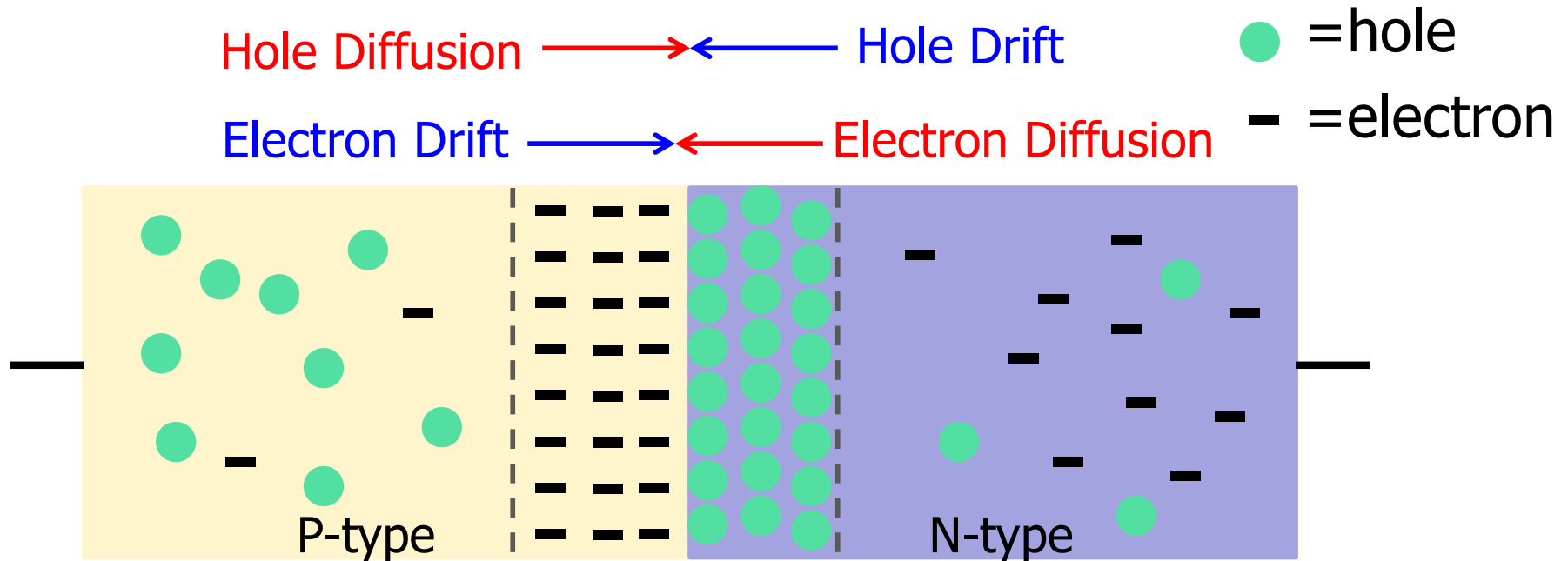
# PN Junction



- PN junction causes a depletion region to form
  - Electrons diffuse from N-type to P-type
  - Holes diffuse from P-type to N-type
    - Diffusion current caused by diffusion of carriers



# PN Junction



- PN junction causes a depletion region to form
  - Electrons diffuse from N-type to P-type
  - Holes diffuse from P-type to N-type
    - Diffusion current caused by diffusion of carriers
- Equilibrium achieved when  $V_{bi}$ , built-in potential, is formed across the depletion region
  - Drift current caused by E-field due to  $V_{bi}$  to counteract diffusion current

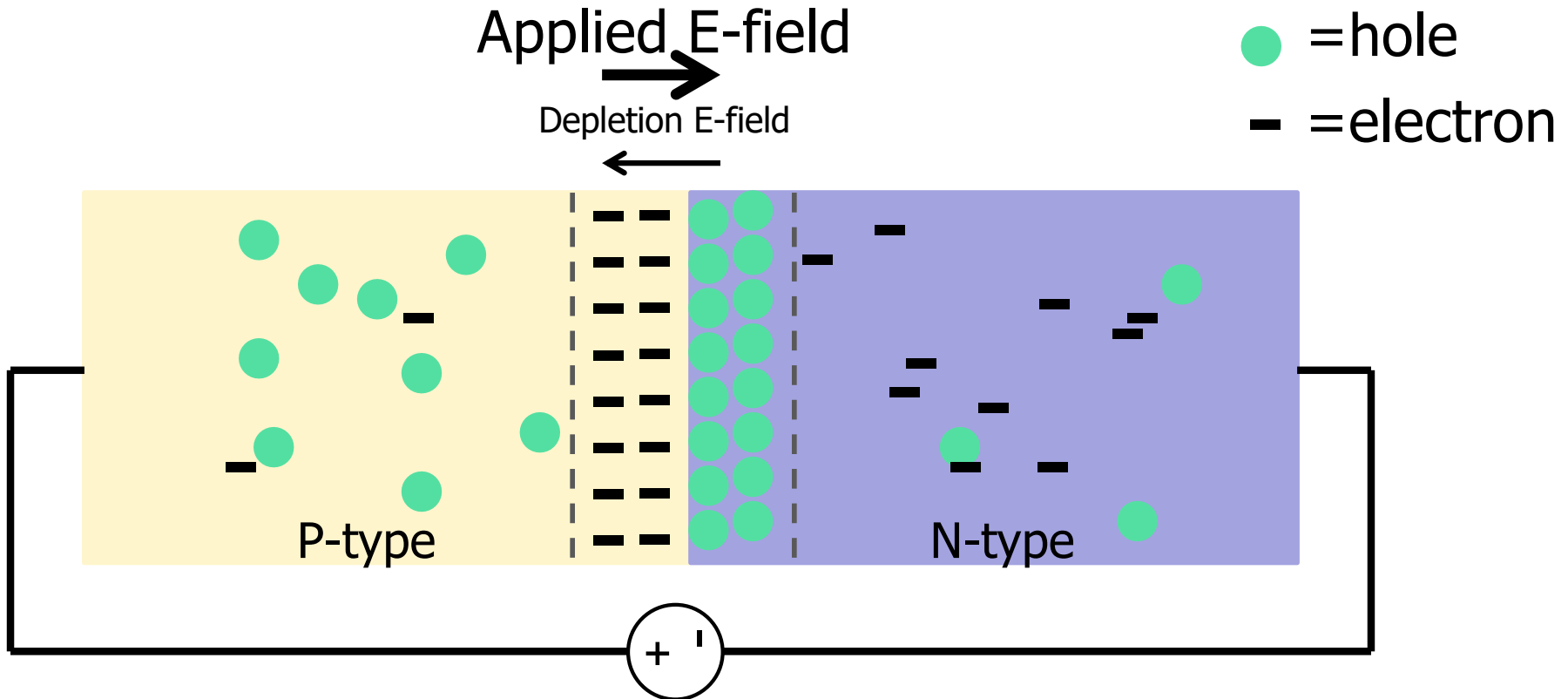


# Drift/Diffusion Currents

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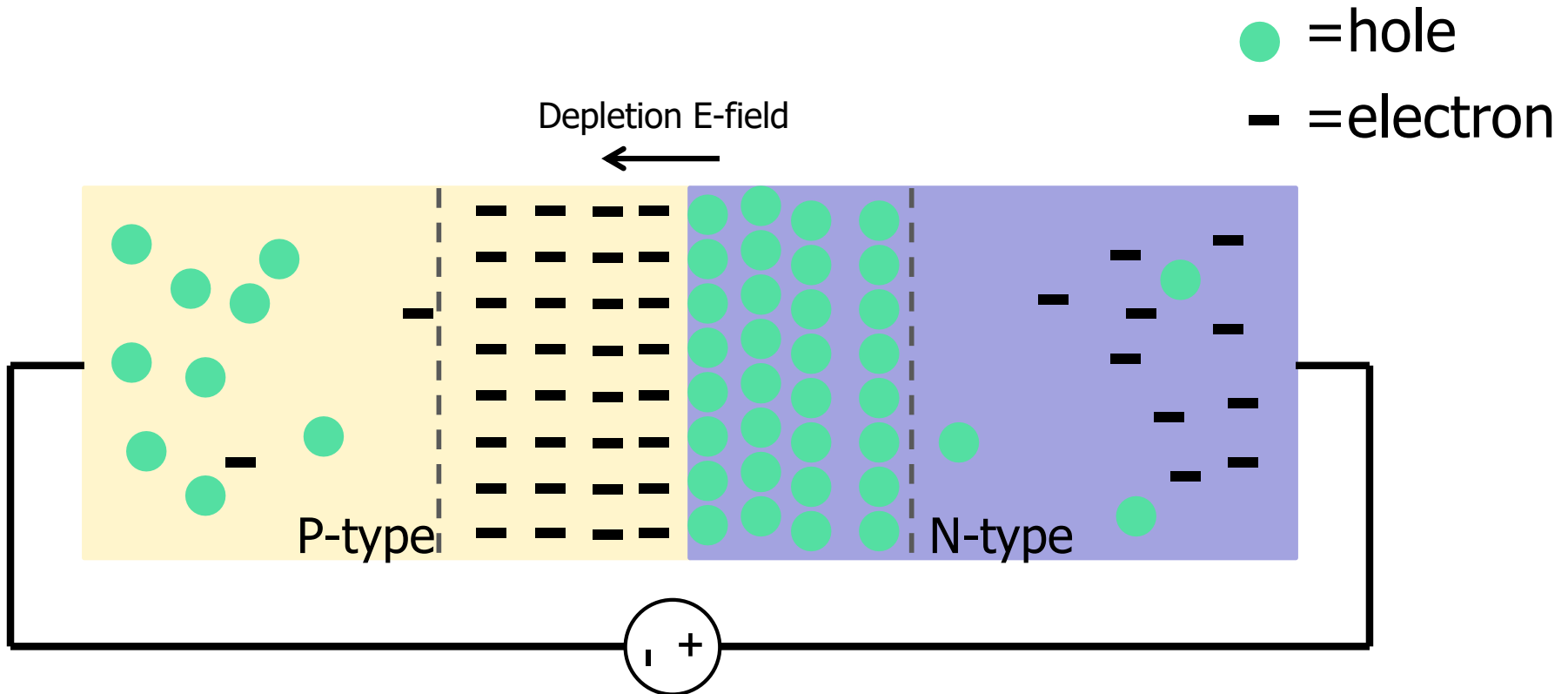
- Diffusion current
  - Current caused by semiconductor diffusion of holes and electrons
  
- Drift current
  - Current due to movement of holes and electrons caused by force from potential difference induced e-field

# PN Junction – Forward Biasing



- Forward biasing connect positive terminal to p-type and negative terminal to n-type
  - Holes/electrons pushed towards depletion region, causing it to narrow
  - The applied voltage e-field continues to narrow the depletion region (i.e reduce the depletion e-field)
  - current flows through the device from p-type to n-type

# PN Junction – Reverse Biasing

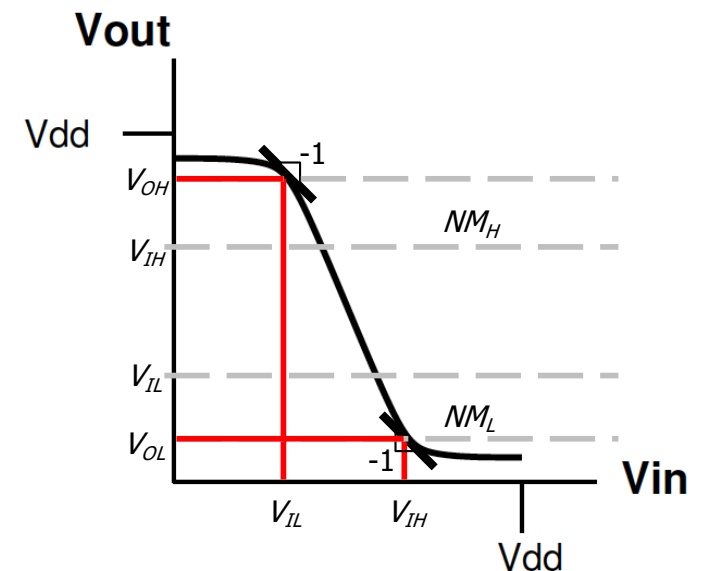


- Reverse biasing connect positive terminal to n-type and negative terminal to p-type
  - Holes/electrons attracted away from depletion region, causing it to widen
    - No current flows through the device
  - If reverse bias increases past breakdown voltage, the depletion e-field increases until breakdown occurs and reverse biased current flows causing thermal damage to junction



# Big Idea

- Need robust logic
  - Can design into any (feed forward) graph with logic gates and tolerate loss and noise, while maintaining digital abstraction
- Regeneration and noise margins
  - Every gate makes signal “better”
  - Design level of noise tolerance







# Admin

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- ❑ Homework 2 out now due Friday 2/9
  - Will take time to install and learn ngspice
    - START EARLY!!! For troubleshooting
    - Use Ed discussion and office hours for technical difficulty questions
    - Highly recommend you work in study groups/pairs for trouble shooting
  - Tool guides/tutorials on course webpage for help
    - <https://www.seas.upenn.edu/~ese3700/#tools>
  - Get your design workflow set up now
    - EVERYONE will be required to verify this on HW 3



# Acknowledgement

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- ❑ Prof. André DeHon (University of Pennsylvania)
- ❑ Prof. Jing Li (University of Pennsylvania)