

ESE5320: System-on-a-Chip Architecture

Day 7: September 26, 2022
Pipelining



Penn ESE5320 Fall 2022 -- DeHon

1

Previously

- Pipelining in the large
 - Not just for gate-level circuits
- Throughput and Latency
- Pipelining as a form of parallelism

Penn ESE5320 Fall 2022 -- DeHon

2

2

Today

Pipelining details (for gates, primitive ops)

- Systematic Approach (Part 1)
- Justify Operator and Interconnect Pipelining (Part 2)
- Loop Bodies
- Cycles in the Dataflow Graph (Part 3)
- C-slow [supplemental recording] (Part 4)

Penn ESE5320 Fall 2022 -- DeHon

3

3

Message

- Pipelining is an efficient way to reuse hardware to perform the **same** set of operations at high throughput

Penn ESE5320 Fall 2022 -- DeHon

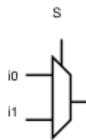
4

4

Multiplexer Gate

- MUX

- When $S=0$, output= i_0
- When $S=1$, output= i_1



S	i0	i1	Mux2(S,i0,i1)
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

Penn ESE5320 Fall 2022 -- DeHon

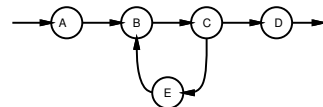
5

5

Cycle

Two uses of term in this lecture:

- Repetitive waveform
 - E.g. sine wave or square wave
- Graph cycle



Penn ESE5320 Fall 2022 -- DeHon

6

6

Waveform Cycles

- How many cycles showing of sine wave?

- How many cycles of square wave?

Penn ESE5320 Fall 2022 -- DeHon

7

Waveform Cycles

- How many cycles showing of sine wave?
- How many cycles of square wave?

- Note: clock on which we pipeline is a square wave
 - Talk about what happens in a clock cycle
 - Talk about number of clock cycles

Penn ESE5320 Fall 2022 -- DeHon

8

Latch

- Element that can hold a previous value of an input

Penn ESE5320 Fall 2022 -- DeHon

9

Register

- Use a pair to create a flip-flop
 - Also call register
- What happens when
 - CLK is low (0) ?
 - CLK is high (1) ?
 - CLK transitions from 0 to 1 ?
- What output Q until next 0 to 1 CLK transition?

Penn ESE5320 Fall 2022 -- DeHon

10

Register

- Use a pair to create a flip-flop
 - Also call register
- Sample D input on 0→1 transition of clock (CLK)
- Never an open path from D→Q
 - One of the mux latches always in hold state

Penn ESE5320 Fall 2022 -- DeHon

11

Synchronous Circuit Discipline

- Registers that sample inputs at clock edge and hold value throughout clock period
- Compute from registers-to-registers
- Clock Cycle time large enough for longest logic path between registers
- Min cycle = Max path delay between registers

Penn ESE5320 Fall 2022 -- DeHon

12

Preclass 1

- Delay between registers as shown?

The diagram shows three parallel data paths. Each path starts with a red square representing a register. The top path has two registers in series before an adder. The middle path has one register before an adder. The bottom path has no register before an adder. All paths then feed into a final adder.

Penn ESE5320 Fall 2022 -- DeHon

13

Preclass 1

- Move registers so can clock at adder delay?
(which is of choices at bottom is correct = same behavior)

Diagram A: Registers are placed before each adder stage. Diagram B: Registers are placed after each adder stage. Diagram C: Registers are placed before the first and after the second adder stage.

Penn ESE5320 Fall 2022 -- DeHon

14

Pipeline Reuse

- Lower delay between clocks
 - Higher clock rate
 - Higher potential throughput
 - Faster we reuse our logic
 - More capacity get out of design
 - Assuming registers cheap in area and time overhead
 - $T_{\text{setup}}, T_{\text{clk} \rightarrow q} \sim 20\text{ps}$, $T_{\text{add}} \sim 500\text{ps}$
 - Registers ~ 10 transistors/bit
 - Adder $\sim 40\text{--}50$ transistors/bit

Penn ESE5320 Fall 2022 -- DeHon

15

Preclass 2: What Happens?

- What would be wrong with this pipelining?

The diagram shows a pipeline where the output of the second adder is fed back into the first adder. This creates a loop that could cause a race condition or a hold violation.

- For this initial design:

Penn ESE5320 Fall 2022 -- DeHon

16

Behavior

The diagrams show the state of registers (A, B, C) and the output of adders over four clock cycles. In Cycle 0, registers A0, B0, and C0 are initialized. In Cycle 1, A1 and B1 are updated, and the first adder outputs (B0+C0). In Cycle 2, A2, B2, and C2 are updated, and the second adder outputs (A0+(B0+C0)). In Cycle 3, A3, B3, and C3 are updated, and the first adder outputs (A1+(B1+C1)).

17

Equations

The equations describe the state of registers and the output of adders at cycle i :

$$A_i = A_{i-1}$$

$$B_i = B_{i-1}$$

$$C_i = C_{i-1}$$

$$A_{i-1} = A_{i-2} + (B_{i-2} + C_{i-2})$$

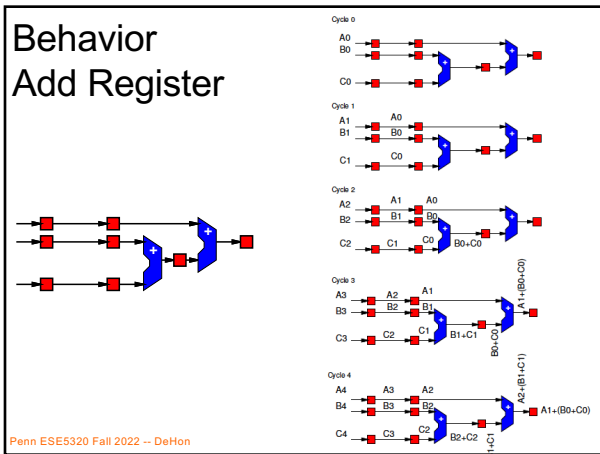
$$B_{i-1} = B_{i-3} + (B_{i-3} + C_{i-3})$$

$$C_{i-1} = C_{i-3} + (B_{i-3} + C_{i-3})$$

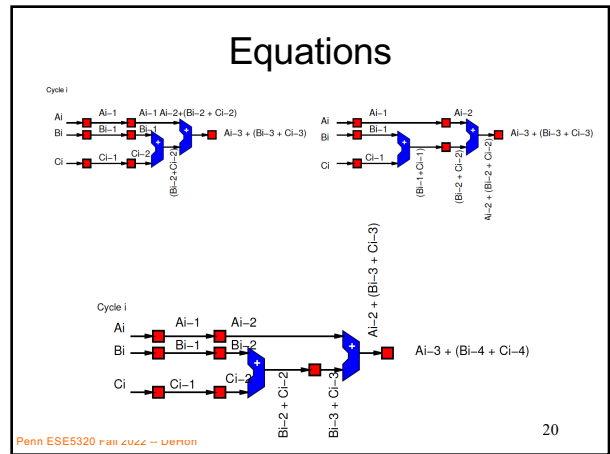
$$A_{i-2} = A_{i-3} + (B_{i-3} + C_{i-3})$$

Penn ESE5320 Fall 2022 -- DeHon

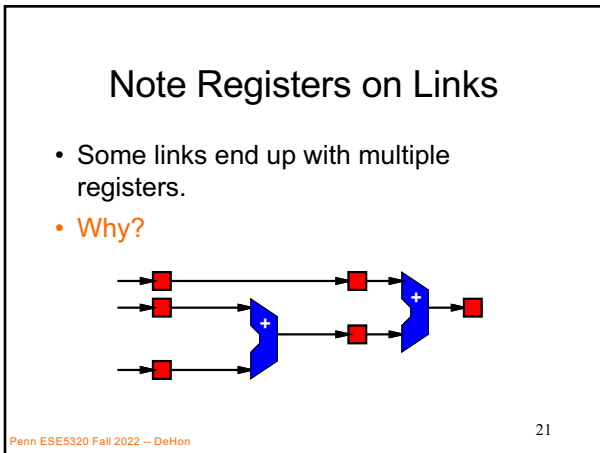
18



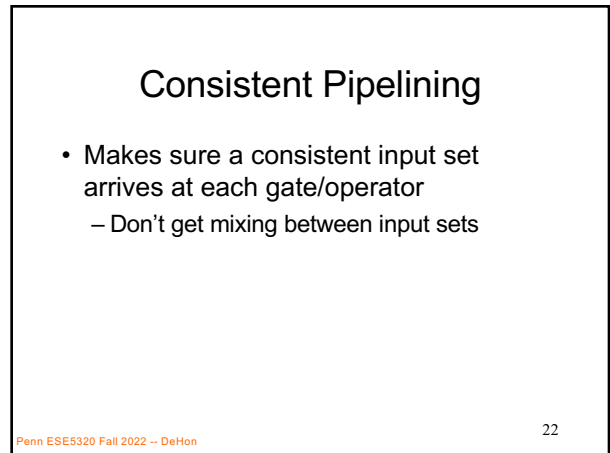
19



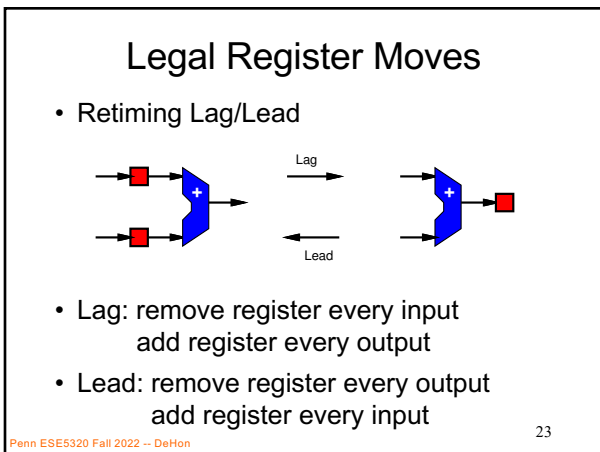
20



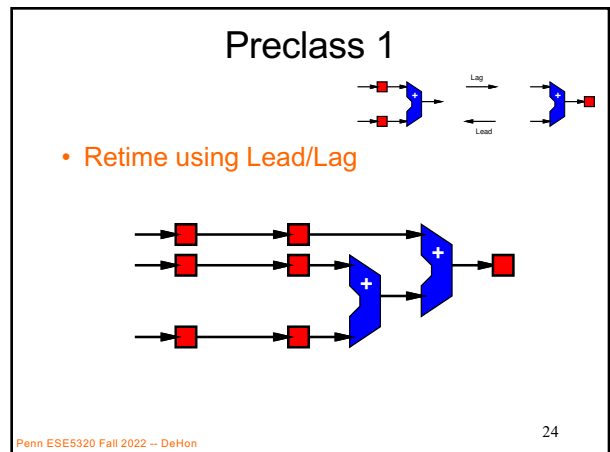
21



22

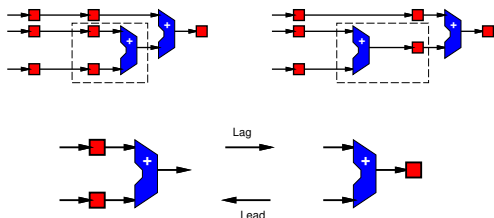


23



24

Preclass 1 (revisited)



Penn ESE5320 Fall 2022 -- DeHon

25

25

Add Registers and Move

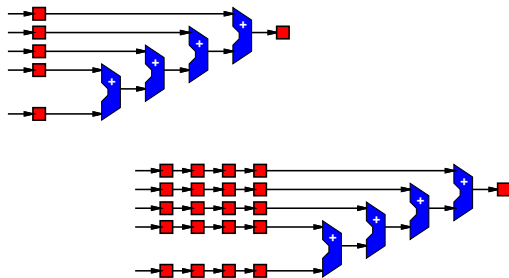
- If we're willing to add pipeline delay
 - Add any number of pipeline registers at input
 - Move registers into circuit to reduce cycle time
 - Reduce max delay between registers

Penn ESE5320 Fall 2022 -- DeHon

26

26

Add Registers at Input

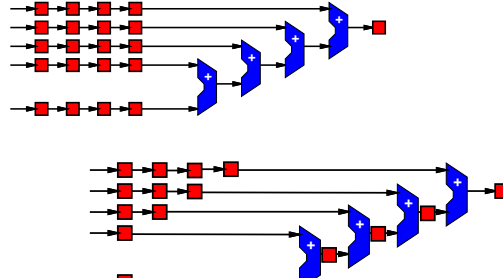


Penn ESE5320 Fall 2022 -- DeHon

27

27

Add Registers at Input and Retime



Penn ESE5320 Fall 2022 -- DeHon

28

28

Add Register and Retime

- Add chain of registers on every input
- Retime registers into circuit
 - Minimizing delay between registers

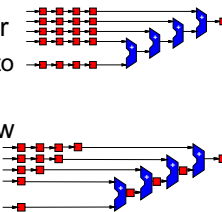
Penn ESE5320 Fall 2022 -- DeHon

29

29

Add Registers and Retime

- Lets us think about behavior
 - What the pipelining is doing to cycles of delay
- Separate from details of how redistribute registers
- Behavioral equivalence between the registers-at-front and properly retimed version of circuit



Penn ESE5320 Fall 2022 -- DeHon

30

30

Justify Pipelining

(or composing pipelined operators)
Part 2

Penn ESE5320 Fall 2022 -- DeHon

31

31

Handling Pipelined Operators

- Given a pipelined operator
 - (or a pipelined interconnect)
- Discipline of picking a frequency target and designing everything for that
 - May be necessary to pipeline operator since its delay is too high
- Due to hierarchy
 - Pipelined this operator and now want to use it as a building block

Penn ESE5320 Fall 2022 -- DeHon

32

32

Examples

- Run at 500MHz
- Floating-point unit that takes 9ns
 - Can pipeline into 5, 2ns stages
- Multiplier that takes 6ns
- Memory can access in 2ns
 - Only if registers on address/inputs and output
 - i.e. exist in own clock stage

Penn ESE5320 Fall 2022 -- DeHon

33

33

Interconnect Delay

- Chips >> Clock Cycles
- May have chip 100s of Operators wide
- May only be able to reach across 10 operators in a 2ns cycle
- Must pipeline long interconnect links

Penn ESE5320 Fall 2022 -- DeHon

34

34

Interconnect Example

Penn ESE5320 Fall 2022 -- DeHon

35

35

Methodology: Pipelined Operator Graph

- Start with logical, unpipelined graph
- Treat each pipelined operator as a set of unit-delay operators of mandatory depth
- Treat each interconnect pipeline stage as a unit-delay buffer
- Add registers at input
- Retime into graph

Penn ESE5320 Fall 2022 -- DeHon

36

36

Model

- 3-stage Multiplier
- Interconnect Delay

37

37

Pipeline Loop

(and use for justify pipeline example)

38

38

Preclass 4

- Logical (unpipelined) dataflow graph for loop body

39

39

Example Operators

- Operator and Interconnect delays
 - Multiplier 3 cycles
 - Reading from Input array
 - Memory op is cycle after computing address
 - Takes one cycle delay bring data back to multiplier (or adder)

40

40

Illustrate Need

- What happens if just use graph as is (with operators pipelined as required)?

41

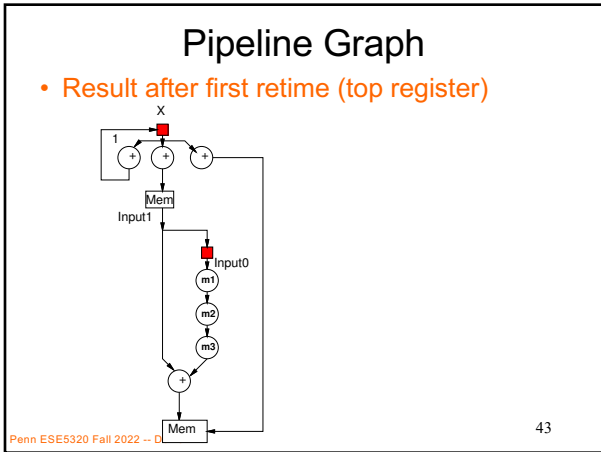
41

Model Graph

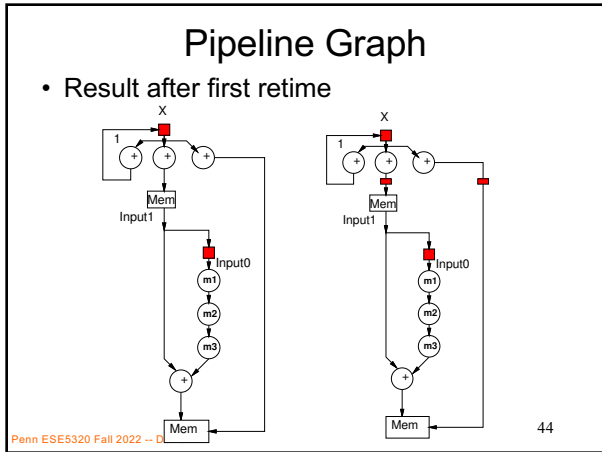
- Revised graph for modeling

42

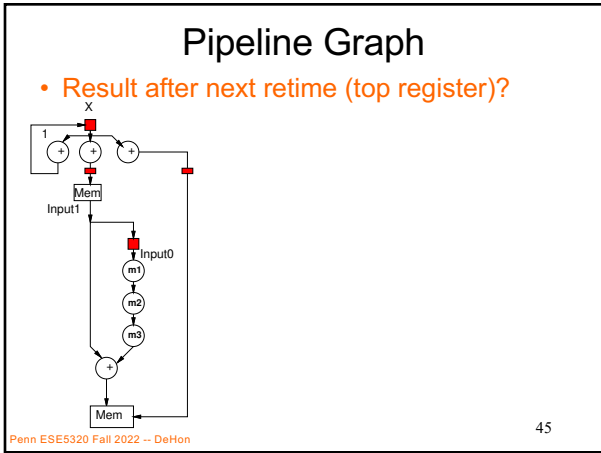
42



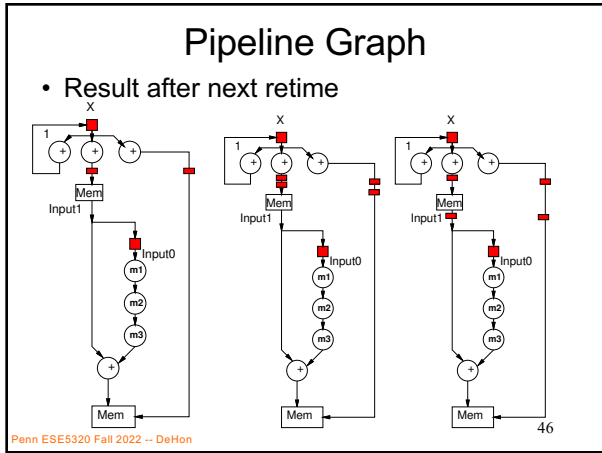
43



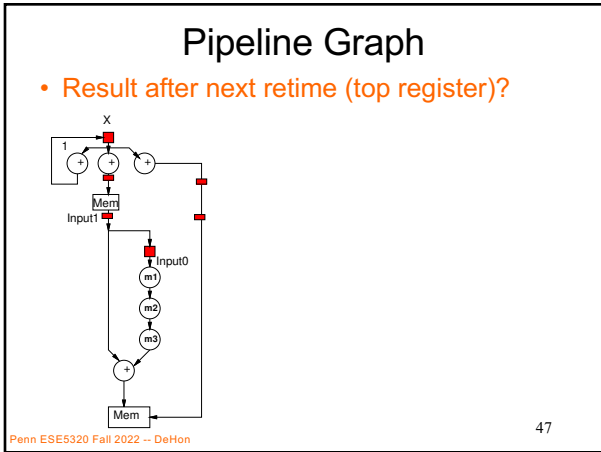
44



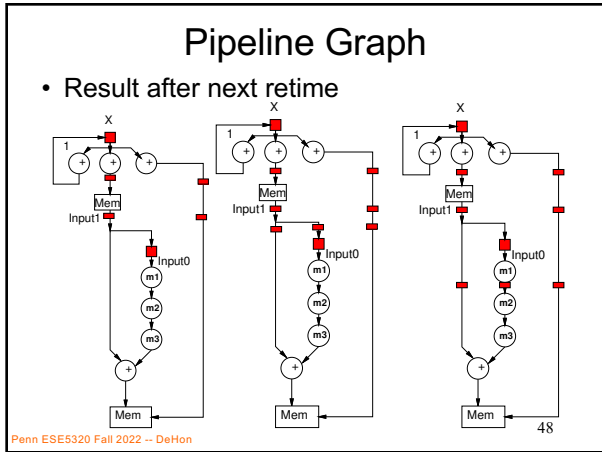
45



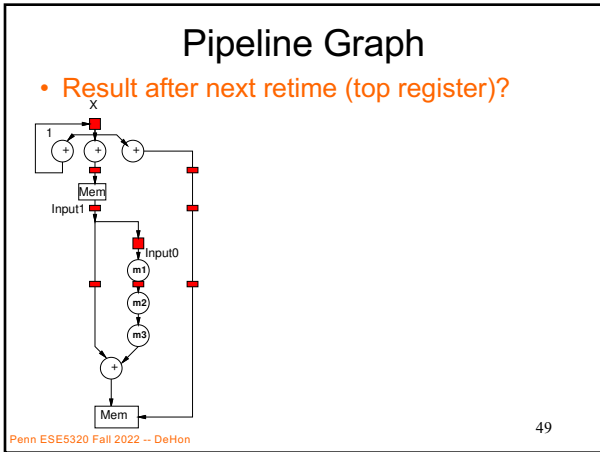
46



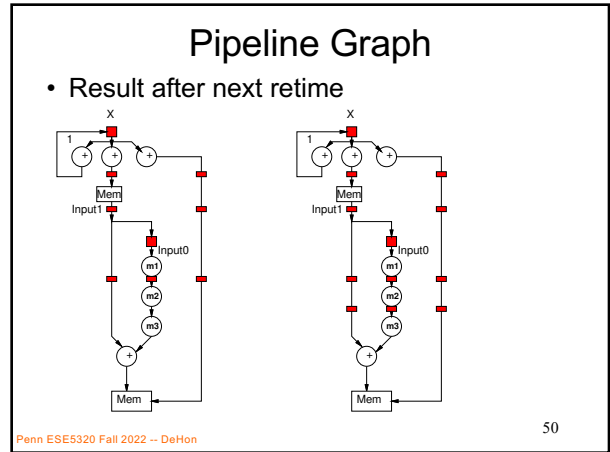
47



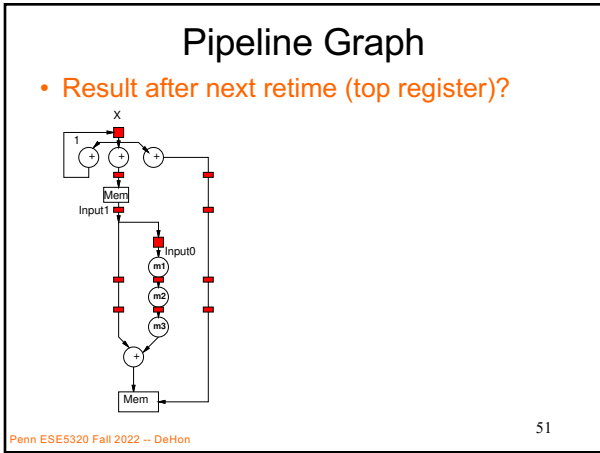
48



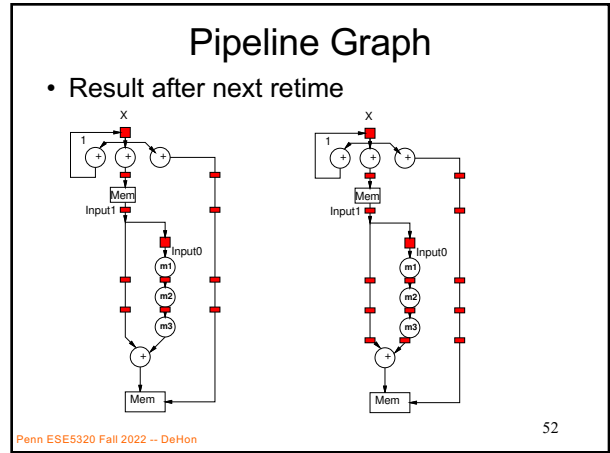
49



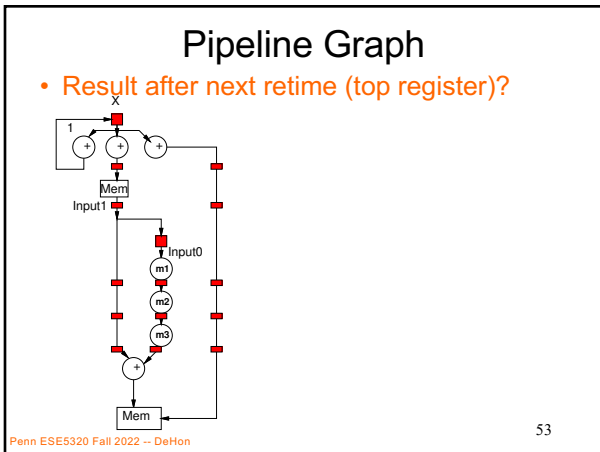
50



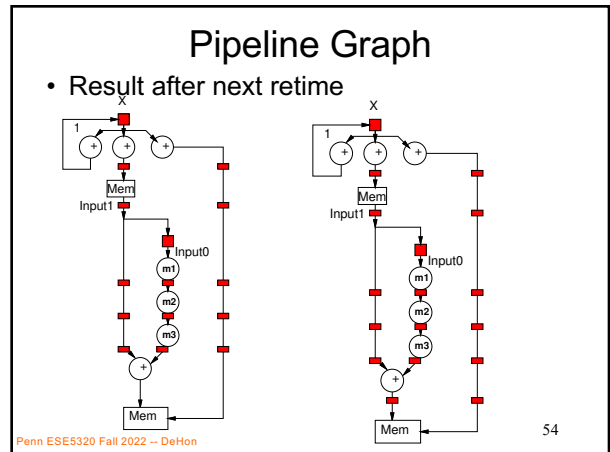
51



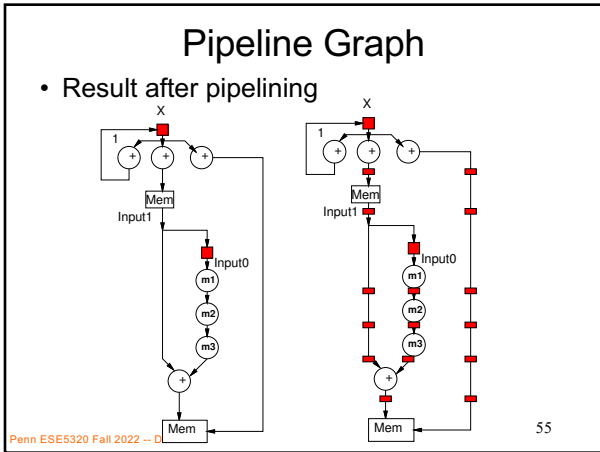
52



53



54



55

Pipelining Lesson

- Can always pipeline an **acyclic** graph (no graph cycles) to fixed frequency target
 - fixed pipelining of primitive operators
 - Pipeline interconnect delays
- Need to keep track of registers to balance paths
 - So see consistent delays to operators

Penn ESE5320 Fall 2022 -- DeHon

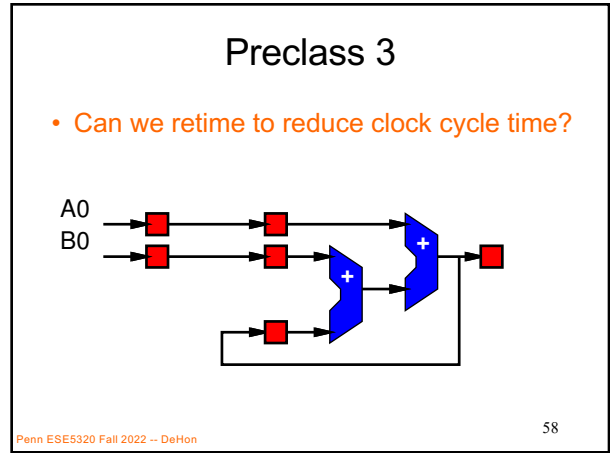
56

Graph Cycles

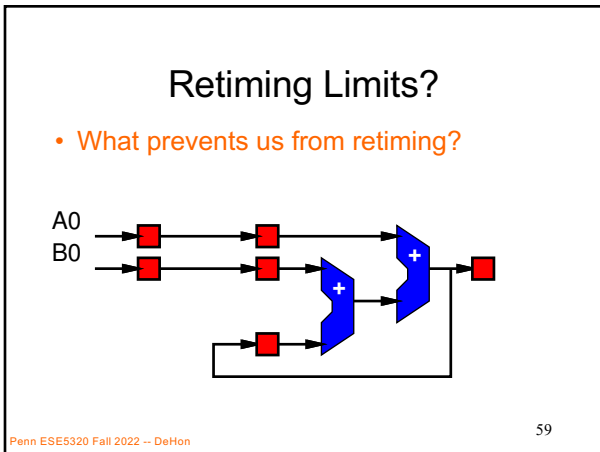
Watch: Clock cycle
Cycle time
Cycle in Graph
Part 3

Penn ESE5320 Fall 2022 -- DeHon

57



58



59

(Graph) Cycle Observation

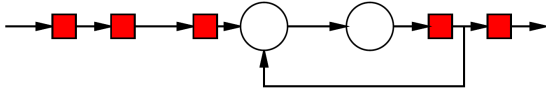
- Retiming does not allow us to change the *number of registers inside a graph cycle*.
- Limit to **clock cycle time**
 - Max delay in **graph cycle** / Registers in **graph cycle**
- Pipelining doesn't help inside **graph cycle**
 - Cannot push registers into **graph cycle**

Penn ESE5320 Fall 2022 -- DeHon

60

Simple Graph Cycle

- Delay of graph cycle?
- Registers in graph cycle?
- What happens to graph cycle if try to apply lead/lag?

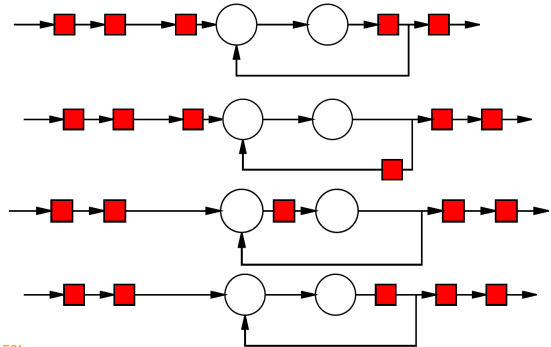


Penn ESE5320 Fall 2022 -- DeHon

61

61

Retiming

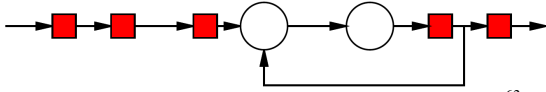


Penn ESE5320 Fall 2022 -- DeHon

62

Initiation Interval (II)

- Cyclic dependencies in a dataflow graph can limit throughput
- Due to data-dependent cycles in graph,
 - May not be able to initiate a new computation on every clock cycle
- II – clock cycles (delay) before can initiate
- Throughput = $1/II$



Penn ESE5320 Fall 2022 -- DeHon

63

63

Loop

- Consider
 - [multiply and mod each take 3 cycles]
- For ($i=0; i<N; i++$)
 - $C[i] = (C[i-1] * A[i]) \% N;$

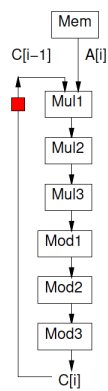
Penn ESE5320 Fall 2022 -- DeHon

64

64

Loop

- For ($i=0; i<N; i++$)
 - $C[i] = (C[i-1] * A[i]) \% N;$



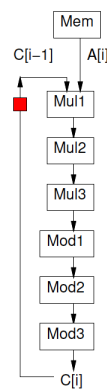
Penn ESE5320 Fall 2022 -- DeHon

65

65

Loop

- For ($i=0; i<N; i++$)
 - $C[i] = (C[i-1] * A[i]) \% N;$
- Initiation Interval?



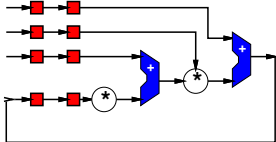
Penn ESE5320 Fall 2022 -- DeHon

66

66

Initiation Interval

- Delay Around graph cycle?
 - Assume multiply 3, add 1
- Registers in graph cycle?
- Retiming clock cycle bound = II ?
- Achievable?

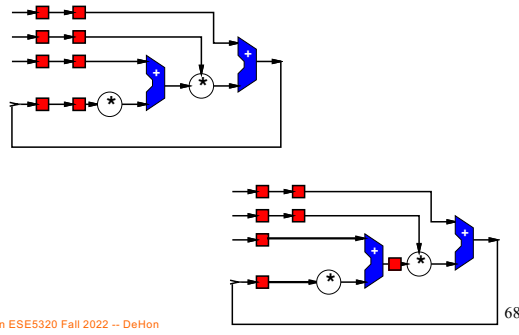


Penn ESE5320 Fall 2022 -- DeHon

67

67

Retimed



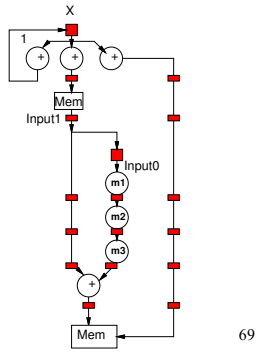
Penn ESE5320 Fall 2022 -- DeHon

68

68

II and Latency

- Actually is a cycle
 - II?
 - Latency?



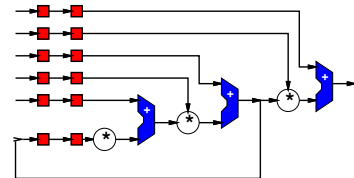
Penn ESE5320 Fall 2022 -- DeHon

69

69

II and Latency

- II? (assume willing to pipeline inputs)
- Latency?

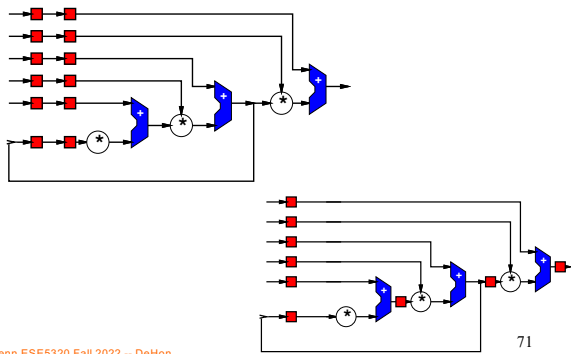


Penn ESE5320 Fall 2022 -- DeHon

70

70

II and Latency



Penn ESE5320 Fall 2022 -- DeHon

71

71

Lesson

- Cyclic dependencies limit throughput on single task or data stream
 - Cycle-length / registers-in-cycle

Penn ESE5320 Fall 2022 -- DeHon

72

72

Vector Pipelines

- Data Parallel Vector Operations are interesting even when Vector Lanes < Vector Length
- Within Vector operation, data parallel so no cyclic dependencies
 - So get an $II=1$ issuing Vector Lane operations
 - May have data dependences between Vector operations

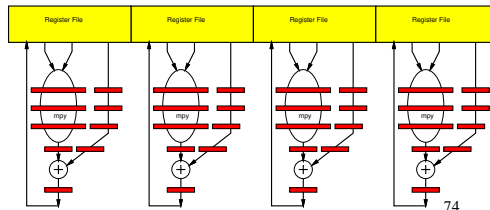
Penn ESE5320 Fall 2022 -- DeHon

73

73

Vector Pipeline Example

```
for (int i=0; i<32; i++)
    d[i]=a[i]*b[i]+c[i]
```



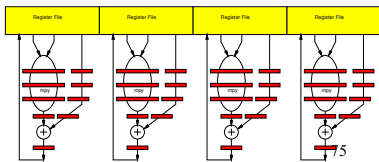
Penn ESE5320 Fall 2022 -- DeHon

74

74

Dependence between Vector Operations

```
for (int i=0; i<32; i++)
    d[i]=a[i]*b[i]+c[i]
for (int i=0; i<32; i++)
    g[i]=e[i]*f[i]+d[i]
```



Penn ESE5320 Fall 2022 -- DeHon

75

75

Big Ideas

- Pipeline computations to reuse hardware and maximize computational capacity
- Can compose pipelined operators and accommodate fixed-frequency target
 - Be careful with data retiming
- Graph cycles limit pipelining on single stream – II (Initiation Interval)
- C-slow to share hardware among multiple, data-parallel streams (part 4)

Penn ESE5320 Fall 2022 -- DeHon

76

76

Admin

- Remember Feedback form
 - Including HW3
- Reading for Day 8 on web
- HW4 due Friday

Penn ESE5320 Fall 2022 -- DeHon

77

77

C-Slow

(See uploaded recording)
Part 4

Penn ESE5320 Fall 2022 -- DeHon

78

78

Problem

- Pipelining cannot push registers into a **graph cycle**
- **Graph cycles** can prevent running at full pipeline target (maximum **clock** frequency)
- If not reusing operators at full pipeline target are underutilizing resources
- **Can we use the resources for something?**

79

79

C-Slow

- **Observation:** if we have data-level parallelism, can use to solve independent problems on same hardware
- **Transformation:** make C copies of each register
- **Guarantee:** C computations operate independently
 - Do not interact with each other

80

80

2-Slow Simple Cycle

- Replace register with pair
- Retime

81

81

2-Slow Simple Cycle

- Replace register with pair
- Retime
- Observe independence of red/blue computations

82

82

Equivalence

- The 2-slow operator is equivalent to two data parallel operators running at half the speed
 - E.g. processing separate audio channels

83

83

Automation

- No mainstream tool today will perform C-slow transformation for you automatically
- Synthesis tools will retime registers

84

84

Lesson

- Cyclic dependencies limit throughput on single task or data stream
 - $II = \text{Cycle-length} / \text{registers-in-cycle}$
- Can use on C ($C \leq II$) independent (data parallel) tasks

Penn ESE5320 Fall 2022 -- DeHon

85

85

Big Ideas

- Pipeline computations to reuse hardware and maximize computational capacity
- Can compose pipelined operators and accommodate fixed-frequency target
 - Be careful with data retiming
- Graph cycles limit pipelining on single stream -- II
- C -slow ($C \leq II$) to share hardware among multiple, data-parallel streams (part 4)

Penn ESE5320 Fall 2022 -- DeHon

86

86