

# ESE5320: System-on-a-Chip Architecture

Day 7: September 25, 2023  
Pipelining



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## Previously

- Pipelining in the large
  - Not just for gate-level circuits
- Throughput and Latency
- Pipelining as a form of parallelism

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## Today

Pipelining details (for gates, primitive ops)

- Systematic Approach (Part 1)
- Justify Operator and Interconnect Pipelining (Part 2)
- Loop Bodies
- Cycles in the Dataflow Graph (Part 3)
- C-slow [supplemental recording] (Part 4)

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## Message

- Pipelining is an efficient way to reuse hardware to perform the **same** set of operations at high throughput

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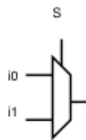
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## Multiplexer Gate

- MUX

- When  $S=0$ , output= $i_0$
- When  $S=1$ , output= $i_1$



S	i0	i1	Mux2(S,i0,i1)
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

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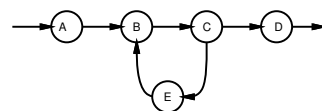
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## Cycle

Two uses of term in this lecture:

- Repetitive waveform
  - E.g. sine wave or square wave
- Graph cycle



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## Waveform Cycles

- How many cycles showing of sine wave?

- How many cycles of square wave?

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## Waveform Cycles

- How many cycles showing of sine wave?
- How many cycles of square wave?

- Note: clock on which we pipeline is a square wave
  - Talk about what happens in a clock cycle
  - Talk about number of clock cycles

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## Latch

- Element that can hold a previous value of an input

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## Register

- Use a pair to create a flip-flop
  - Also call register
- What happens when
  - CLK is low (0) ?
  - CLK is high (1) ?
  - CLK transitions from 0 to 1 ?
- What output Q until next 0 to 1 CLK transition?

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## Register

- Use a pair to create a flip-flop
  - Also call register
- Sample D input on 0→1 transition of clock (CLK)
- Never an open path from D→Q
  - One of the mux latches always in hold state

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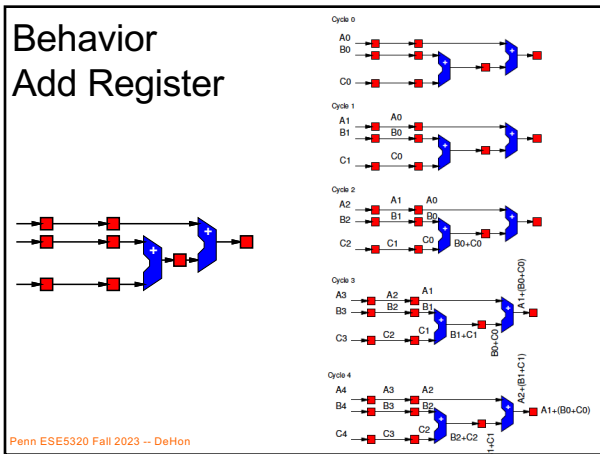
## Synchronous Circuit Discipline

- Registers that sample inputs at clock edge and hold value throughout clock period
- Compute from registers-to-registers
- Clock Cycle time large enough for longest logic path between registers
- Min cycle = Max path delay between registers

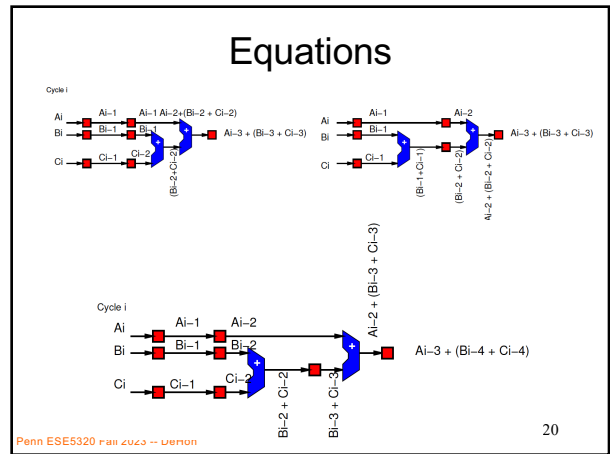
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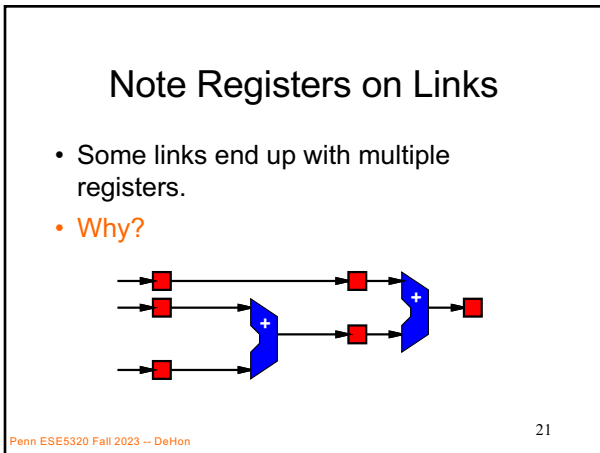




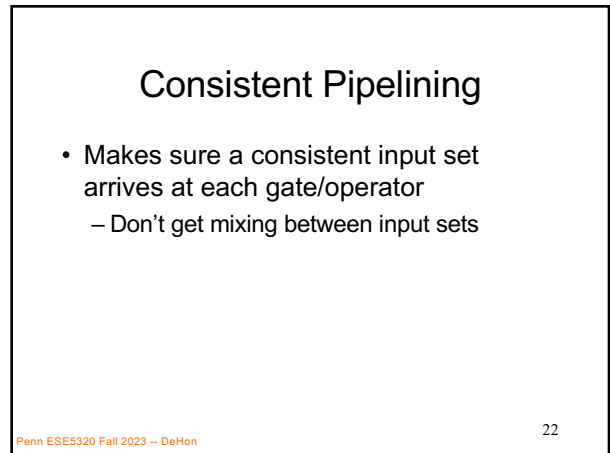
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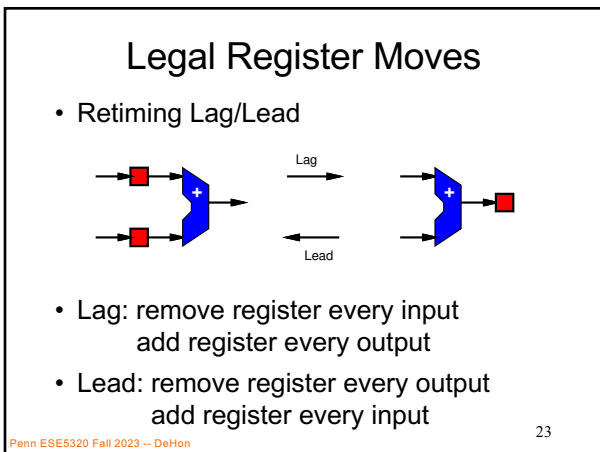
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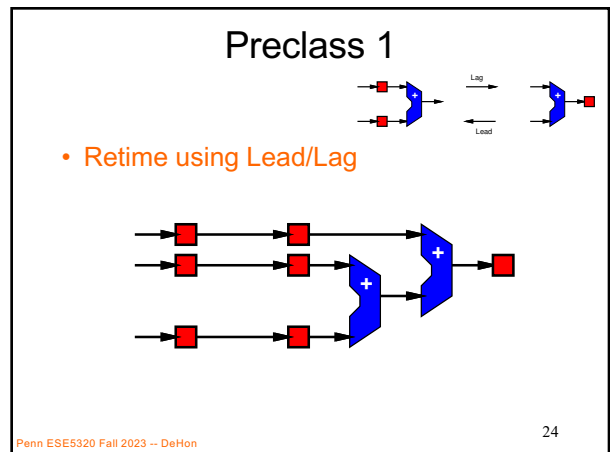
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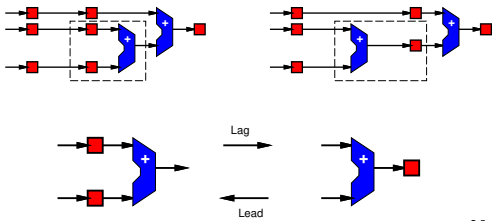


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## Preclass 1 (revisited)



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## Add Registers and Move

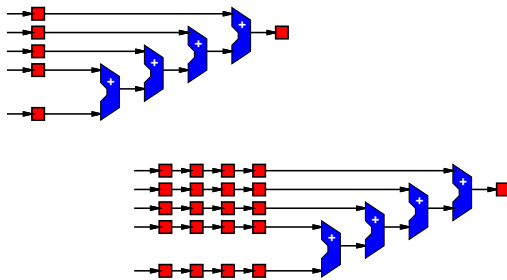
- If we're willing to add pipeline delay
  - Add any number of pipeline registers at input
  - Move registers into circuit to reduce cycle time
    - Reduce max delay between registers

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## Add Registers at Input

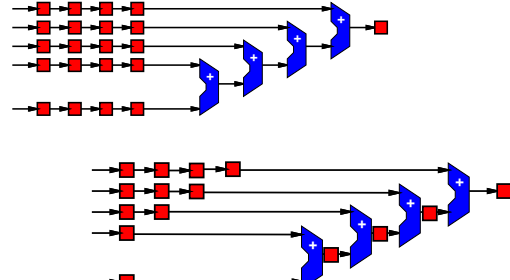


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## Add Registers at Input and Retime



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## Add Register and Retime

- Add chain of registers on every input
- Retime registers into circuit
  - Minimizing delay between registers

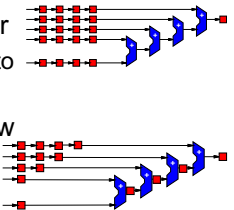
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## Add Registers and Retime

- Lets us think about behavior
  - What the pipelining is doing to cycles of delay
- Separate from details of how redistribute registers
- Behavioral equivalence between the registers-at-front and properly retimed version of circuit



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## Justify Pipelining

(or composing pipelined operators)  
Part 2

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## Handling Pipelined Operators

- Given a pipelined operator
  - (or a pipelined interconnect)
- Discipline of picking a frequency target and designing everything for that
  - May be necessary to pipeline operator since its delay is too high
- Due to hierarchy
  - Pipelined this operator and now want to use it as a building block

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## Examples

- Run at 500MHz
- Floating-point unit that takes 9ns
  - Can pipeline into 5, 2ns stages
- Multiplier that takes 6ns
- Memory can access in 2ns
  - Only if registers on address/inputs and output
  - i.e. exist in own clock stage

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## Interconnect Delay

- Chips >> Clock Cycles
- May have chip 100s of Operators wide
- May only be able to reach across 10 operators in a 2ns cycle
- Must pipeline long interconnect links

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## Interconnect Example

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## Methodology: Pipelined Operator Graph

- Start with logical, unpipelined graph
- Treat each pipelined operator as a set of unit-delay operators of mandatory depth
- Treat each interconnect pipeline stage as a unit-delay buffer
- Add registers at input
- Retime into graph

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### Model

- 3-stage Multiplier
- Interconnect Delay

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### Pipeline Loop

(and use for justify pipeline example)

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### Preclass 4

- Logical (unpipelined) dataflow graph for loop body

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### Example Operators

- Operator and Interconnect delays
  - Multiplier 3 cycles
  - Reading from Input array
    - Memory op is cycle after computing address
    - Takes one cycle delay bring data back to multiplier (or adder)

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### Illustrate Need

- What happens if just use graph as is (with operators pipelined as required)?

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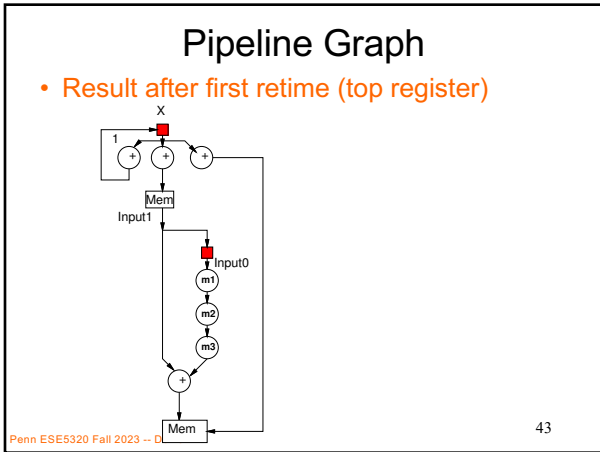
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### Model Graph

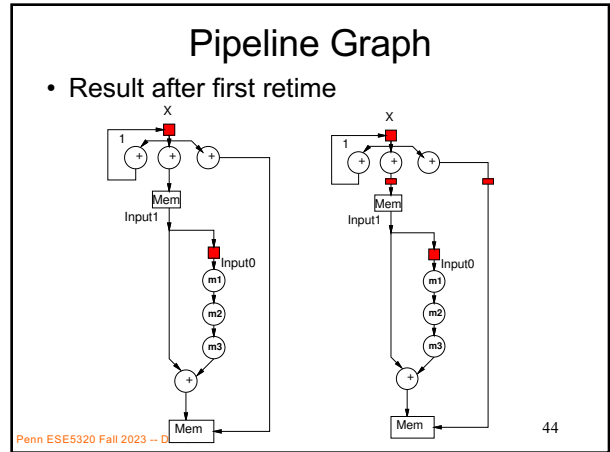
- Revised graph for modeling

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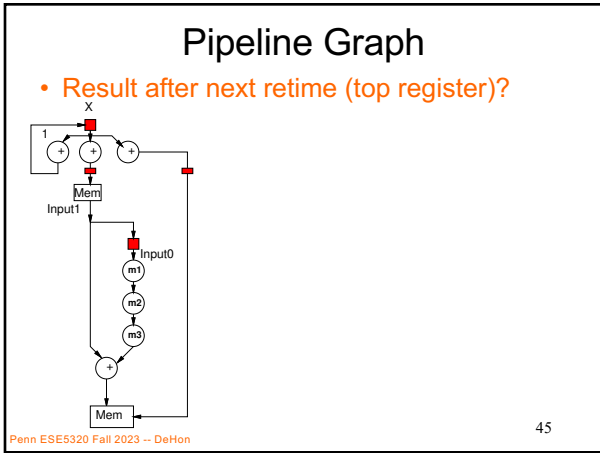
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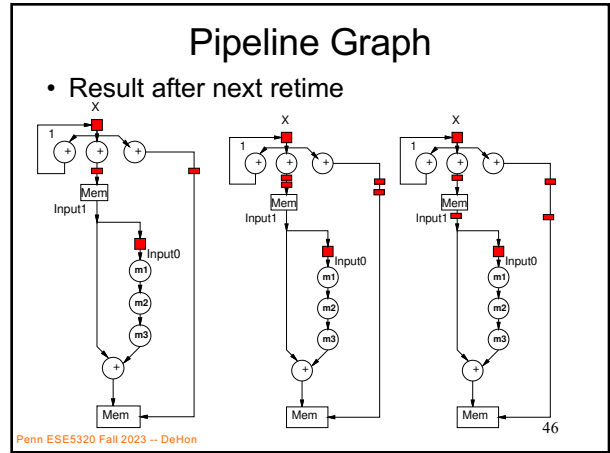
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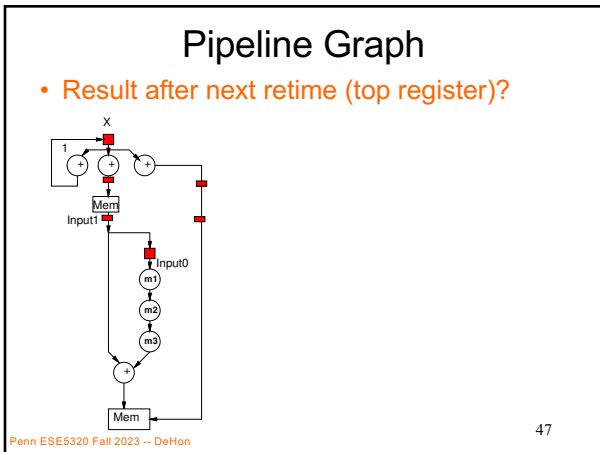
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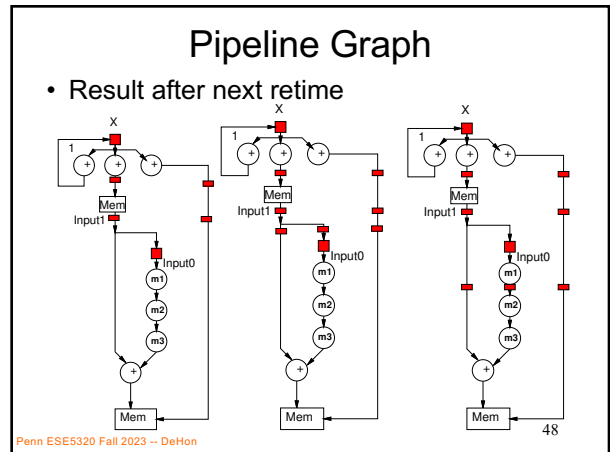
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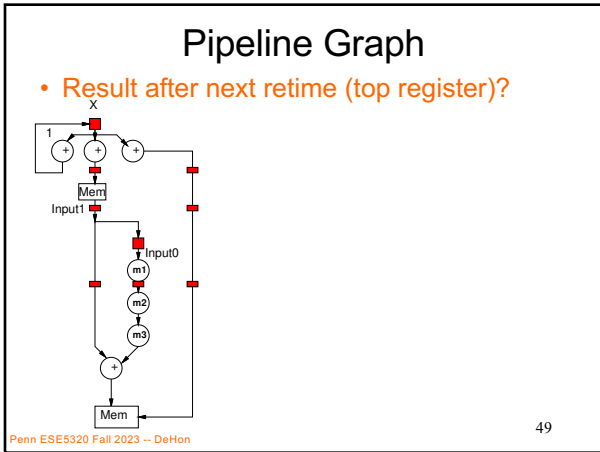


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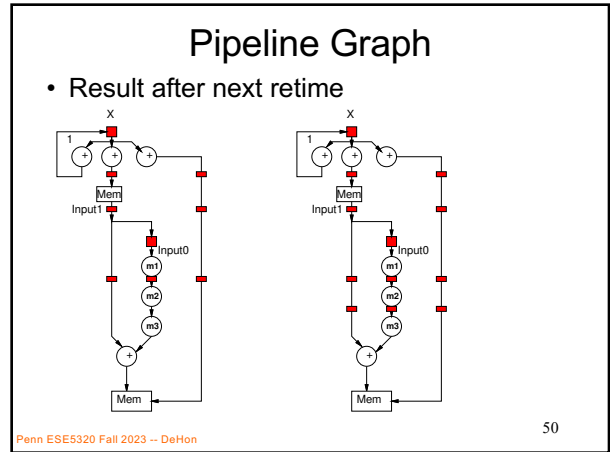


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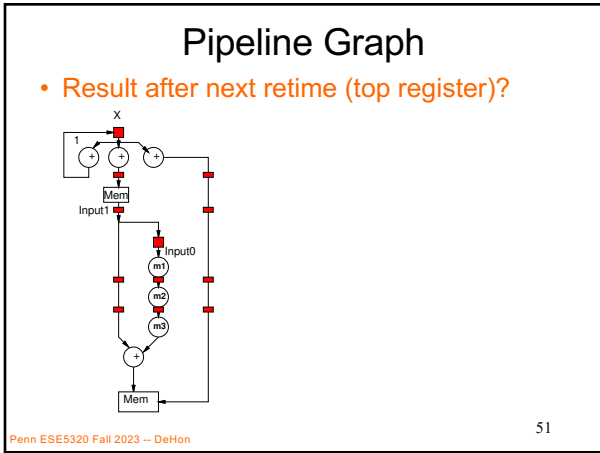




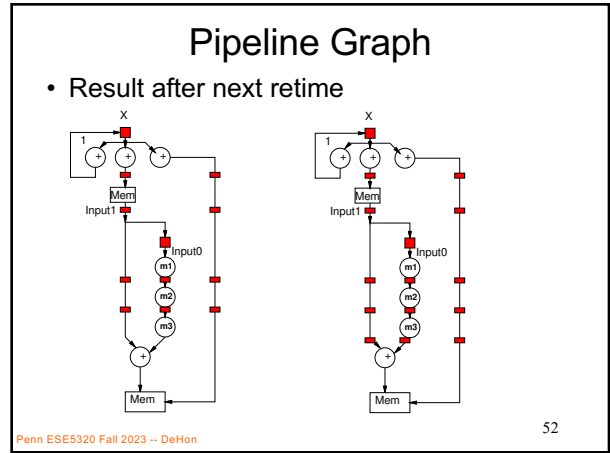
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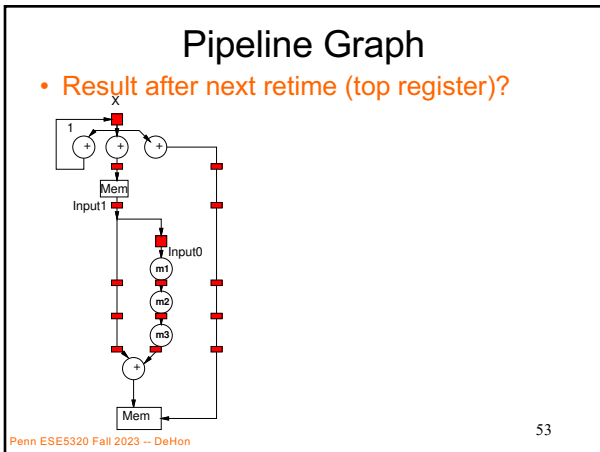
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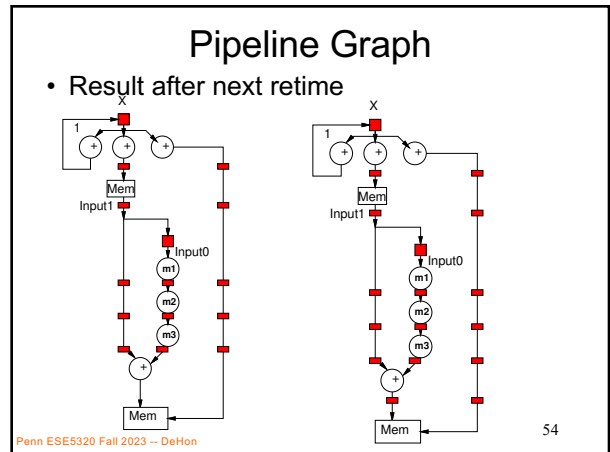
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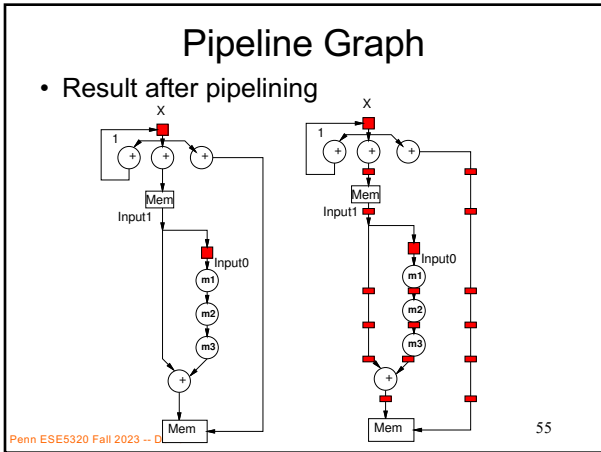
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### Pipelining Lesson

- Can always pipeline an **acyclic** graph (no graph cycles) to fixed frequency target
  - fixed pipelining of primitive operators
  - Pipeline interconnect delays
- Need to keep track of registers to balance paths
  - So see consistent delays to operators

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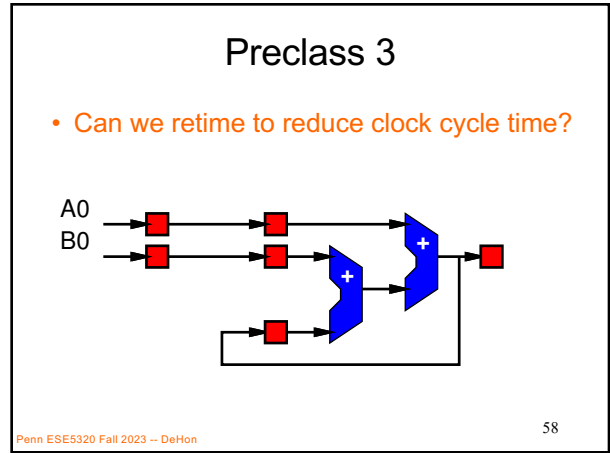
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### Graph Cycles

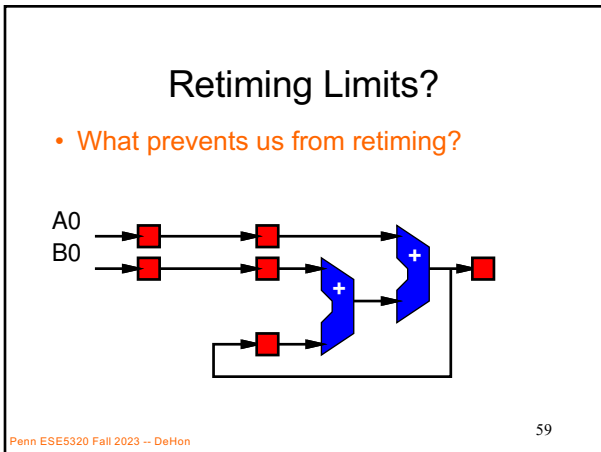
Watch: Clock cycle  
Cycle time  
Cycle in Graph  
Part 3

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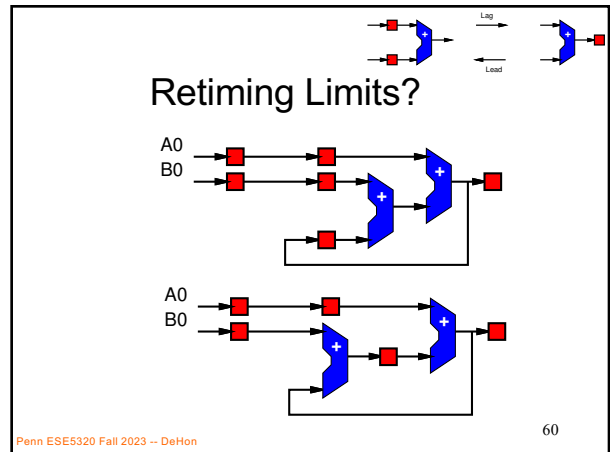
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## (Graph) Cycle Observation

- Retiming does not allow us to change the *number of registers inside a graph cycle*.
- Limit to **clock cycle** time
  - Max delay in **graph cycle** / Registers in **graph cycle**
- Pipelining doesn't help inside **graph cycle**
  - Cannot push registers into **graph cycle**

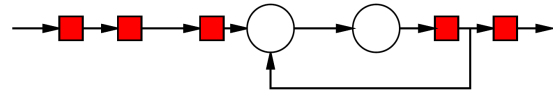
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## Simple Graph Cycle

- Delay of graph cycle?
- Registers in graph cycle?
- What happens to graph cycle if try to apply lead/lag?

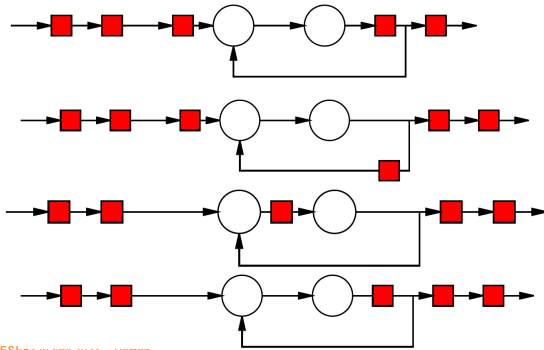


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## Retiming



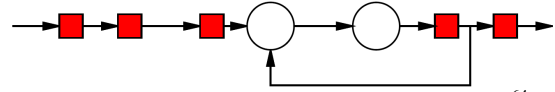
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## Initiation Interval (II)

- Cyclic dependencies in a **dataflow graph** can limit throughput
- Due to data-dependent **cycles in graph**,
  - May not be able to initiate a new computation on every **clock cycle**
- II – **clock cycles (delay)** before can initiate
- Throughput = 1/II



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## Loop

- Consider
  - [multiply and mod each take 3 cycles]
- For (i=0;i<N;i++)  
 $C[i] = (C[i-1] * A[i]) \% N;$

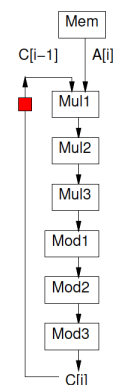
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## Loop

- For (i=0;i<N;i++)  
 $C[i] = (C[i-1] * A[i]) \% N;$



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### Loop

- For (i=0;i<N;i++)  
 $C[i] = (C[i-1] * A[i]) \% N;$
- Initiation Interval?

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### Initiation Interval

- Delay Around graph cycle?  
 - Assume multiply 3, add 1
- Registers in graph cycle?
- Retiming clock cycle bound = II ?
- Achievable?

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### Retimed

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### II and Latency

- Actually is a cycle  
 - II?  
 - Latency  
 x to mem write?

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### II and Latency

- II? (assume willing to pipeline inputs)
- Latency?

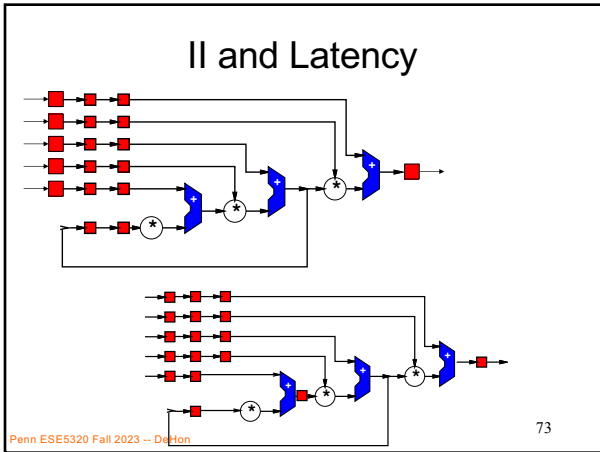
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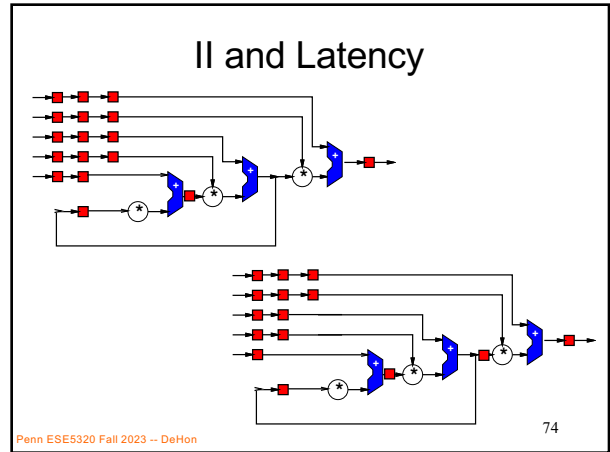
### II and Latency

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### Lesson

- Cyclic dependencies limit throughput on single task or data stream
  - Cycle-length / registers-in-cycle

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### Big Ideas

- Pipeline computations to reuse hardware and maximize computational capacity
- Can compose pipelined operators and accommodate fixed-frequency target
  - Be careful with data retiming
- Graph cycles limit pipelining on single stream – II (Initiation Interval)
- C-slow to share hardware among multiple, data-parallel streams (part 4)

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### Admin

- Remember Feedback form
  - Including HW3
- Reading for Day 8 on web
- HW4 due Friday

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### C-Slow

(See uploaded recording)  
Part 4

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### Problem

- Pipelining cannot push registers into a **graph cycle**
- **Graph cycles** can prevent running at full pipeline target (maximum **clock** frequency)
- If not reusing operators at full pipeline target are underutilizing resources
- **Can we use the resources for something?**

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### C-Slow

- **Observation:** if we have data-level parallelism, can use to solve independent problems on same hardware
- **Transformation:** make C copies of each register
- **Guarantee:** C computations operate independently
  - Do not interact with each other

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### 2-Slow Simple Cycle

- Replace register with pair
- Retime

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### 2-Slow Simple Cycle

- Replace register with pair
- Retime
- Observe independence of red/blue computations

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### Equivalence

- The 2-slow operator is equivalent to two data parallel operators running at half the speed
  - E.g. processing separate audio channels

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### Automation

- No mainstream tool today will perform C-slow transformation for you automatically
- Synthesis tools will retime registers

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## Lesson

- Cyclic dependencies limit throughput on single task or data stream
  - $II = \text{Cycle-length} / \text{registers-in-cycle}$
- Can use on  $C$  ( $C \leq II$ ) independent (data parallel) tasks

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## Big Ideas

- Pipeline computations to reuse hardware and maximize computational capacity
- Can compose pipelined operators and accommodate fixed-frequency target
  - Be careful with data retiming
- Graph cycles limit pipelining on single stream --  $II$
- $C$ -slow ( $C \leq II$ ) to share hardware among multiple, data-parallel streams (part 4)

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