Previously

• Used Rent's Rule characterization to understand wire growth
  \[ IO = c N^p \]

  • Top bisections will be \( \Omega(N^p) \)
  • 2D wiring area
  \[ \Omega(N^p) \times \Omega(N^p) = \Omega(N^{2p}) \]

We Know

• How we avoid \( O(N^2) \) wire growth for “typical” designs
• How to characterize locality
• How we might exploit that locality to reduce wire growth
• Wire growth implied by a characterized design

Today

• Switching
  – Implications
  – Options

Observation

• Locality that saved us wiring, also saves us switching
  \[ IO = c N^p \]
Consider

- Crossbar case to exploit wiring:
  - split into two halves, connect with limited wires
  - $N/2 \times N/2$ crossbar each half
  - $N/2 \times (N/2)^p$ connect to bisection wires
  - $2(N^2/4) + 2(N/2)^{p+1} < N^2$

Recurse

- Repeat at each level
  - form tree

Result

- If use crossbar at each tree node
  - $O(N^{2p})$ wiring area
    - for $p>0.5$, direct from bisection
  - $O(N^{2p})$ switches
    - top switch box is $O(N^{2p})$
      - $2 \cdot W_{top} \times W_{bot} + (W_{bot})^2$
      - $2 \cdot (N^p \times (N/2)^p + (N/2)^{2p})$
      - $N^{2p}(1/2^p + 1/2^{2p})$
  - switches at one level down is
    - $2(1-2^p) \times$ previous level
    - coefficient < 1 for $p>0.5$

Result

- If use crossbar at each tree node
  - $O(N^{2p})$ wiring area
    - for $p>0.5$, direct from bisection
  - $O(N^{2p})$ switches
    - top switch box is $O(N^{2p})$
      - $N^{2p}(1/2^p + 1/2^{2p})$
    - switches at one level down is
      - $2 \times (N/2)^{2p}(1/2^p + 1/2^{2p})$
      - $2 \times (1/2^{p})^2 \times (N^{2p}(1/2^p + 1/2^{2p}))$
      - $2 \times (1/2^{p})^2 \times$ previous level
  - Total switches:
    - $N^{2p} \times (1 + 2^{1-2p} + 2^{2(1-2p)} + 2^{3(1-2p)} + \ldots)$
    - get geometric series; sums to $O(1)$
    - $N^{2p} \times ((1 - 2^{1-2p})^{-1})$
    - $= 2(2p^{-1})/(2(2p^{-1}) - 1) \times N^{2p}$
Good News

- Good news
  - asymptotically optimal
  - Even without switches, area $O(N^{2p})$
    - so adding $O(N^{2p})$ switches not change

Bad News

- Switches area >> wire crossing area
  - Consider $8\lambda$ wire pitch $\Rightarrow$ crossing $64\lambda^2$
  - Typical (passive) switch $\Rightarrow$ $2500\lambda^2$
  - Passive only: $40\times$ area difference
    - worse once rebuffer or latch signals.
  - and switches limited to substrate
    - whereas can use additional metal layers for wiring area

Additional Structure

- This motivates us to look beyond crossbars
  - can depopulate crossbars on up-down connection without loss of functionality?

Can we do better?

- Crossbar too powerful?
  - Does the specific down channel matter?
- What do we want to do?
  - Connect to any channel on lower level
  - Choose a subset of wires from upper level
    - order not important

$N$ choose $K$

- Exploit freedom to depopulate switchbox
- Can do with:
  - $K \times (N-K+1)$ switches
  - $Vs. \ K \times N$
  - Save $\sim K^2$

$N$-choose-$M$

- Up-down connections
  - only require concentration
    - choose $M$ things out of $N$
      - i.e. order of subset irrelevant.
  - Consequent:
    - can save a constant factor $\sim 2^p/(2^p-1)$
      - $(N/2)^p \times N^p \ vs \ (N^p - (N/2)^p+1)(N/2)^p$
      - $p=2/3 \Rightarrow 2^{2p}/(2^p-1) \approx 2.7$
  - Similary, Left-Right
    - order not important $\Rightarrow$ reduces switches
Multistage Switching

- We can route any permutation w/ less switches than a crossbar
- If we allow switching in stages
  - Trade increase in switches in path
  - For decrease in total switches

Butterfly

- Log stages
- Resolve one bit per stage

What can a Butterfly Route?

- 0000 → 0001
- 1000 → 0010

Butterfly Routing

- Cannot route all permutations
  - Get internal blocking

What required for non-blocking network?
Decomposition

- Pick a link to route.
- Route to destination over red network
- At destination,
  - What can we say about the link which shares the final stage switch with this one?
  - What can we do with this link?
- Route that link
  - What constraint does this impose?
  - So what do we do?

Decomposed Routing

Switches: $N/2 \times 2 \times 4 + (N/2)^2 < N^2$

Recurse

If it works once, try it again…

Result: Beneš Network

- $2\log_2(N)$ stages (switches in path)
- Made of $N/2$ 2x2 switchpoints
  - (4 switches)
- $4N \times \log_2(N)$ total switches
- Compute route in $O(N \log(N))$ time

Beneš Network Wiring

- Bisection: $N$
- Wiring $\Rightarrow O(N^2)$ area (fixed wire layers)
Beneš Switching

• Beneš reduced switches
  – $N^2$ to $N\log(N)$
  – using multistage network
• Replace crossbars in tree with Beneš switching networks

Beneš Switching

• Implication of Beneš Switching
  – still require $O(W^2)$ wiring per tree node
    • or a total of $O(N^2p)$ wiring
  – now $O(W \log(W))$ switches per tree node
    • converges to $O(N)$ total switches!
  – $O(\log^2(N))$ switches in path across network
    • strictly speaking, dominated by wire delay ~$O(N^p)$
    • but constants make of little practical interest except for very large networks 😊

Better yet…

• Believe do not need Beneš on the up paths
• Single switch on up path
• Beneš for crossover
• Switches in path:
  - $\log(N)$ up
  - $\log(N)$ down
  - $2\log(N)$ crossover
  - Total switches: $4\log(N)$
  - $O(\log(N))$

Linear Switch Population

• Can further reduce switches
  – connect each lower channel to $O(1)$ channels in each tree node
  – end up with $O(W)$ switches per tree node

Linear Switch Population

• Linear Switch (p=0.5)
Linear Population and Beneš

- Top-level crossover of $p=1$ is Beneš switching

Beneš Compare

- Can permute stage switches so local shuffles on outside and big shuffle in middle

Linear Consequences: Good News

- Linear Switches
  - $O(\log(N))$ switches in path
  - $O(N^{2p})$ wire area
  - $O(N)$ switches
  - More practical than Beneš crossover case

Linear Consequences: Bad News

- Lacks guarantee can use all wires
  - as shown, at least mapping ratio > 1
  - likely cases where even constant not suffice
  - expect no worse than logarithmic
- Finding Routes is harder
  - no longer linear time, deterministic
  - open as to exactly how hard

Mapping Ratio

- Mapping ratio says
  - if I have W channels
    - may only be able to use $W/MR$ wires
      - for a particular design's connection pattern
    - to accommodate any design
      - for all channels
  - physical wires $\geq MR \times$ logical
- Example:
  - Shows MR=3/2
  - For Linear Population, 1:1 switchbox
Area Comparison

Both:
p = 0.67
N = 1024

M-choose-N
perfect map
Linear
MR = 2

Area Comparison

• Since
  – switch >> wire
• may be able to
tolerate MR > 1
• reduces switches
  – net area savings
• Empirical:
  – Never seen greater than 1.5

Expander Theory

\((\alpha, \beta)\)-expansion
– Any group of size \(k = \alpha N\) will expand
  connect to a group of size \(\beta k = \beta \alpha N\) in each
  logical direction

[Arora, Leighton, Maggs
SIAM Journal of Comp. v25n3p600 1996]

Expander Idea

• IF we can achieve expansion
  – Can guarantee non-blocking at each stage
• i.e.
  – Guarantee use less than \(\alpha N\)
  – Guarantee connections to more stuff in
    next level
  – Since \(\beta \alpha N > \alpha N\) available in next level
    • Guaranteed to be an available switch

Dilated Switches

• Have multiple outputs per logical
direction
  – Dilation: number of outputs per direction
  – E.g. radix 2 switch w/ 4 outputs
    • 2 per direction
    • Dilation 2

Up (0)
Down (1)
Dilated Switches allow Expansion

- On Right
  - Any pair of nodes connects to 3 switches
- Strictly speaking must have $d>2$ for expansion

Random Wiring

- Random, dilated wiring for butterfly can achieve

$$d > \beta + 1 + \frac{\beta + 1 + \ln 2\beta}{\ln \left(\frac{2\beta}{\beta + 1}\right)}$$

$$2d > 2\beta + 1 + \frac{2\beta + 1 + \ln 2\beta}{\ln \left(\frac{2\beta}{\beta + 1}\right)}$$

- For tree... $2 \rightarrow 2^p$ (?)

Constraints

- Total load should not exceed $\alpha$ of net
  - $L = \text{mapping ratio} (\text{light loading factor})$
  - $\alpha LW = \text{number into each subtree}$
  - $L \geq 1/(2\alpha)$
- Cannot expand past the size of subtree
  - $\beta \leq N/2^p$
  - $\beta \alpha \leq \frac{W}{2^p}$

Extra Switches

- Extra switch factor: $d \geq L$
- Try:
  - $\beta = 2, \alpha = 1/10$
  - $d = 8$
  - $dL = 40$ (p=1)
- Try:
  - $\beta = 1.01, \alpha = 1/4, d = 6, L = 2$  $dL = 40$ (p=1)
  - $\beta = 1.01, \alpha = 1/4, d = 6, L = 2.8$  $dL = 40$ (p=0.5)

Putting it Together

- Base, linear-population trees have $O(N)$ switches
- Make larger by a factor of $L$ (linear factor)
- Dilated version have a factor of $d$ more switches
- Randomly wired expander
  - Can have $O(N)$ switches
  - Guarantee routes
  - Constants < 100 (looks like < 20)
  - Open: how tight can make it?

Big Ideas

- In addition to wires, must have switches
  - Have significant area and delay
- Rent’s Rule locality reduces
  - both wiring and switching requirements
- Naïve switches match wires at $O(N^{2p})$
  - switch area $>>$ wire area
  - prevent benefit from multiple layers of metal
Big Ideas
[MSB Ideas]

• Can achieve O(N) switches
  – plausibly O(N) area with sufficient metal layers
• Switchbox depopulation
  – save considerably on area (delay)
  – will waste wires
  – May still come out ahead (evidence to date)