Today

- Partitioning
  - why important
  - practical attack
  - variations and issues

Motivation (1)

- Divide-and-conquer
  - trivial case: decomposition
  - smaller problems easier to solve
    - net win, if super linear
    - \( \text{Part}(n) + 2 \times T(n/2) < T(n) \)
  - problems with sparse connections or interactions
  - Exploit structure
    - limited cutsize is a common structural property
    - random graphs would not have as small cuts

Motivation (2)

- Cut size (bandwidth) can determine area
- Minimizing cuts
  - minimize interconnect requirements
  - increases signal locality
- Chip (board) partitioning
  - minimize IO
- Direct basis for placement

Bisection Bandwidth

- Partition design into two equal size halves
- Minimize wires (nets) with ends in both halves
- Number of wires crossing is **bisection bandwidth**
  - lower bw = more locality

Interconnect Area

- Bisection is lower-bound on IC width
  - Apply wire dominated
  - (recursively)
Classic Partitioning Problem

- **Given:** netlist of interconnect cells
- Partition into two (roughly) equal halves (A,B)
- minimize the number of nets shared by halves
- “Roughly Equal”
  - balance condition: \((0.5-\delta)N \leq |A| \leq (0.5+\delta)N\)

Balanced Partitioning

- NP-complete for general graphs
  - [ND17: Minimum Cut into Bounded Sets, Garey and Johnson]
  - Reduce SIMPLE MAX CUT
  - Reduce MAXIMUM 2-SAT to SMC
  - Unbalanced partitioning poly time
- Many heuristics/attacks

KL FM Partitioning Heuristic

- Greedy, iterative
  - pick cell that decreases cut and move it
  - repeat
- small amount of non-greediness:
  - look past moves that make locally worse
  - randomization

Fiduccia-Mattheyses (Kernighan-Lin refinement)

- Start with two halves (random split?)
- Repeat until no updates
  - Start with all cells free
  - Repeat until no cells free
    - Move cell with largest gain (balance allows)
    - Update costs of neighbors
    - Lock cell in place (record current cost)
    - Pick least cost point in previous sequence and use as next starting position
- Repeat for different random starting points

Efficiency

Tricks to make efficient:
- Expend little (O(1)) work picking move candidate
- Update costs on move cheaply [O(1)]
- Efficient data structure
  - update costs cheap
  - cheap to find next move

Ordering and Cheap Update

- Keep track of Net gain on node == delta net crossings to move a node
  - cut cost after move = cost - gain
- Calculate node gain as \(\Sigma\) net gains for all nets at that node
  - Each node involved in several nets
- Sort nodes by gain
FM Cell Gains
Gain = Delta in number of nets crossing between partitions
= Sum of net deltas for nets on the node

After move node?
- Update cost
  - Newcost = cost - gain
- Also need to update gains
  - on all nets attached to moved node
  - but moves are nodes, so push to
    • all nodes affected by those nets

Composability of Net Gains

FM Recompute Cell Gain
- For each net, keep track of number of cells in each partition [F(net), T(net)]
- Move update: (for each net on moved cell)
  - if T(net) = 0, increment gain on F side of net
    • (think -1 => 0)
  - if T(net) = 1, decrement gain on T side of net
    • (think 1 => 0)
  - decrement F(net), increment T(net)
FM Recompute Cell Gain

- Move update: (for each net on moved cell)
  - if T(net) == 0, increment gain on F side of net
  - if T(net) == 1, decrement gain on T side of net
  - decrement F(net), increment T(net)
  - if F(net) == 1, increment gain on F cell

FM Recompute Cell Gain

- Move update: (for each net on moved cell)
  - if T(net) == 0, increment gain on F side of net
  - if T(net) == 1, decrement gain on T side of net
  - decrement F(net), increment T(net)
  - if F(net) == 1, increment gain on F cell
  - if F(net) == 0, decrement gain on all cells (T)

FM Recompute Cell Gain

- For each net, keep track of number of cells in each partition [F(net), T(net)]
- Move update: (for each net on moved cell)
  - if T(net) == 0, increment gain on F side of net
  - decrement F(net), increment T(net)

FM Recompute (example)

[note markings here are deltas...earlier pix were absolutes]
FM Recompute (example)

FM Optimization Sequence (ex)

FM Data Structures
- Partition Counts A,B
- Two gain arrays
  - One per partition
  - Key: constant time cell update
- Cells
  - successors (consumers)
  - inputs
  - locked status

Binned by cost → constant time update

FM Running Time?
- Randomly partition into two halves
- Repeat until no updates
  - Start with all cells free
  - Repeat until no cells free
    - Move cell with largest gain
    - Update costs of neighbors
    - Lock cell in place (record current cost)
  - Pick least cost point in previous sequence and use as next starting position
- Repeat for different random starting points
**FM Running Time**

- **Claim**: small number of passes (constant?) to converge
- Small (constant?) number of random starts
- N cell updates each round (swap)
- Updates K + fanout work (avg. fanout K)
  - assume K-LUTs
- Maintain ordered list O(1) per move
  - every io move up/down by 1
- Running time: O(K^2N)
  - Algorithm significant for its speed (more than quality)

**Weaknesses?**

- Local, incremental moves only
  - hard to move clusters
  - no lookahead
- Looks only at local structure

**Improving FM**

- Clustering
- Technology mapping
- Initial partitions
- Runs
- Partition size freedom
- Replication

Following comparisons from Hauck and Boriello ’96

**Clustering Benefits**

- Catch local connectivity which FM might miss
  - moving one element at a time, hard to see move whole connected groups across partition
- Faster (smaller N)
  - METIS -- fastest research partitioner exploits heavily
  - FM work better w/ larger nodes (?)?
How Cluster?

- Random
  - cheap, some benefits for speed
- Greedy “connectivity”
  - examine in random order
  - cluster to most highly connected
  - 30% better cut, 16% faster than random
- Spectral (next time)
  - look for clusters in placement
  - (ratio-cut like)
- Brute-force connectivity (can be $O(N^2)$)

LUT Mapped?

- Better to partition before LUT mapping.
  - When IO limited

Today: maybe a case for crude placement before LUT mapping? --- something to explore.

Initial Partitions?

- Random
  - Pick Random node for one side
    - start imbalanced
    - run FM from there
  - Pick random node and Breadth-first search to fill one half
  - Pick random node and Depth-first search to fill half
  - Start with Spectral partition

Initial Partitions

- If run several times
  - pure random tends to win out
  - more freedom / variety of starts
  - more variation from run to run
  - others trapped in local minima

Number of Runs

- 2 - 10%
- 10 - 18%
- 20 <20% (2% better than 10)
- 50 (4% better than 10)
- …but?
FM Starts?

21K random starts, 3K network -- Alpert/Kahng

Unbalanced Cuts

• Increasing slack in partitions
  – may allow lower cut size

Unbalanced Partitions

Following comparisons from Hauck and Boriello '96

Small/large is benchmark size not small/large partition IO.

Replication

• Trade some additional logic area for smaller cut size
  – Net win if wire dominated

Replication data from: Enos, Hauck, Sarrafzadeh '97

What Bisection doesn't tell us

• Bisection bandwidth purely geometrical
• No constraint for delay
  – I.e. a partition may leave critical path weaving between halves

Replication

• 5% ➔ 38% cut size reduction
• 50% ➔ 50+% cut size reduction
Critical Path and Bisection

Minimum cut may cross critical path multiple times.
Minimizing long wires in critical path => increase cut size.

So...

• Minimizing bisection
  – good for area
  – oblivious to delay/critical path

Partitioning Summary

• Decompose problem
• Find locality
• NP-complete problem
• linear heuristic (KLFM)
• many ways to tweak
  – Hauck/Boriello, Karypis
• even better with replication
• only address cut size, not critical path delay

Admin

• Assignment 3
  – Start early
  – Select a time on Friday to meet?
• No class Monday (2/25)
  – Next class Wednesday

Today's Big Ideas:

• Divide-and-Conquer
• Exploit Structure
  – Look for sparsity/locality of interaction
• Techniques:
  – greedy
  – incremental improvement
  – randomness avoid bad cases, local minima
  – incremental cost updates (time cost)
  – efficient data structures