Previously

- Cover (map) LUTs for minimum delay
  - solve optimally for delay → flowmap
- Retiming for minimum clock period
  - solve optimally

Today

- Solving cover/retime separately not optimal
- Cover+retime

Example

Cover with 4-LUTs

Example: Retimed
Example: Retimed

Note: only 4 signals here
(2 signals with 2 delays each)

Example 2

Cover with 4-LUTs

Example 2

Cycle Bound: 2

Example 2: retimed

Cycle Bound: 1

Basic Observation
- Registers break up circuit, limiting coverage
  - fragmentation
  - prevent grouping
Phase Ordering Problem

- General problem
  - don’t know effect of other mapping step
  - Will see this many places
- Here
  - don’t know delay if retime first
    - don’t know what can be packed into LUT
  - If we do not retime first
    - fragmentation: forced breaks at bad places

Observation #1

- Retiming flops to input of (fanout free) subgraph is trivial (and always doable)
  - Does not change I/O into subgraph

Observation #1: Consequence

- Can cover ignoring flop placement
- Then retime flops to input of gates

Fanout Problem?

Cannot retime without replicating.
Replicating increases I/O (so cut size).
…but I/O is what defined feasible covers for LUTs
3 cut → 5 cut

Observation #2

- Retiming flops to input of a subgraph with fanout may change the subgraph I/O
Different Replication Problem

What does this do to I/O?

Can retime and cover with single 4-LUT.

Different Replication Problem

Replication

- Once add registers
  - can’t just grab max flow and get replication
    - (compare flowmap)
  - Or, can’t just ignore flop placement when have reconvergent fanout through flop

Replication

- Key idea:
  - represent timing paths in graph
  - differentiating based on number of registers in path
  - new graph: all paths from node to output have same number of flip-flops
  - label nodes \( u^d \) where \( d \) is flip-flops to output

Deal with Replication

- Expanded Graph:
  - start with target output node
  - for each input \( u^d \) to current expanded graph
    - grab its input edge \((x \rightarrow u)\) with weight \((w(e))\)
    - add node \(x^{(d+w(e))}\) to graph (if necessary)
    - add edge \( x^{(d+w(e))} \rightarrow u^d\) with weight \((w(e))\)
  - continue breadth first until have enough
    - at most \( k \times n \) node depth required
Example

Build expanded graph

Example

Retime

4-LUT Cover
Expanded Graph

- Expanded graph does not have fanout of different flip-flop depths from the same node.
  - Captures IO after register retiming
- Can now cover ignoring flip-flops and trivially retime.

Labeling

- Key idea #1:
  - compute distances/delay like flowmap
    - Try collapse and compute flow cut
    - Dynamic programming to compute min delay covers
- Key idea #2:
  - count distance from register
    - like G-1/c graph

Labeling: Edge Weights

- To target clock period c
  - use graph G-1/c
  - paper:
    • assign weight -c*w(e)+1
    • (same thing scaled by c and negated)

Labeling: Edge Weight Idea

- same idea:
  - will need register ever c LUT delays
  - credit with registers as encounter
  - charge a fraction (1/c) every LUT delay
  - know net distance at each point
  - if negative (delays > c*registers)
    • cannot distribute to achieve c
  - otherwise
    • labeling tells where to distribute
Labeling: Flow cut

- Label node as before (flowmap)
  - $L(v) = \min\{l(u) + w(e) | \exists u \rightarrow v\}$
  - trivially can be $L(v) - 1/c \Rightarrow$ new LUT
- Correspond to flowmap case: $L(v) + 1$
- note min vs. max and $-1/c$ vs. $+1$ due to rescaling to match retiming formulation and $G^{-1}/c$ graph
- in this formulation, a combinational circuit of depth 4 would have $L(v) = -4/c$
- if can put this and all $L(v)$’s in one LUT
  - this can be $L(v)$
  - construct and compute flow cut to test

LUT Map and Retime

- Start with outputs
- Cover with LUT based on cut
  - move flip-flops to inputs of LUT
  - don’t have meaningful labels for covered nodes
  - Know can do this by expanded graph construction
- Recursively cover inputs
- Use label to retime
  - $r(v) = \lceil l(v) \rceil - 1$

Target Clock Period $c$

- As before (retiming)
  - binary search to find optimal $c$

Summary

- Can optimally solve
  - LUT map for delay
  - retiming for minimum clock period
- But, solving separately does not give optimal solution to problem
- Can solve problems together
  - Account for registers on paths
  - Label based on register placement and (flow) cover ignoring registers
  - Labeling gives delay, covering, retiming

Admin

- Office hours pushed back this week
  - T5:30pm
- Wednesday reading online

Today’s Big Ideas

- Exploit freedom
- Cost of decomposition
  - benefit of composite solution
- Technique:
  - dynamic programming
  - network flow