Address Allocation Models

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Clean Slate Research “Agenda”

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Some Level Setting

• What does “clean slate research” have to do with allocation of IP addresses?
  – Learn from past mistakes and try to avoid repeating them
  – Identify existing weaknesses and figure out how to exploit them to displace IP
  – As a matter of fact the IPv6 vs. IPv4 story holds many lessons that clean slate proposals can benefit from

• Any clean slate proposal will have to
  – Articulate its value over the incumbent technology
  – Evaluate the need for and trade-off associated with gateways
  – Set its price so as to be competitive
  – Understand the dynamics and ultimate targets (equilibrium points) of technology migration
Truth in Advertising

• I don’t claim to
  – Be an expert in clean slate proposals (no crystal ball, yet!)
  – Understand all/any of the issues associated with allocation of addresses, be they IPv4 or IPv6
  – Have a specific proposal for allocating new IPv6 addresses or the remaining IPv4 addresses
  – Understand the many ways in which address allocation can affect network neutrality
• I will attempt to
  – Outline a FIND (clean slate) project whose motivations may be relevant to the issue of address allocation
    • Present some initial findings that illustrate the kind of issues one may face when dealing with IPv4 and IPv6 address allocation
      • “Dynamics of technology diffusion in the presence of network externalities” – Joint work with K. Hosanagar, Y. Jin, A. Odlyzko, S. Sen and Z.-L. Zhang

Project Motivations

• Success of new technologies depends not only on their technical advantages, but also on economic factors
  – Many technologies have failed to widely deploy
  – Ex: IPv6, various QoS services

• How do we assess (design?) new network architectures that are not only technically superior but also economically viable?
Grand Objectives

• Identify key economic factors that influence design choices and trade-offs in developing, deploying and evolving network architectures

• Model the functional relationships between the economic factors and new technologies in network architecture designs

• Compare alternative network architectures in terms of their economic viability

Actual Research Topics

Identify the impact of incumbent technology on new network architecture adoption
  – Model the dynamics of technology adoption in a network setting

• Quantify the trade-off between diversity and integration in network offerings
  – Many services over a single network vs.
  – Many services over separate multiple networks and the impact of virtualization on those choices

• Assess the benefits of open and flexible network architectures
Parameters of Interest

- Intrinsic benefits of an architecture/technology
- Network externalities
  - From users of the same technology
  - Across technologies when converters/gateways are available
- Costs
  - Fixed cost: deployment cost
  - Variable cost: operation and maintenance cost
  - Switching costs (getting to learn a new technology)
  - How they vary over time (learning curve) and as a function of technology complexity
- Pricing
  - Initial settings and dynamic strategies
- Many if not all of these apply equally to IPv4→IPv6 migration

Our Focus and Initial Goals

- Develop a quantitative understanding of what can happen (dynamics and possible equilibria) when introducing a new network technology aimed at displacing an incumbent
  - And yes, that means models
- Identify possible outcomes and the parameters affecting them
  - How many equilibria, are they stable or not?
  - Effect of incumbent market penetration
  - Need for seeding of new technology
  - Sensitivity to initial pricing
  - Dynamics of technology adoption
  - When do technologies coexist vs. having one dominate?
- Non Goals
  - Not seeking "recipes" that can be readily applied to predict the outcome for specific technology configurations, e.g., IPv6 at $2/address wipes out IPv4 but at $5/address it never takes off...
First Step

- What does it take to displace a (strong) incumbent (IPv4) with a new, niftier (clean slate or IPv6) technology
  - Each technology delivers a certain intrinsic utility \((q_i, i=1,2)\) with presumably \(q_1 \leq q_2\), and charges a certain price \((p_i, i=1,2)\)
    - All these are \textit{generic} quantities with a common unit (no attempt – yet – at “dollaring” these quantities)
  - Users have individual preferences \((\theta)\) that shape their technology adoption behavior
    - User preferences have a certain (known) distribution, e.g., uniform
  - Technology 1 enjoys an existing market penetration when technology 2 is first introduced at time \(t=0\) \((x_1(0)>0, x_2(0)=0)\)
    - Network externalities increase utility of each technology in “proportion” to its number of adopters

- Model should capture the dynamics of technology adoption in this scenario
  - Identification of (stable) equilibrium points
  - Trajectory of equilibrium
  - More importantly, we should extract a better understanding/insight of what can happen and the key parameters affecting the outcome

Basic Notation

- Basic parameters
  - \(x_i\): fraction of technology \(i\) adopters \((0 \leq x_i \leq 1, i=1,2; x_1 + x_2 \leq 1)\)
  - \(\theta\): individual user preference (uniformly distributed in \([0,1]\))
  - \(q_i\): utility of technology \(i\)
  - \(v(x_i)\): network externality (assume \(v(x_i) = x_i\))
  - \(p_i\): price of technology \(i, i=1,2\)

- Utility of technology \(i\): \(U_i(\theta, x_i) = \theta q_i + x_i \cdot p_i, i=1,2\)

- User behavior (rational decision)
  \[
  \begin{cases} 
  \text{no technology} & \text{if } U_i < 0 \text{ for } i = 1,2 \\
  \text{technology } 1 & \text{if } U_1 > 0 \text{ and } U_1 > U_2 \\
  \text{technology } 2 & \text{if } U_2 > 0 \text{ and } U_2 > U_1 
  \end{cases}
  \]
Technology Adoption Model

- Indifference points for technology adoption
  \[ \theta_1^0 : U_1(\theta) \geq 0 \text{ if } \theta \geq \theta_1^0, \text{ users adopt technology 1} \]
  \[ \theta_2^0 : U_2(\theta) \geq 0 \text{ if } \theta \geq \theta_2^0, \text{ users adopt technology 2} \]
  \[ \theta_1^1 : U_2(\theta) \geq U_1(\theta) \text{ if } \theta \geq \theta_1^1, \text{ users prefer technology 2} \]

- Sample outcome

Technology Diffusion

- Let \( H(x), i=1,2 \) and \( x= (x_1, x_2) \) denote the "measure" of the set of adopters of technology \( i \) (how many users have adopted it)
  - At equilibrium, we must have \( H(x_i) = x_i, i=1,2 \)

- Diffusion of technology proceeds iteratively
  - In each interval of duration \( \Delta t \), \( x(t + \Delta t) = H(x(t)) \)
  - This can be used to define a differential equation

- Solution identifies different "regions" of the parameter space \( (p_1, q_1, p_2, q_2) \)
  - In each region we can
    - Characterize and validate candidate equilibrium points and determine if they are stable or unstable
    - Solve the differential equation to identify the trajectory of technology diffusion

- Most importantly, use this machinery to gain some insight into possible behaviors of technology competition
  - Some representative examples to follow
The Impact of Pricing – (1a)

- Two technologies
  - $q_1 = 2.95, p_1 = 1.01$
  - $q_2 = 5.5, p_2 = 2.57$
- Technology 2 prices itself out of (eventual) existence
  - Note that it does take off and gain some traction, but technology 1 is still growing faster and eventually wins
  - Outcome is independent of initial technology 1 penetration

The Impact of Pricing – (1b)

- Two technologies
  - $q_1 = 2.95, p_1 = 1.01$
  - $q_2 = 5.5, p_2 = 2.55$
- Technology 2 prices itself competitively
  - The two technologies converge to unhappy coexistence (roughly equal market shares)
  - Outcome is again independent of initial technology 1 penetration
The Impact of Pricing – (1c)

- Two technologies
  - \( q_1 = 2.95, p_1 = 1.01 \)
  - \( q_2 = 5.5, p_2 = 2.54 \)
- Technology 2 prices itself to win
  - Technology 1 continues growing for some time after the introduction of technology 2, but is eventually wiped out
  - Outcome is again independent of initial technology 1 penetration

Taking Stock – (1)

- A better technology does not always win
  - No surprise there
- A full range of possible outcomes
  - Either or both technology can survive
- Rapid transitions between different outcomes based on small price changes
  - \( p_2 = 2.54 \): only technology 1 survives
  - \( p_2 = 2.55 \): both technologies survive
  - \( p_2 = 2.57 \): only technology 2 survives
- The initial penetration of technology 1 did not affect the outcome
- Are these general conclusions or can we see different behaviors?
The Impact of Pricing – (2a)

- Two technologies
  - \( q_1 = 2.95, p_1 = 1.2 \) (higher)
  - \( q_2 = 5.1, p_2 = 2.7 \)
- Technology 2 again prices itself out of (eventual) existence
  - As before, it takes off, but grows more slowly than technology 1 which eventually wins
  - Outcome still independent of the initial penetration of technology 1

The Impact of Pricing – (2b)

- Two technologies
  - \( q_1 = 2.95, p_1 = 1.2 \) (higher)
  - \( q_2 = 5.1, p_2 = 2.55 \)
- The outcome now depends on the initial penetration of technology 1
  - Above a certain threshold (~0.4), it eventually prevails
  - Below the threshold, only technology 2 survives in spite of continued robust growth of technology 1 after technology 2 is first introduced
  - Note the presence of an unstable equilibrium \( \triangle \), where both technologies would have survived
The Impact of Pricing – (2c)

- Two technologies
  - \( q_1 = 2.95, p_1 = 1.2 \) (higher)
  - \( q_2 = 5.1, p_2 = 2.4 \)
- As in scenario 1, technology 2 now prices itself to be the only one to survive
  - Technology 1 continues to grow (slowly) for some time after the introduction of technology 2, but is eventually wiped out
  - Outcome is back to being independent of initial technology 1 penetration

Taking Stock – (2)

- We have now seen scenarios where both pricing and the initial penetration of the incumbent play a role
  - Technology 1 was the same as before, just a bit more expensive
  - Technology 2 was slightly less performant than in the previous examples
- Basically we went, using two mostly similar configurations, from an environment with a stable equilibrium where both technology co-existed to one where only one of the two technologies survived
One Last Example – (3a)

- Two technologies
  - \( q_1 = 0.3, p_1 = 0.5 \)
  - \( q_2 = 9.6, p_2 = 5.2 \)
- We now have a low-quality but cheap technology competing against a high-quality but expensive one
- The outcome depends on the initial penetration of the cheaper technology
  - Above a threshold, both technologies end-up coexisting and achieve full market penetration
  - Below the threshold only the better technology survives

What Is Missing? Lots of Things

- Time-varying technology quality and price
  - It gets better and cheaper over time
- Pricing that depends on the number of adopters
  - The more people are using the technology the cheaper it gets
- Profit model and profit maximization strategies
  - How to charge to maximize profit over a certain time period
- Dynamic pricing strategies
  - How does each technology react to maximize its chances of survivals and/or its profit
- Addition of gateways that deliver cross-technology externalities
- Validation
  - Identify existing/ongoing deployment scenarios on which to try to apply this, i.e., examples of prices, costs, qualities, etc.
  - And yes that means that we need more DATA!

- And the list goes on…
Conclusion

- Interactions of competing technologies with network externalities can give rise to a wide range of outcomes based on
  - Pricing, technology quality, level of penetration of the incumbent, etc.
- We are starting to develop some basic models to explore these complex interactions
  - Much work remains, but the end-result should offer improved insight of what to watch for or take into account when assessing how to best introduce new network technologies
- And yes, this might be applicable to IPv4-IPv6 migration