University of Pennsylvania

ESE 605 Modern Convex Optimization
Homework 7

Due on Thursday March 26 in class

1. *Robust least-squares with interval coefficient matrix:* An interval matrix in $R^{m\times n}$ is a matrix whose entries are intervals:

$$\mathcal{A} = \{A \in \mathbb{R}^{m\times n} || A_{ij} - \bar{A}_{ij}| \leq R_{ij}, \quad i = 1, \cdots, m, \;\; j = 1, \cdots, n\}.$$

The matrix $\bar{A} \in \mathbb{R}^{m\times n}$ is called the nominal value or center value, and $R \subset \mathbb{R}^{m\times n}$, which is element-wise nonnegative, is called the radius. The robust least-squares problem, with interval matrix, is

$$\text{minimize} \;\; sup_{A\in\mathcal{A}} ||Ax - b||_2,$$

with optimization variable $x \in \mathbb{R}^n$. The problem data are $\mathcal{A}$ (i.e., $\bar{A}$ and R) and $b \in \mathbb{R}^m$. The objective, as a function of x, is called the worst-case residual norm. The robust least-squares problem is evidently a convex optimization problem. (a) Formulate the interval matrix robust least-squares problem as a standard optimization problem, e.g., a QP, SOCP, or SDP. You can introduce new variables if needed. Your reformulation should have a number of variables and constraints that grows linearly with m and n, and not exponentially. (b) Consider the specific problem instance with $m = 5$, $n = 10$, with A generated in matlab as `Ā=round(10*rand(5,10)), b=round(5*randn(5,1))` . The matrix R should be chosen in such a way that the uncertainty in each entry of A is ± 0.1.

Find the solution x_{ls} of the nominal problem (i.e., minimize $||\bar{A}x - b||_2$), and robust least-squares solution x_{rls}. For each of these, find the nominal residual norm, and also the worst-case residual norm. Make sure the results make sense. You need to use CVX for this problem.

2. Assume the set $\mathcal{A}$ in problem 1 is given by $\bar{A} + U$, where $||U||_2 \leq 0.1$. Find the robust least squares solution with the same problem data as before. Find the worst case residual norm.