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Robust non-fragile LQ controllers: the static state feedback case

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This paper describes the synthesis of non-fragile or *resilient* regulators for linear systems. A general framework for fragility is described using state-space methodologies, and the LQ/$\mathcal{H}_2$ static state-feedback problem is examined in detail. We discuss the multiplicative structured uncertainties case, and propose remedies of the fragility problem using an optimization programming framework via matrix inequalities. A special case that leads to a convex optimization framework via linear matrix inequalities (LMIs) will be considered. The benchmark problem is taken as an example to show how special controller gain variations can affect the performance of the closed-loop system.

1. Introduction

In the literature there are different theoretical approaches and several computational techniques which treat the classical problem shown in figure 1:

Given a linear plant P with additive uncertainties ΔP, find a feedback controller K which internally stabilizes the family $P + \Delta P$ and satisfies a given performance measure.

In a recent paper, however, Keel and Bhattacharyya (1997) have shown that, in the case of unstructured uncertainties in the plant, the controllers resulting from either weighted $\mathcal{H}_\infty$, l or l_1 synthesis techniques exhibit a poor stability margin if not implemented exactly. This so-called 'fragility' is displayed even though these controllers are optimal when implemented using their nominal parameters. Another example of a compensator that cannot be exactly implemented is from Rosenthal and Wang (1996) where a dynamic controller is going to be designed in order to place the closed loop poles of a linear plant: it can be easily shown that, in one numerical example, the 15th digits numerical implementation of the controller matrices results in an unstable closed-loop system!

Keel and Bhattacharrya (1997) give the following suggestions to overcome the fragility problem:

(1) Develop synthesis algorithms which take into account the uncertainties in the controllers and search for the 'best' solution that guarantees a compromise between optimality and fragility;

(2) Parametrize the controller in an appropriate way (lower-order or fixed-structure controllers).

Haddad and Corrado (1997) address and solve a special case of the fragility problem by considering a *structured uncertain* dynamic compensator for a noise-driven linear plant: with the use of classical quadratic Lyapunov bounds (Bernstein and Haddad 1990), Haddad and Corrado (1997) obtain a controller which is proven to be 'resilient' in the sense that stability and some measure of performance are maintained even when the controller is not exactly implemented.

It is true that other authors have hinted at the problem of fragility (see, for example, Ackermann 1993, p. 75) and that many critics have dismissed the issue, since robust controllers are not designed to be resilient. On the other hand, the problem is reminiscent of the linear quadratic Gaussian (LQG) optimal controllers which were only useful when implemented on the exact plant, and had no guaranteed robustness margins if the plant was uncertain. This lack of robustness was corrected using linear quadratic Gaussian synthesis with loop transfer recovery (LQG/LTR) (Dorato *et al.* 1995). In addition, even robust controllers will eventually have to be implemented on an actual system using digital hardware, and should be resilient both to implementation errors and to tuning (Ackermann 1993).

The aim of this paper is to extend the ideas in Keel and Bhattacharyya (1997), Haddad and Corrado (1997) and, with reference to the scheme of figure 2, to analyse the robust fragility problem for a static full-state feedback controller synthesis problem by considering the combined effect of structured uncertainties in the plant and in the compensator. Note that it is reasonable to consider only structured uncertainties in the controller since the designer can exactly choose the structure even though he may not be able to implement that nominal configuration. The basic idea is that, instead of computing the controller as a single point in the parameters space, we look for a set of controllers allowing the par-

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