

Mathematical Foundations of Computer Science

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Homework 1

September 14, 2010; **Revised Due Date:** September 28, 2010

Beginning of class.

Problem 1. Give a proof of the proposition $Q \Rightarrow (P \Rightarrow (Q \Rightarrow P))$ in the system $\mathcal{N}_m^{\Rightarrow}$.

Problem 2. (1) Give a proof of the proposition $Q \Rightarrow (((P \Rightarrow Q) \Rightarrow P) \Rightarrow P)$ in the system $\mathcal{N}_m^{\Rightarrow}$.

(2) Give a proof of the proposition $P \Rightarrow (((P \Rightarrow Q) \Rightarrow P) \Rightarrow P)$ in the system $\mathcal{N}_m^{\Rightarrow}$.

What happens if you try to prove $((P \Rightarrow Q) \Rightarrow P) \Rightarrow P$? Keep your answer short!

Problem 3. Prove that if $P \Rightarrow Q$ and $Q \Rightarrow R$ are provable from the set of premises Γ in the system $\mathcal{N}_m^{\Rightarrow}$, then $P \Rightarrow R$ is also provable from Γ in $\mathcal{N}_m^{\Rightarrow}$.

What this means is that you have to describe a procedure that takes a deduction of $P \Rightarrow Q$ from the set of premises Γ and a deduction of $Q \Rightarrow R$ from the set of premises Γ and produces a deduction of $P \Rightarrow R$ from the set of premises Γ .

Problem 4. Give a proof of the proposition $(P \Rightarrow Q) \Rightarrow ((P \Rightarrow (Q \Rightarrow R)) \Rightarrow (P \Rightarrow R))$ in the system $\mathcal{N}_m^{\Rightarrow}$.