$\boxed{\textbf{Fall 2010 \quad CIS 160}}$

Mathematical Foundations of Computer Science
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Homework 2

September 28, 2010; Due October 5, 2010
Beginning of class

Problem 1. (a) Prove the "de Morgan" laws in classical logic:

$$\neg(P \wedge Q) \equiv \neg P \vee \neg Q$$
$$\neg(P \vee Q) \equiv \neg P \wedge \neg Q.$$

(b) Prove that $\neg(P \vee Q) \equiv \neg P \wedge \neg Q$ is also provable in intuitionistic logic.

(c) Prove that the proposition $(P \wedge \neg Q) \Rightarrow \neg(P \Rightarrow Q)$ is provable in intuitionistic logic and $\neg(P \Rightarrow Q) \Rightarrow (P \wedge \neg Q)$ is provable in classical logic.

Problem 2. (a) Prove that $P \Rightarrow \neg\neg P$ is provable in intuitionistic logic.

(b) Prove that $\neg\neg\neg P$ and $\neg P$ are equivalent in intuitionistic logic.

Problem 3 (a) Prove that the proposition

$$(P \vee \neg P) \Rightarrow (((P \Rightarrow Q) \Rightarrow P) \Rightarrow P)$$

is provable in intuitionistic logic.

(b) Prove that the proposition $((P \Rightarrow Q) \Rightarrow P) \Rightarrow P$ is classically provable.

Problem 4 (a) Prove that the proposition

$$\neg\neg(((P \Rightarrow Q) \Rightarrow P) \Rightarrow P)$$

is provable in intuitionistic logic.

(b) Use (a) to prove that the proposition $((P \Rightarrow Q) \Rightarrow P) \Rightarrow P$ is classically provable.

Problem 5. Prove that the rule

$$
\frac{\begin{array}{cc} \Gamma & \Delta \\ \mathcal{D}_1 & \mathcal{D}_2 \\ P \Rightarrow Q & \neg Q \end{array}}{\neg P}
$$

can be derived from the other rules of intuitionistic logic. This means that you have to describe how a deduction tree $\mathcal{D}_1$ for $P \Rightarrow Q$ from Γ and a deduction tree $\mathcal{D}_2$ for $\neg Q$ from Δ can be combined to obtain a deduction of $\neg P$ from $\Gamma \cup \Delta$.