Fall 2010 CIS 160

Mathematical Foundations of Computer Science
Jean Gallier

Homework 3

October 5, 2010 Due October 14, 2010
Beginning of class

Problem 1. (a) Show that if we assume that the proposition

$$P \Rightarrow (Q \Rightarrow R)$$

is an axiom for *all* propositions, P, Q, R, then *every proposition* becomes provable!

(b) Show that if P is provable (intuitionistically or classically), then $Q \Rightarrow P$ is also provable for *every* Q.

Problem 2. (a) Give intuitionistic proofs for the equivalences

$$\begin{aligned} P \vee P &\equiv P \\ P \wedge P &\equiv P \\ P \vee Q &\equiv Q \vee P \\ P \wedge Q &\equiv Q \wedge P. \end{aligned}$$

(b) Give intuitionistic proofs for the equivalences

$$\begin{aligned} P \wedge (P \vee Q) &\equiv P \\ P \vee (P \wedge Q) &\equiv P. \end{aligned}$$

Problem 3. Give intuitionistic proofs for the propositions

$$\begin{aligned} &P \Rightarrow (Q \Rightarrow (P \wedge Q)) \\ &(P \Rightarrow Q) \Rightarrow ((P \Rightarrow \neg Q) \Rightarrow \neg P) \\ &(P \Rightarrow R) \Rightarrow ((Q \Rightarrow R) \Rightarrow ((P \vee Q) \Rightarrow R)). \end{aligned}$$

Problem 4. Give intuitionistic proofs for

$$\begin{aligned} &(P \Rightarrow Q) \Rightarrow \neg\neg(\neg P \vee Q) \\ &\neg\neg(\neg\neg P \Rightarrow P). \end{aligned}$$