Fall 2010 CIS 160

Mathematical Foundations of Computer Science
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Homework 4

October 14, 2010; Due October 26, 2010, beginning of class

Problem 1. (a) Give intuitionistic proofs for the distributivity of $\wedge$ over $\vee$ and of $\vee$ over $\wedge$:

$$P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$$
$$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R).$$

(b) Give intuitionistic proofs for the associativity of $\wedge$ and $\vee$:

$$P \wedge (Q \wedge R) \equiv (P \wedge Q) \wedge R$$
$$P \vee (Q \vee R) \equiv (P \vee Q) \vee R.$$

Problem 2. Recall that in HW1 you proved that if $P \Rightarrow Q$ and $Q \Rightarrow R$ are provable, then $P \Rightarrow R$ is provable (intuitionistically). Deduce from this fact that if $P \equiv Q$ and $Q \equiv R$ hold, then $P \equiv R$ holds (intuitionistically or classically).

Prove that if $P \equiv Q$ holds then $Q \equiv P$ holds (intuitionistically or classically). Finally, check that $P \equiv P$ holds (intuitionistically or classically).

Problem 3. (a) Prove that if $P_1 \Rightarrow Q_1$ and $P_2 \Rightarrow Q_2$ are provable intuitionistically, then

1. $(P_1 \wedge P_2) \Rightarrow (Q_1 \wedge Q_2)$
2. $(P_1 \vee P_2) \Rightarrow (Q_1 \vee Q_2)$

are provable intuitionistically.

(b) Prove that if $Q_1 \Rightarrow P_1$ and $P_2 \Rightarrow Q_2$ are provable intuitionistically, then

1. $(P_1 \Rightarrow P_2) \Rightarrow (Q_1 \Rightarrow Q_2)$
2. $\neg P_1 \Rightarrow \neg Q_1$

are provable intuitionistically.

(c) Prove that if $P \Rightarrow Q$ is provable intuitionistically, then

1. $\forall t P \Rightarrow \forall t Q$

2. $\exists t P \Rightarrow \exists t Q$

are provable intuitionistically.

(d) Prove that if $P_1 \equiv Q_1$ and $P_2 \equiv Q_2$ are provable intuitionistically, then

1. $(P_1 \wedge P_2) \equiv (Q_1 \wedge Q_2)$

2. $(P_1 \vee P_2) \equiv (Q_1 \vee Q_2)$

3. $(P_1 \Rightarrow P_2) \equiv (Q_1 \Rightarrow Q_2)$

4. $\neg P_1 \equiv \neg Q_1$

5. $\forall t P_1 \equiv \forall t Q_1$

6. $\exists t P_1 \equiv \exists t Q_1$

are provable intuitionistically.