$\boxed{\textbf{Fall 2010} \quad \textbf{CIS 160}}$

Mathematical Foundations of Computer Science
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Midterm

October 14, 2010; 75mn

Problem 1. (20 points) Prove the proposition

$$\neg P \Rightarrow [(\neg P \Rightarrow (P \Rightarrow Q)) \wedge ((P \Rightarrow Q) \Rightarrow \neg P)],$$

intuitionistically.

Problem 2 (20 points). Prove the following fact: if

$$\begin{array}{c} \Gamma \\ \mathcal{D}_1 \\ P \vee Q \end{array} \qquad \text{and} \qquad \begin{array}{c} \Gamma, R \\ \mathcal{D}_2 \\ Q \end{array}$$

are deduction trees provable intuitionistically, then there is a deduction tree

$$\begin{array}{c} \Gamma, P \Rightarrow R \\ \mathcal{D}_3 \\ Q \end{array}$$

which is also provable intuitionistically.

Problem 3. (20 points) Give a classical proof of

$$\neg(P \Rightarrow \neg Q) \Rightarrow (P \wedge Q).$$

Problem 4. (20 points) An integer, $n \in \mathbb{Z}$, is divisible by 3 iff $n = 3k$, for some $k \in \mathbb{Z}$. Thus (by the division theorem), an integer, $n \in \mathbb{Z}$, is not divisible by 3 iff it is of the form $n = 3k + 1, 3k + 2$, for some $k \in \mathbb{Z}$ (You don't have to prove this.).

Prove that for any integer, $n \in \mathbb{Z}$, if n^2 is divisible by 3, then n is divisible by 3.

Hint. Prove the contrapositive: If n of the form $n = 3k + 1, 3k + 2$, then so is n^2 (for a different k).