Problem 1. (20 points) Prove the proposition

\[ \neg P \Rightarrow [ (\neg P \Rightarrow (P \Rightarrow Q)) \land ((P \Rightarrow Q) \Rightarrow \neg P) ] , \]

intuitionistically.

Problem 2 (20 points). Prove the following fact: if

\begin{align*}
\Gamma & D_1 P \lor Q \\
\Gamma R & D_2 Q
\end{align*}

are deduction trees provable intuitionistically, then there is a deduction tree

\begin{align*}
\Gamma, P & \Rightarrow R \\
D_3 & Q
\end{align*}

which is also provable intuitionistically.

Problem 3. (20 points) Give a classical proof of

\[ \neg(P \Rightarrow \neg Q) \Rightarrow (P \land Q) . \]

Problem 4. (20 points) An integer, \( n \in \mathbb{Z} \), is divisible by 3 iff \( n = 3k \), for some \( k \in \mathbb{Z} \). Thus (by the division theorem), an integer, \( n \in \mathbb{Z} \), is not divisible by 3 iff it is of the form \( n = 3k + 1, 3k + 2 \), for some \( k \in \mathbb{Z} \) (You don’t have to prove this.).

Prove that for any integer, \( n \in \mathbb{Z} \), if \( n^2 \) is divisible by 3, then \( n \) is divisible by 3.

*Hint.* Prove the contrapositive: If \( n \) of the form \( n = 3k + 1, 3k + 2 \), then so is \( n^2 \) (for a different \( k \)).