$\boxed{\textbf{Fall 2010} \quad \textbf{CIS 160}}$

Mathematical Foundations of Computer Science

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Practice Midterm

October 8, 2010; 75mn

Problem 1. (15 points) Prove the proposition

$$\neg P \Rightarrow (P \Rightarrow P \wedge Q),$$

intuitionistically or classically, whichever is easier for you.

Problem 2 (20 points). Prove the following fact: if

$$\frac{\Gamma}{P \vee Q}$$

is a deduction tree that is provable intuitionistically, then the deduction tree

$$\frac{\Gamma, \neg P}{Q}$$

is also provable intuitionistically.

Problem 3. (25 points) Give a classical proof of

$$\neg(\neg P \vee Q) \Rightarrow P.$$

Problem 4. (20 points) An integer, $n \in \mathbb{Z}$, is divisible by 5 iff $n = 5k$, for some $k \in \mathbb{Z}$. Thus (by the division theorem), an integer, $n \in \mathbb{Z}$, is not divisible by 5 iff it is of the form $n = 5k+1, 5k+2, 5k+3, 5k+4$, for some $k \in \mathbb{Z}$ (You don't have to prove this.).

Prove that for any integer, $n \in \mathbb{Z}$, if n^2 is divisible by 5, then n is divisible by 5.

Hint. Prove the contrapositive: If n of the form $n = 5k+1, 5k+2, 5k+3, 5k+4$, then so is n^2 (for a different k).