CIS160
Typesetting Proof Trees Using prooftree.tex

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1 Building a Proof Tree

The TeX macros in prooftree.tex can be used to construct proof trees. They have been adpated from macros created by Frank Pfenning (from CMU).

Proof trees are built up from the top down, starting with the leaves and ending with the root (at the bottom). To start growing a tree, use the command

\starttree{argument}

where argument is some expression, possibly in math mode. For example,

```
\starttree{A}
```

will create a one-node tree labeled A.

To actually make a tree, use the command

\settreee

You need to give an integer as argument to \settreee, which identifies the tree that you are building and then pass an actual tree as argument. For example,

```
\settree1{
  \starttree{A}
}
```

will create tree number 1 consisting of the node A.

The command

\settree i{

$\vdots$

}

builds a tree in a laTeX register called \box$i$. To actually display a tree, say tree number 1, use the command

\ligne{\hfill\box1\hfill}

This command displays the contents of \box1. The purpose of the \hfill's is to center the tree. Don't forget the \ligne command or else you will build a tree but you will never see it!

For example,

```
\settree1{
  \starttree{A}
}
\ligne{\hfill\box1\hfill}
```

will display the one-node tree labeled A:

A

It is usually preferable to add extra space just above the tree as follows:

```
\settree1{
  \starttree{A}
}

\bigskip
\ligne{\hfill\box1\hfill}
```

to get the tree:

A

Looking at a tree from the bottom-up, every non-leaf node has either 1, 2, or 3 immediate successors. Since trees are grown from top down, there are three kinds of instructions to construct a bigger tree from smaller trees.

(1) To add a new root node, use

\step

For example,

```
\settree1{
  \starttree{A}
  \step{B}
}

\bigskip
\ligne{\hfill\box1\hfill}
```

yields the tree:

$$\frac{\mathrm{A}}{\mathrm{B}}$$

and

```
\settree1{
  \starttree{A}
  \step{B}
  \step{C}
}

\bigskip
\ligne{\hfill\box1\hfill}
```

yields the tree:

$$\begin{array}{c}\mathrm{A}\\ \hline \mathrm{B}\\ \hline \mathrm{C}\end{array}$$

There are options to \step:

If you want to omit the horizontal bar, use \istep. Here is an example:

```
\settree1{
  \starttree{A}
  \istep{B}
  \step{C}
}

\bigskip
\ligne{\hfill\box1\hfill}
```

yields the tree:

$$\begin{array}{c}\mathrm{A}\\ \mathrm{B}\\ \hline \mathrm{C}\end{array}$$

If you want a justified step, use \jstep, with an argument. Here is an example:

```
\settree1{
  \starttree{A}
  \jstep{B}{a big step}
  \step{C}
}

\bigskip
\ligne{\hfill\box1\hfill}
```

yields the tree:

$$\begin{array}{c} \mathrm{A} \\ \hline \mathrm{B} \\ \hline \mathrm{C} \end{array} \quad \text{a big step}$$

(2) Construct a tree by joining two subtrees and adding a new root. For this, use \jointrees, with two integer arguments.

For example,

```
\settree1{
  \starttree{A}
  \jstep{B}{a step}
  \step{C}
}

\settree2{
 \starttree{D}
 \jstep{E}{a funny step}
}

\settree3{
\jointrees{1}{2}
\jstep{E}{voila}
}

\bigskip
\ligne{\hfill\box3\hfill}
```

yields the tree:

$$\dfrac{\dfrac{\dfrac{A}{B}}{C}\ \text{a step} \qquad \dfrac{D}{E}\ \text{a funny step}}{E}\ \text{voila}$$

Note that you can overwrite a tree with another tree. For example, instead of using the new tree numbered 3, we could have reused tree number 1, and then printed \box1 instead of \box3 as shown below:

```
\settree1{
  \starttree{A}
  \jstep{B}{a step}
  \step{C}
}

\settree2{
 \starttree{D}
 \jstep{E}{a funny step}
}

\settree1{
\jointrees{1}{2}
\jstep{E}{voila}
}

\bigskip
\ligne{\hfill\box1\hfill}
```

However, one has to be careful not to destroy subtrees that might be needed later. The normal amount of space between the two subtrees is 60pt. This can be reduced using:

\njointrees, a narrow join of 25pt

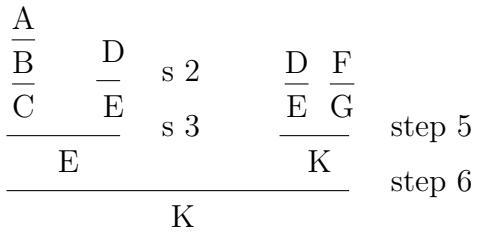
\vnjointrees, a very narrow join of 8pt

\snjointrees, a super narrow join of 2pt

Here is an example using these various commands:

```
\settree1{
 \starttree{A}
 \step{B}
 \step{C}
}

\settree2{
 \starttree{D}
 \jstep{E}{s 2}
}

\settree1{
 \njointrees{1}{2}
 \jstep{E}{s 3}
}

\settree2{
 \starttree{D}
 \step{E}
}

\settree3{
 \starttree{F}
 \step{G}
}

\settree2{
  \vnjointree{2}{3}
  \jstep{K}{step 5}
}

\settree1{
  \jointrees{1}{2}
  \jstep{K}{step 5}
}

\bigskip
\ligne{\hfill\box1\hfill}
```

yields



(3) Construct a tree by joining three subtrees and adding a new root. For this, use \tjointrees, with three integer arguments.

Here is an example:

```
\settree1{
  \starttree{A}
  \jstep{B}{a step}
  \step{C}
}

\settree2{
 \starttree{D}
  \jstep{E}{s 1}
}

\settree1{
\jointrees{1}{2}
\jstep{E}{s 2}
}

\settree2{
 \starttree{D}
  \jstep{E}{s 3}
}

\settree3{
 \starttree{F}
  \jstep{G}{step 4}
}

\settree4{
  \tjointrees{1}{2}{3}
  \jstep{K}{step 5}
}
```

```
\bigskip
\ligne{\hfill\box4\hfill}
```

yields the tree:

$$\dfrac{\dfrac{\dfrac{A}{B}\ \text{a step}}{C}\qquad \dfrac{D}{E}\ \text{s 1}}{E}\ \text{s 2}\qquad \dfrac{D}{E}\ \text{s 3}\qquad \dfrac{F}{G}\ \begin{matrix}\text{step 4}\\ \text{step 5}\end{matrix}$$
$$K$$

Observe that it was necessary to create another copy of tree 2 (as the second tree in joining three subtrees) because the previous copy was "consumed" when \jointrees{1}{2} was used to create the updated tree 1. Also note that \tjointrees does not leave much space for the justification of the subtrees. The normal spacing is 35pt. The spacing can be reduced using:

\tnjointrees, a very narrow ternary join of 20pt

\tsnjointrees, a super narrow ternary join of 10pt

Fine Points: When a variable is used, in math mode, as justification, it may appear to big. In this case, use

$\scriptstyle $

Here is an example illustrating this point:

```
\settree1{
     \starttree{$((R\impl R) \impl Q)^x$}
}
\settree2{
     \starttree{$R^z$}
     \step{$R$}
     \jstep{$R\impl R$}{$\scriptstyle z$}
}
\settree1{
     \njointrees{1}{2}
     \step{$Q$}
     \jstep{$((R\impl R)\impl Q) \impl Q$}{$\scriptstyle x$}
}

\bigskip
\ligne{\hfill\box1\hfill}
```

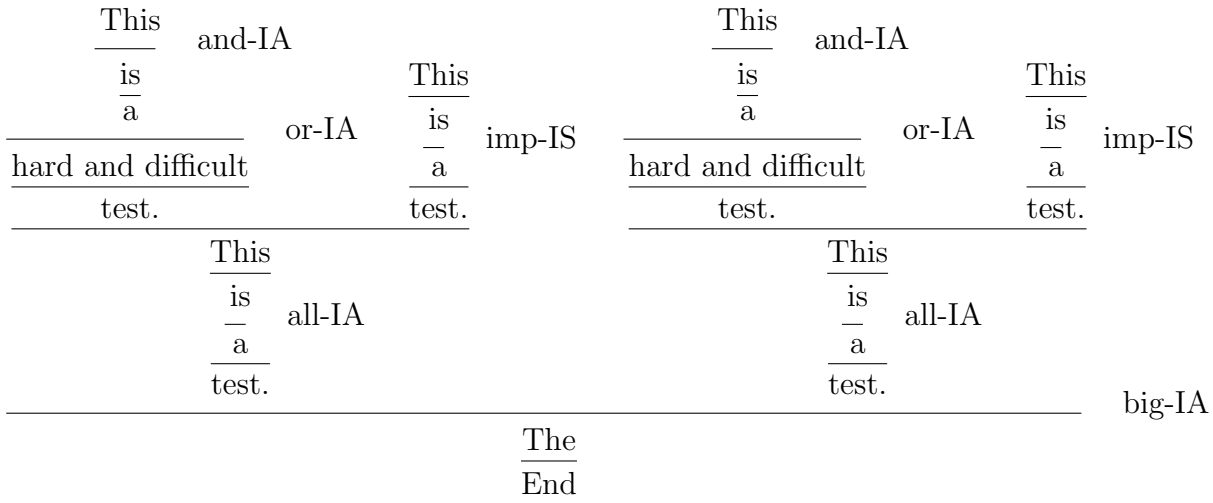
which yields the tree:

$$\dfrac{\dfrac{((R \Rightarrow R) \Rightarrow Q)^{x} \qquad \dfrac{\dfrac{R^{z}}{R}}{R \Rightarrow R}\ {\scriptstyle z}}{Q}}{((R \Rightarrow R) \Rightarrow Q) \Rightarrow Q}\ {\scriptstyle x}$$

Here is one more example taken from the file prooftree.tex:

```
\settree1{
       \starttree{This}
       \jstep{is}{and-IA}
       \step{a}
       \jstep{hard and difficult}{or-IA}
       \step{test.}}
\settree2{
       \starttree{This}
       \step{is}
       \jstep{a}{imp-IS}
       \step{test.}}
\settree1{
       \jointrees{1}{2}
       \step{This}
       \step{is}
       \jstep{a}{all-IA}
       \step{test.}}
\setbox2\copy1
\settree1{
       \jointrees{1}{2}
       \jstep{The}{big-IA}
       \step{End}}

\bigskip
\ligne{\hfill\box1\hfill}
```

which yields



This example uses the \setbox2\copy1 command which makes a copy of a tree (tree 1) as another tree (tree 2).