TCOM 370
Homework Set 4 1999

The following problems are to be turned in for grading on **Thursday March 4**

Problem 1

1997 Exam 1, # 6

Consider the following (5,2) linear block code. For each binary message $[d_1, d_0]$ the corresponding codeword is $[d_1, d_0, p_2, p_1, p_0]$. The appended bits are computed as:

$p_2=d_1+d_0$, $p_1=d_1$, $p_0=d_0$ ("+" is a modulo-2 sum)

(a) Write down all the codewords for this code, and determine the minimum Hamming distance d_{min} for this code.

(b) How many errors in a codeword is it guaranteed to correct?

(c) The bit error probability on the link used to transmit the codewords is $p_e=10^{-3}$. Find the maximum probability of error in decoding a 2-bit message using this code.

Problem 2

1997 Final Exam, # 5

(a) 8-bit data words are protected by a single parity check bit.
What is the minimum distance d_{min} for this block code?

(b) Another code for 8-bit data words uses one parity bit (bit 9) for the first 4 bits and a second parity bit (bit 10) for the next 4 bits of the data word.
What is the rate of this code, what is d_{min}, how many errors is it guaranteed to detect?

(c) Suppose that two errors occur in the transmission of a codeword.
What is the probability that the code of part (a) will detect the errors?
What is the probability that the code of part (b) will detect the errors?
(Each transmitted bit is subject to error with a probability of 10^{-4}, independently of other bits)

Problem 3
1998 Exam 2, # 5

The systematic Hamming (7,4) code for which d_{min}=3 is modified to generate codewords for a **(11,4) linear block code** as follows:

If a codeword of the (7,4) code is [**d**, **p**] = [d_3, d_2, d_1, d_0, p_2, p_1, p_0], then the codeword of the (11,4) code is obtained by *preceding* it with the message bits **d**=[d_3, d_2, d_1, d_0], forming codeword [**d**, **d**, **p**]

Let the *received* word be [**r**, **s**, **q**]. Here **r** and **s** are each 4 bits long, and **q** is 3 bits long. Without errors, [**r**, **s**, **q**] =[**d**, **d**, **p**]. Consider the following decoding rule:

Decode using *either* the last 7 bits [**s**, **q**] only, *or* the first 4 and last 3 bits [**r**, **q**] only, depending on which of these two 7-bit words is closer to a legitimate Hamming (7,4) codeword. If both are equally close to different codewords, an error is declared to have been detected only.

(a) For this rule, what patterns of two errors within its 11-bit codeword will the (11,4) code be able to *correct* ?

(Example answer: "one error in the first 2 bits and 1 error in the next 5 bits, or 2 errors in the 6th through 10th bits, will be corrected." *This is just an illustration, to indicate the nature of the answer that you should provide.)*

(b) Given that each bit is subject to error with a fixed probability independently of other bits, and *given* that 2 errors have occurred in a codeword, what is the probability that the 2 errors will be corrected?

Problem 4
1998 Final Exam, #4

Consider the systematic Hamming (7,4) linear block code for which d_{min}=3.
A (9,4) block code may be generated from this as follows:
If [**d**, **p**] = [d_3, d_2, d_1, d_0, p_2, p_1, p_0] is a codeword of the (7,4) code for message word **d**, then the codeword of the (9,4) code is [**d**, **p**, b_1, b_0], formed by appending bits b_1 and b_0. These bits are such as to make [**d**, b_1] and [**p**, b_0] both have even parity.

(a) Is the (9,4) code a linear block code? (Justify your answer.)

(b) What is d_{min} for this (9,4) code? (Justify your answer.)

(c) Suppose we define a (7,3) block code as follows. Each codeword [**d**, **p**] of the Hamming (7,4) code for which d_3=0 is used; the d_3 bit is converted into a parity check bit p_3 for the other 6 bits. The codeword is now [p_3, d_2, d_1, d_0, p_2, p_1, p_0] corresponding to message [d_2, d_1, d_0].
What is d_{min} for this code? Is the (9,4) code better than the (7,3) code? (Explain your answers).

Problem 5

Consider the Hamming (7,4) code. Let the probability of a bit error during transmission be given by p. Bits are treated independently by the channel, so that the correct or incorrect reception of one bit is independent of the outcome for any other bit.

(a) Suppose no coding is used, and let $p=10^{-3}$. Find in this case the probability that a block of 4 data bits will be received in error (one or more bits are erroneous).

(b) Now suppose the (7,4) Hamming code is used for the same link. Find the probability that the 4 data bits cannot be recovered correctly from a code-word that is transmitted (two or more bits in the codeword are erroneous.)

(c) By what factor must the message data rate be reduced, to allow use of this code compared to the uncoded case, for a fixed transmission-rate link?

Problem 6

The **extended** Hamming (8,4) code is obtained from the Hamming (7,4) code by adding an overall even parity bit to each codeword.

(a) What is the G matrix for this new code?

(b) What is dmin for this code?

(c) How many errors can it correct? How many can it detect?