TCOM 370 **April 23, 1999**

Solutions to Homework 99-8

Problem 1

$\Pr\{empty\ slot\} = 0.1 = e^{-G} \Rightarrow G = 2.3$

$\Pr\{collided\ slots\} = 0.6$

$\Pr\{success\} = 1 - 0.1 - 0.6 = 0.3$

But we know that $\Pr\{success\} = Ge^{-G}$, and if we use the value of G (=2.3) calculated using Pr{empty slot}, we get $\Pr\{success\} = 2.3e^{-2.3} \approx 0.23$ which does not equal 0.3. Thus Poisson process assumption is not valid.

Problem 2

(a)

$$\eta = \Pr\{success\} = \Pr\{\text{A transmits and B does not transmit}\} \text{ or } \{\text{A does not transmit and B transmits}\}$$
$$= p_a(1-p_b) + (1-p_a)p_b = p_a + p_b - 2p_a p_b$$

(b) The maximum value of η is achieved when p_a=1 and p_b=0, or p_a=0 and p_b=1. In either case, $\eta = 1$. This is not a desirable operating point since only one of the workstations can transmit.

(c) $\eta\big|_{p_a=0.3} = 0.3 + p_b - 0.6p_b = 0.3 + 0.4p_b$

The function is linear, so the maximum is achieved for $p_b = 1$. Thus $\eta_{\max} = 0.7$. B can get through in 70% of the slots (whenever A is not transmitting). A can never transmit successfully, and its buffer will overflow eventually.

(d) $\eta\big|_{p_a=0.5} = 0.5 + p_b - 2 \times 0.5 \times p_b = 0.5$ η does not depend on p_b. If p_b=0, then 50% of the slots are used by A, and 50% of the slots are empty, so $\eta = 50\%$. As p_b increases, some transmission attempts by A will result in collision, but some transmission attempts by B will be successful, and the overall η will remain at 50%. In the extreme case in which p_b=1, 50% of the slots will result in collision, and 50% of B's transmission attempts will be successful.

Problem 3

Pure ALOHA

$Total\ load = load\ per\ station \times number\ of\ stations$

$\frac{30\,\text{frames/sec} \times 2000\,\text{bits/frame}}{100\,\text{Mbps}} \times 1000 = 0.6$ Therefore, $\eta_{requested} = 0.6 > \eta_{\max} = 0.18$

Pure ALOHA cannot be used.

Slotted ALOHA

$\eta_{requested} = 0.6 > \eta_{\max} = 0.368$

Slotted ALOHA also cannot be used. Recall that in a slotted ALOHA system, all nodes must be synchronized. In this setup, we assume that all nodes are synchronized such that frames sent by different nodes arrive within one frame time at the receiver (different nodes may have different starting times for the same frame).

Ethernet

First make sure that collision can be detected in this system.

Propagation time $\tau = \dfrac{1000m}{2.25\times10^{8}\, m/\sec} = 4.44\mu\sec$

Transmission time $T_{ix} = \dfrac{2000\,\text{bits}}{100\,\text{Mbps}} = 20\mu\sec$

Since $T_{ix} > 2\tau$, all collisions will be detected.

$$\eta_{Ethernet} = \frac{1}{1+5a} = \frac{1}{1+5\frac{T_p}{T_{ix}}} = \frac{1}{1+0.56} = 0.64 > \eta_{requested}$$

So Ethernet can be used.

Problem 4

Propagation delay $T_p = \dfrac{1500\text{m}}{2.25\times10^{8}\,\text{m/sec}} = 6.67\mu\sec$

Transmission time $T_{ix} = \dfrac{800\text{bits}}{10\times10^{6}\,\text{bits/sec}} = 80\mu\sec$

$$a = \frac{T_p}{T_{ix}} = \frac{6.67}{80} = 8.33\times10^{-2} \Rightarrow \eta_{\max} = \frac{1}{1+5a} \approx 0.706 \Rightarrow \text{Effective throughput} = 7.06\text{Mbps}$$

Total offered load must be less than the effective throughput of the link. Thus,

$$200\times800\times\lambda \le 7.06\times10^{6}$$

$$\lambda \le 44.125\,\text{frames/sec}$$

Problem 5

New system

Propagation delay $T_p = \dfrac{2400\text{m}}{2.25\times10^{8}\,\text{m/sec}} = 10.67\mu\sec$

Transmission time $T_{ix} = \dfrac{800\text{bits}}{20\times10^{6}\,\text{bits/sec}} = 40\mu\sec$

$$a_{new} = 0.266 \Rightarrow R_{eff,new} = 20\frac{1}{1+5a_{new}} = 8.58\text{Mbps}$$

Original system

Propagation delay $T_p = \dfrac{2400\text{m}}{2.25\times10^{8}\,\text{m/sec}} = 10.67\mu\sec$

Transmission time $T_{ix} = \dfrac{800\text{bits}}{10\times10^{6}\,\text{bits/sec}} = 40\mu\sec$

$$a_{orig} = 0.133 \Rightarrow R_{eff,orig} = 10\frac{1}{1+5a_{orig}} = 6.0\text{Mbps}$$

$$\frac{R_{eff,new}}{R_{eff,orig}} = \frac{8.58}{6.0} = 1.43$$

So the increase in throughput is only 43% not close to 100% as claimed by the engineer.

Problem 6

(a) With only 1 uplink, the system is standard slotted ALOHA.

Maximum efficiency = $\frac{1}{e} \approx 0.368$ Therefore, maximum throughput = $2\,Mbps \times 0.368 = 0.736\,Mbps$

This occurs with G=1 packet/slot average load

$$\Pr\{\text{empty slot}\} = e^{-G} = \Pr\{1\,\text{packet in slot}\} = Ge^{-G} = e^{-1} \approx 0.368$$

$$\Pr\{\text{collision}\} = 1 - 2e^{-1} \approx 0.264$$

(b) The answers would not change. The throughput is determined by one uplink only. The PC's will offer twice as many packets to reach the maximum throughput.

(c)

$$\Pr\{\text{success}\} = \Pr\{\text{both uplinks successful}\} + 2\Pr\{\text{one slot successful and one slot not successful}\}$$

$$= P_s^{\,2} + 2P_s(1-P_s)$$

where Ps is the maximum probability of success (0.368 calculated earlier). Thus $\Pr\{\text{success}\} \approx 0.6$, and the maximum throughput = $2\,Mbps \times 0.6 = 1.2\,Mbps$

Problem 7

(a) LAN efficiency = $\frac{1}{1+5a}$ $\quad a = \frac{T_p}{T_{ix}}$ $\quad T_p = \frac{2250}{2.25\times10^8} = 10^{-5}\text{ sec}$

$$T_{ix} = \frac{1000}{10\times10^6} = 10^{-4}\text{ sec}$$

$$\eta_{LAN} = 2/3$$

Effective bit rate on LAN = $\frac{2}{3}\times 10\,Mbps$

$\therefore$ average # packets/sec = $\frac{2}{3}\times\frac{10\times10^6}{10^3} \approx 6666\,\text{packets/sec}$

(b) Idle RQ utilization = $\frac{T_{ix}}{T_{ix}+T_a+2T_p} = \frac{1}{1+0.5+2a}$

$T_p = \frac{15\times10^3}{3\times10^8} = 5\times10^{-5}\text{ sec}$ $\qquad T_{ix} = 10^{-4}$ $\qquad a = \frac{T_p}{T_{ix}} = 0.5$

$U = \frac{0.9}{2.5} = 0.36 \Rightarrow$ effective throughput $= 0.36 \times 10\,Mbps = 3.6\,Mbps$

$\therefore$ wireless link is the bottleneck.

(c) Suppose the distance is "d" giving some "a" for wireless link. We want utilization U of the wireless link to be 2/3.

If $K \geq 1 + 2a\,(a \leq 1 \text{since } K = 3)$, then $U = \frac{1 - P_f}{1 + 2aP_f} = \frac{0.9}{1 + 0.2a} = \frac{2}{3} \Rightarrow a = \frac{0.35}{0.2}$ not £ 1 as we assumed.

So try $K < 1 + 2a\,(a > 1)$, then $U = \frac{K}{1+2a}\frac{1-P_f}{1-P_f + KP_f} = \frac{3}{1+2a}\frac{0.9}{0.9+0.3} = \frac{2}{3} \Rightarrow a = \frac{19}{16}$

$\therefore\ T_p = \frac{19}{16}T_{ix} = \frac{19}{16}\times 10^{-4} \qquad d \leq \frac{19}{16}\times 3\times 10^{4} \approx 35625m$

(d) If R has frames to send, then S must use the LAN. Thus the rate at which S receives packets (from 100 nodes) is smaller. The wireless link will have less forward traffic, so S and R can be further apart.