TCOM 370 Homework 5 Solutions March 16, 1999

Problem 1 (#3 in 1998 Exam 2)

(Hand-out in class)

Problem 2 (#1 in 1997 Exam 2)

(a) Generate the FCS using long division:

0000000 $\rightarrow$ no remainder

1101000 $\rightarrow X^6 + X^5 + X^3$

$$\begin{array}{rl} & X^3 \\ X^3 + X^2 + 1 \big) & \overline{X^6 + X^5 + X^3} \\ & \underline{X^6 + X^5 + X^3} \\ & \qquad 0 \end{array}$$

1001000 $\rightarrow X^6 + X^3$

$$\begin{array}{rl} & X^3 + X^2 + X + 1 \\ X^3 + X^2 + 1 \big) & \overline{X^6 + 0X^5 + 0X^4 + X^3} \\ & \underline{X^6 + X^5 \qquad\qquad + X^3} \\ & \quad X^5 \\ & \quad \underline{X^5 + X^4 + X^2} \\ & \qquad X^4 + X^2 \\ & \qquad \underline{X^4 + X^3 + X} \\ & \qquad\quad X^3 + X^2 + X \\ & \qquad\quad \underline{X^3 + X^2 + 1} \\ & \qquad\qquad\quad X + 1 \quad (\textit{remainder}) \end{array}$$

0000	$\rightarrow$	0000000
1101	$\rightarrow$	1101000
1001	$\rightarrow$	1001011

(b) Note that CRC code is cyclic. All cyclic shifts of 1101000 generate total of 7 codewords, all of Hamming weight 3. All shifts of 1001011 generate 7 codewords of Hamming weight 4. All-zero codeword is the 15th codeword. The last codeword comes from all-one message 1111 for which the corresponding codeword has Hamming weight of at least 4. Therefore, the minimum Hamming weight of non-zero codewords is 3, as is the minimum Hamming distance (since CRC code is a linear block code).

(c)

(i) Received 0001111 $\rightarrow X^3 + X^2 + X + 1$

$$
\begin{array}{r}
1 \\
X^3 + X^2 + 1 \,\big)\, X^3 + X^2 + X + 1 \\
\underline{X^3 + X^2 + 1} \\
X \;\; (\textit{remainder}\; 010)
\end{array}
$$

Error detected!

(ii) In general, all error bursts of length 3 (degree of G(X)) or less will be detected.

Problem 3

$M(X) = X^{15}$

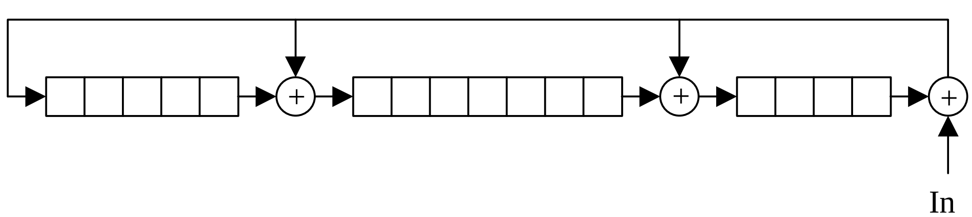
$G(X) = X^{16} + X^{12} + X^5 + 1$

$X^{16} M(X) = X^{31}$

$$
\begin{array}{l}
\qquad\qquad\qquad\qquad\qquad X^{15} + X^{11} + X^7 + X^4 + X^3 \\
X^{16} + X^{12} + X^5 + 1 \,\big)\, X^{31} \\
\underline{X^{31} + X^{27} + X^{20} + X^{15}} \\
X^{27} + X^{20} + X^{15} \\
\underline{X^{27} + X^{23} + X^{16} + X^{11}} \\
X^{23} + X^{20} + X^{16} + X^{15} + X^{11} \\
\underline{X^{23} + X^{19} + X^{12} + X^7} \\
X^{20} + X^{19} + X^{16} + X^{15} + X^{12} + X^{11} + X^7 \\
\underline{X^{20} + X^{16} + X^9 + X^4} \\
X^{19} + X^{15} + X^{12} + X^{11} + X^9 + X^7 + X^4 \\
\underline{X^{19} + X^{15} + X^8 + X^3} \\
X^{12} + X^{11} + X^9 + X^8 + X^7 + X^4 + X^3
\end{array}
$$

Thus, FCS=0001101110011000

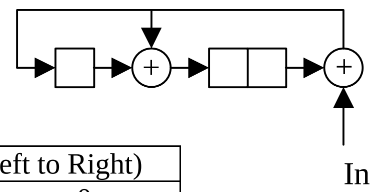
(We can use the following feedback shift register to generate the FCS.)



Problem 4

We can easily calculate FCS to be 101 using long division.

(a)



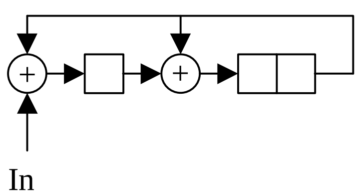
Encoder

Input	Feedback	Shift Register Contents (Left to Right)		
		0	0	0
0	0	0	0	0
1	1	1	1	0
0	0	0	1	1
1	0	0	0	1
0	1	1	1	0
1	1	1	0	1

FCS is again 101.

(b)

Decoder



Input	Feedback	Shift Register Contents (Left to Right)		
		0	0	0
0	0	0	0	0
1	0	1	0	0
0	0	0	1	0
1	0	1	0	1
0	1	1	0	0
1	0	1	1	0
1	0	1	1	1
0	1	1	0	1
1	1	0	0	0

Note that the remainder is 000, as there is no error.

Solution of Problem 3.12 in Textbook (was not assigned for homework)

(a)

0000 $\rightarrow$ 0000000
Any all-zero sequence divided by any G(X) results in remainder of 0

$$
\begin{array}{r|l}
 & \quad 1 \\
\cline{2-2}
X^3+X^2+1 & X^3 \\
 & \underline{X^3+X^2+1} \\
 & \quad X^2+1
\end{array}
$$

$\rightarrow$ 0001101
Data bits: 0001
FCS: 101

$$
\begin{array}{r|l}
 & \qquad X+1 \\
\cline{2-2}
X^3+X^2+1 & X^4 \\
 & \underline{X^4+X^3+X} \\
 & \quad X^3+X \\
 & \quad \underline{X^3+X^2+1} \\
 & \qquad X^2+X+1
\end{array}
$$

$\rightarrow$ 0010111
Data bits: 0010
FCS: 111

(b) Since we are working with a linear block code, we can generate all codewords from the 3 given in the problem. Using 0001101, we can generate 7 codewords all with Hamming weight of 3 by using cyclic shift. Using 0010111, we can generate 7 more codewords all with Hamming weight of 4. The last two codewords are 0000000 and 1111111.

Hamming distance is the minimum difference (in number of bits) between any two codewords. For this code, the Hamming distance is 3 since the minimum Hamming weight is 3.

(c) The code word for 1111 is 1111111.

Single bit error (assume 0111111 received)

$$
\begin{array}{r|l}
 & \qquad\qquad\qquad X^2+1 \\
\cline{2-2}
X^3+X^2+1 & X^5+X^4+X^3+X^2+X^1+1 \\
 & \underline{X^5+X^4+X^2} \\
 & X^3+X+1 \\
 & \underline{X^3+X^2+1} \\
 & X^2+X
\end{array}
$$

The remainder is non-zero, hence the error is detected.

Double bit error (assume 0011111 received)

$$X^3 + X^2 + 1 \overline{\big)\, X^4 + X^3 + X^2 + X + 1}^{\;X}$$

$$\underline{X^4 + X^3 + X}$$

$$X^2 + 1$$

The remainder is non-zero, hence the error is detected.

(d) Recall that the received codeword can be broken up into [transmitted codeword ⊕ E(X)], where E(X) represents the bits that are in error. If you choose E(X) that is a multiple of G(X) (e.g. 0001101), then the error will not be detected. For example, if 1111111 is sent, and errors represented by E(X)=0001101 occurs, then the received codeword is 1110010. The remainder when divided by G(X) is zero, so no error is detected.