TCOM 370 NOTES 99-4B

APPENDIX TO NOTES 4 :

EXAMPLE OFAMPLITUDE/PHASE DISTORTION

Consider the periodic rectangular pulse train of Notes 99-2, page 6, with period T=1, amplitude A=1, and duration of each pulse $t = 0.25$.

For this pulse train we found $a_0 = \frac{At}{T}$ and $a_n = \frac{2A}{n\pi}\sin(\pi n f_0 t)$, with $b_n = 0$

Figure 1 below (from Notes 99-2, page 7) shows the reconstruction of the pulse train using the a_0 + first 9 coefficients a_n of the Fourier series, that is, using up to the term a_9. (These coefficients are: a_0= 0.25 and a_1 through a_9 respectively 0.4502 0.3183 0.1501 0.0000 -0.0900 -0.1061 -0.0643 0.0000 0.0500)

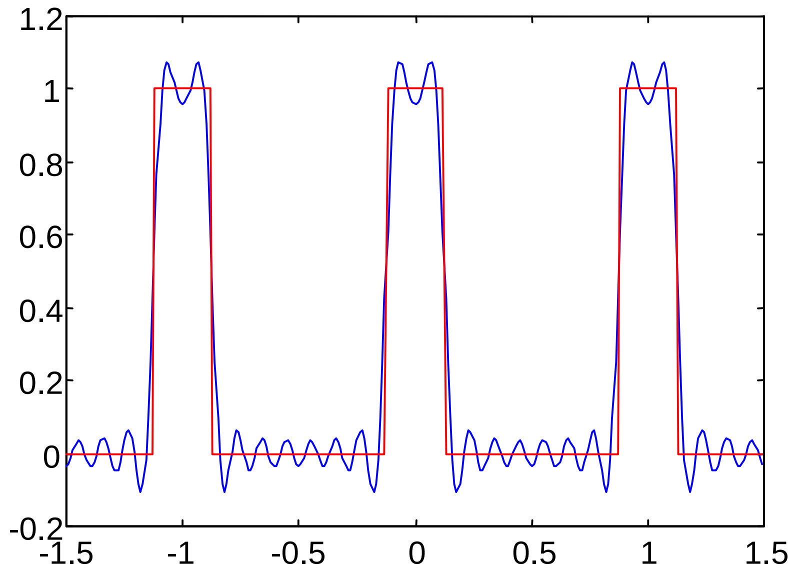
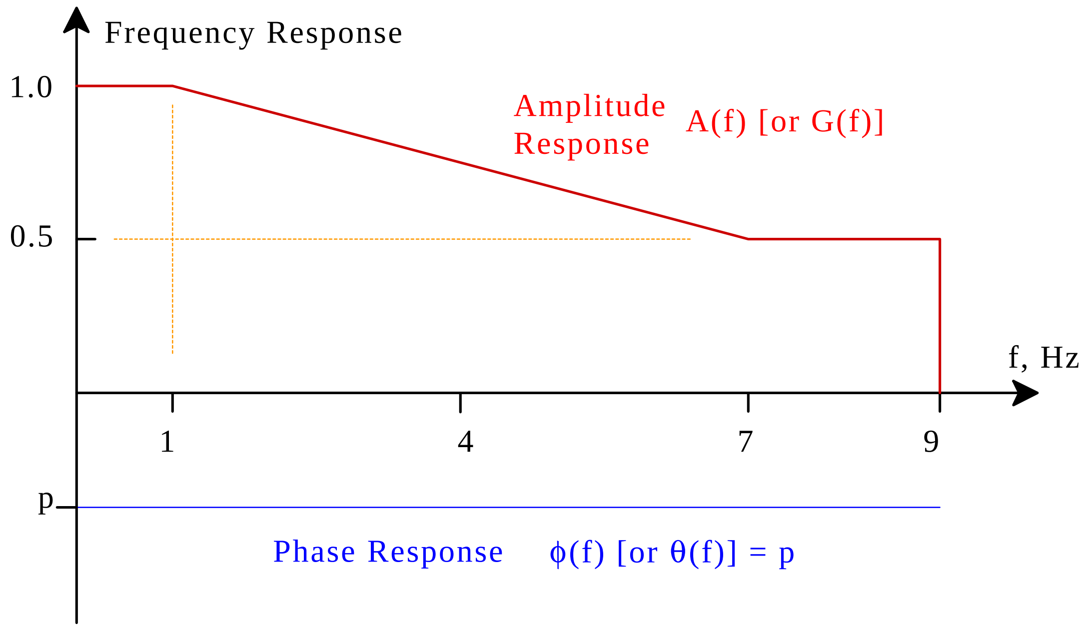


Figure 1

This corresponds to perfect transmission of a band of frequencies between 0 and 9 Hz only of the original pulse train (since f_0=1 Hz).

Amplitude Distortion Only:

Suppose the pulse train is passed through a fixed linear channel for which the frequency response is not ideal. Consider a frequency response shown below which attenuates higher frequencies more than lower ones, with the amplitude response remaining 1 between f=0 and 1 Hz, dropping linearly to 0.5 at f=7 Hz, and staying at this value to f=9 Hz, *beyond* which the response is 0 (i.e. no transmission beyond 9 Hz.).



Suppose that the phase response is ideal, and the constant "p" in the above plot is 0. This corresponds to a special case of a linear phase characteristic, that of a horizontal straight line passing through the origin. The delay is the same (0 delay) for all frequencies in this case.

Now each input frequency $a_n\cos(2\pi nf_0t)$ appears at the output with modified amplitude, as $G_na_n\cos(2\pi nf_0t)$ where G_n is the amplitude response $G(nf_0)$ of the channel at frequency nf_0=n Hz.

(Fine detail: Since the b_n are all 0 the amplitude of the n-th harmonic is $A_n=|a_n|$. Whether we use $a_n\cos(2\pi nf_0t)$ or $A_n\cos(2\pi nf_0t)$ is clearly immaterial when a_n is positive. When a_n is negative then $a_n\cos(2\pi nf_0t)=|a_n|\cos(2\pi nf_0t+\pi)$ and after modifying the amplitude by $G(nf_0)$ we get at the output again the result $G_na_n\cos(2\pi nf_0t)$ by absorbing π as a negative sign outside the cosine.)

For this channel we have G_0=1, G_1=1, G_2=1–0.5/6, G_3=1–1/6, G_4=1–1.5/6, G_5=1–2/6, G_6=1–2.5/6, G_7=0.5, G_8=0.5, G_9=0.5. The dc+9-coefficient reconstruction at the output of the channel is now

$$a_0 + a_1\cos(2\pi t) + G_2 a_2\cos(2\pi 2t) + G_3 a_3\cos(2\pi 3t) + G_5 a_5\cos(2\pi 5t) + G_6 a_6\cos(2\pi 6t) + 0.5 a_7\cos(2\pi 7t) + 0.5 a_9\cos(2\pi 9t);$$

Figure 2 shows this reconstruction. Note the slight widening of the pulses compared to those in Figure 1. The non-ideal amplitude response used here is not severe enough to cause a very significant distortion.

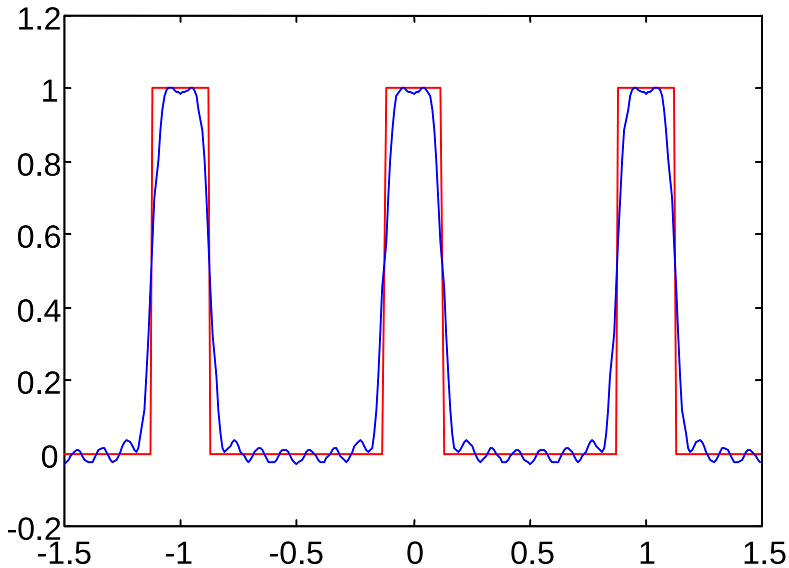


Figure 2

Amplitude and Delay (Phase) Distortion

Suppose the pulse train is passed through the same fixed linear channel as above (frequency response figure on page 2), but now let the constant phase shift imparted by the channel be $p= -0.2\pi$. This means that the delay for the f=1 Hz component is 0.1 and the delay for the f=9 Hz component is 0.011 (Note that $\cos(2\pi f[t-d])=\cos(2\pi ft - 2\pi fd)$, so that delay of d for an f Hz component is a phase shift of $-2\pi fd$. If the phase shift is the same for each frequency, then delay varies inversely with frequency).

The dc+9-coefficient reconstruction at the output of the channel is now

$$a_0 + a_1\cos(2\pi t-0.2\pi) + G_2a_2\cos(2\pi 2t-0.2\pi) + G_3a_3\cos(2\pi 3t-0.2\pi) + G_5a_5\cos(2\pi 5t-0.2\pi) + G_6a_6\cos(2\pi 6t-0.2\pi) + 0.5a_7\cos(2\pi 7t-0.2\pi) + 0.5a_9\cos(2\pi 9t-0.2\pi);$$

Figure 3 shows this reconstruction. The non-ideal phase response here causes a very significant delay distortion in addition to the amplitude distortion.

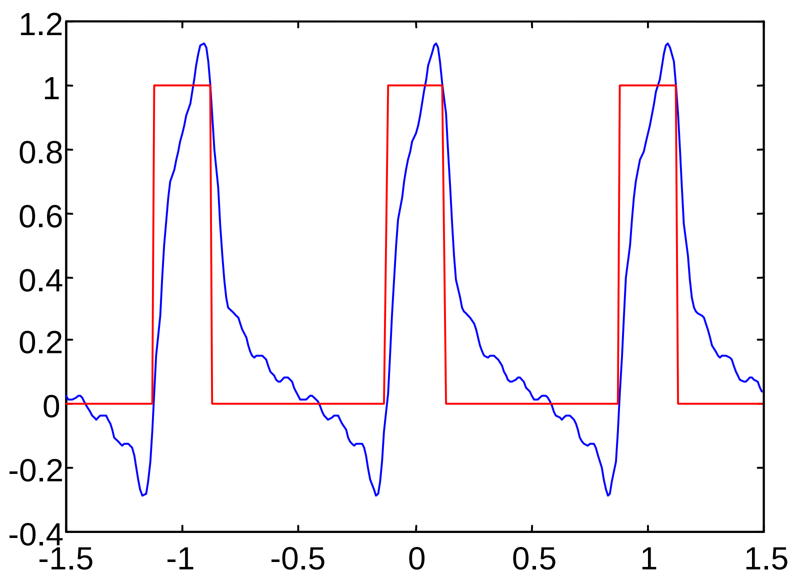


Figure 3