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B. SYMBOLS

<p>c specific heat, J/kg K</p> <p>Ei $Ei(x) \equiv \int_x^{\infty} \frac{e^{-s}}{s} ds, x > 0$</p> <p>$h_{sf}$ latent heat of fusion, J/kg K</p> <p>h enthalpy</p> <p>\bar{h} convective heat transfer coefficient, W/m²s</p> <p>k thermal conductivity, W/m K</p> <p>N_{Ste} Stefan number, $\equiv c(T_0 - T_f)/h_{sf}$</p> <p>$P$ pressure, Pa</p> <p>\mathcal{P} shape coefficient in Plank's equation</p> <p>q heat flux, J/m²s</p> <p>r radius, m</p> <p>R radial position of the melt-front, m</p> <p>\mathcal{R} shape coefficient in Plank's equation</p> <p>t time, s</p> <p>T temperature, K or °C</p>	<p>u velocity of the melt-front due to density change, m/s</p> <p>v specific volume, m³/kg</p> <p>V volume of the body, m³</p> <p>x coordinate</p> <p>X position of the melting front along the x-direction, m</p> <p>Greek Symbols</p> <p>α Thermal diffusivity, m²/s</p> <p>δ Melting front growth rate constant in the integral solution, Eq. (9-58)</p> <p>Δv_{sl} (Specific volume of the liquid phase) - (specific volume of the solid phase), m³/kg</p> <p>$\kappa \equiv \sqrt{\frac{\alpha_l}{\alpha_s}}$</p> <p>$\lambda$ Melting rate parameter, dimensionless</p> <p>$\mu \equiv \rho_l/\rho_s$</p>
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ρ Density, kg/m³
 χ Concentration, kg/kg

Subscripts

0 At the surface ($x=0$)
a Ambient (fluid surrounding the melting object)
f Fusion (melting or freezing)
i Initial (at $t=0$)
 ℓ Liquid
 $s\ell$ Phase-change from solid to liquid
o outer
s Solid
w at wall

C. REFERENCES

There always was an intense interest in predicting melting phenomena as related to such applications as food preservation, climate and its control, navigation, and materials processing, expanding with time into new areas such as power generation and medicine. Increasingly rigorous quantitative predictions started in the 19th century, and the number of published papers is in the tens of thousands. The main books and reviews on the topic, representative key general papers, as well as some of the references on appropriate thermophysical and transport properties, are listed as citations [1]-[191] below. A further rather extensive, yet not complete, list of references is given in subsection IV below under the specific topics in which melting plays an important role. In addition to the identification of past work on specific topics, this extensive list of references also helps identify various applications in which melting plays a role, and the journals which typically cover the field. While encompassing sources from many countries, practically all of the references listed here were selected from the archival refereed literature published in English. Many pertinent publications on this topic also exist in other languages.

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D. INTRODUCTION, APPLICATIONS, AND BASIC PHYSICAL CONCEPTS

Melting occurs naturally, such as with environmental ice and snow, and with the magma in the earth core. It is also a part of many technological processes, such as thawing following freeze-preservation of foodstuffs, snow and ice removal, manufacturing (such as in casting, molding, sintering, combustion synthesis, coating and electro-deposition, soldering, welding, high energy beam cutting and forming, crystal growth, electro-discharge machining, printing), thawing of cryo-preserved or cryosurgically treated organs, and thermal energy storage using solid/liquid phase-changing materials. A bibliography for these applications, with a brief introduction, is given in Subsection IV.

Melting is often accompanied by freezing, and the thermodynamics as well as transport principles of the two processes are very similar. Their mathematical treatment is therefore also similar. Specific discussion of freezing is given in Section 507.8 of the Databook.

In simple thermodynamic systems (i.e., without external fields, surface tension, etc.) of a pure material, melting of a solid occurs at certain combinations of temperature and pressure. Since pressure typically has a relatively smaller influence, only the melting (or "fusion") temperature is often used to identify this phase transition.

The conditions for melting are strongly dependent on the concentration when the material contains more than a single species. Furthermore, melting is also sensitive to external effects, such as electric and magnetic fields, in more complex thermodynamic systems.

The equilibrium thermodynamic system parameters during phase transition can be calculated from the knowledge that the partial molar Gibbs free energies (chemical potentials) of each component in the two phases must be equal (cf. Alexiades and Solomon [1], Hultgren *et al* [37], Kechin [40], Lior [46], Poulidakos [54]). One important result of using this principle for simple single-component systems is the Clapeyron equation relating the temperature (T) and pressure (P) during the transition from the solid to the liquid phase, viz.

$$\frac{dP}{dT} = \frac{h_{sl}}{T\Delta v_{sl}} \quad \text{Eq. (9-1)}$$

where h_{sl} is the enthalpy difference between the phases ($= h_l - h_s > 0$, the latent heat of melting) and Δv_{sl} is the specific volume difference between the phases ($= v_l - v_s$). Examination of Eq. (9-1) shows that increasing the pressure will result in an rise of the melting temperature if $\Delta v_{sl} > 0$ (i.e., when the specific volume of the liquid is higher than that of the solid, which is a property of tin, for example), but will result in a decrease of the melting temperature when $\Delta v_{sl} < 0$ (for water, for example). The latter case

explains why ice may melt under the pressure of a skate blade.

In some materials, called glassy, the phase change between the solid and liquid occurs with a gradual transition of the physical properties, from those of one phase to those of the other. When the liquid phase flows during the process, the flow is strongly affected because the viscosity decreases greatly as the solid changes to liquid. Other materials, such as pure metals and ice, and eutectic alloys, have a definite line of demarcation between the liquid and the solid, the transition being abrupt. This situation is easier to analyze and is therefore more rigorously addressed in the literature.

To illustrate the above-described gradual transition, most distinctly observed in mixtures, consider the equilibrium phase diagram for a binary mixture (or alloy) composed of species a and b , shown in Fig. 9-1. Phase diagrams, or equations describing them, become increasingly complicated as the number of components increases. χ is the concentration of species b in the mixture, ℓ denotes the liquid, s the solid, s_a a solid with a lattice structure of species a in its solid phase but containing some molecules of species b in that lattice, and s_b a solid with a lattice structure of species b in its solid phase but containing some molecules of species a in that lattice. "Liquidus" denotes the boundary above which the mixture is just liquid, and "solidus" is the boundary separating the final solid mixture of species a and b from the solid-liquid mixture zones and from the other zones of solid s_a and solid s_b .

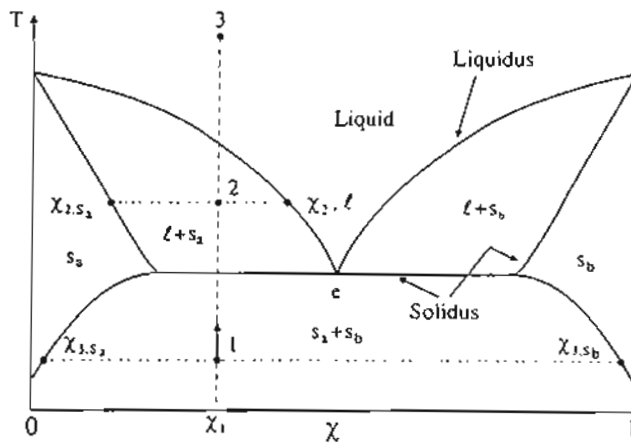


Figure 9-1. A Liquid-Solid Phase Diagram of a Binary Mixture

For illustration (Fig. 9-1), assume that a solid mixture or alloy of components s_a and s_b containing concentrations $(\chi_{1, s_a}$ and $\chi_{1, s_b})$ of species b , respectively is at point 1 and thus at temperature T_1 (Fig. 9-1). The above concentrations are those identified by the intersections of the horizontal dot-dash line passing through point 1, with the left and right wings of the solidus line, respectively. The ratio of the mass of the solid s_a to that of s_b is determined by the *lever rule*,

and is $(\chi_{1, s_b} - \chi_1)/(\chi_1 - \chi_{1, s_a})$ at point 1. This solid is then heated, ascending along the dashed line corresponding to χ_1 . When the temperature rises above the solidus line, melting starts, creating a mixture of liquid and of solid s_a . Such a two-phase mixture is called the *mushy zone*. At point 2 in that mushy zone, for example, the solid phase (s_a) portion contains a concentration χ_{2, s_a} of component b , and the liquid phase portion contains a concentration $\chi_{2, \ell}$ of component b . The ratio of the mass of the solid s_a to that of the liquid can again be determined by the lever rule, and is $(\chi_{2, \ell} - \chi_2)/(\chi_2 - \chi_{2, s_a})$ at point 2. Further heating to above the liquidus line, say to point 3, results in a liquid mixture having concentration χ_1 .

A unique situation occurs for heating along the line of concentration χ_e : the liquid formed has the same concentration as that of the solid mixture of $s_a + s_b$, and a two-phase (*mushy*) zone is not formed during the melting process. χ_e is called the *eutectic concentration*, and the solid mixture (or alloy) having that concentration is called a *eutectic*.

It is obvious from the above that the concentration distribution changes among the phases, which accompany the melting process (Fig. 9-1), are an important factor in the composition of alloys, and are the basis for freeze-separation processes.

The presence of a two-phase mixture zone with temperature-dependent concentration and phase-proportion obviously complicates heat transfer analysis, and requires the simultaneous solution of both the heat and mass transfer equations. Furthermore, the solid usually does not melt on a simple planar surface between the phases. This complicates the mathematical modeling of the process significantly. Further references to melting of multi-component systems are provided in Subsection IV.B.

Flow of the liquid phase often has an important role during melting (cf. Cheung and Epstein [13], Viskanta [69], Yao and Prusa [74], and references [76] - [106]). The flow may be forced, associated with the removal of the melt, and/or may be due to natural convection that arises whenever there are density gradients in the liquid, here generated by temperature and possibly concentration gradients. Under such circumstances, strong coupling may exist between the heat transfer and fluid mechanics, and also with mass transfer when more than a single species is present, and the process must be modeled by an appropriate set of continuity, momentum, energy, mass conservation, and state equations, which need to be solved simultaneously.

E. PREDICTIVE METHODS

The mathematical description of the melting process is characterized by non-linear partial differential equations, which have analytical (closed-form) solutions for only a few simplified cases. As explained above, the problem

becomes even less tractable when flow accompanies the process, or when more than a single species is present. A very large amount of work has been done in developing solution methods for the melting problem (sometimes also called the Stefan problem, after the seminal paper by Stefan [64]), published both as monographs and papers, and included in the list of references to this Section (Alexiades and Solomon [1], Bankoff [4], Chadam and Rasmussen [10], Cheng and Seki [12], Crank [15], Fasano and Primicerio [20], Hill [32], Özisik [50], Tanasawa and Lior [65], Yao and Prusa [74], and references [107] - [191], with emphasis on the reviews by Friedman [129], Fukusako and Seki [130], Meirmanov [153], Ockendon and Hodgkins [162], Rubinshtein [168], and Wilson *et al* [189]. Generically, solutions are obtained either by (1) linearizing the original equations (e.g., perturbation methods) where appropriate, and solving these linear equations, or (2) simplifying the original equations by neglecting terms, such as the neglect of thermal capacity in the "quasi-static method" described in Subsection III. below, or (3) using the "integral method," which satisfies energy conservation over the entire body of interest, as well as the boundary conditions, but is only approximately correct locally inside the body (in Subsection III.A. below), or (4) employing a numerical method.

Many numerical methods have been successfully employed in the solution of melting problems, both of the finite difference and element types, and many well-tested software programs exist that include solutions for that purpose. A significant difficulty in the formulation of the numerical methods is the fact that the liquid-solid interface moves and perhaps changes shape as melting progresses (making this a "moving boundary" or "free boundary" problem). This requires continuous monitoring of the interface position during the solution sequence, and adjustment of the numerical model cell or element properties to those of the particular phase present in them at the time-step being considered. Several formulations of the original equations were developed to simplify their numerical solution. One of them is the popular "enthalpy method" discussed in more detail in Subsection III.B. below.

The predictive equations provided below are all for materials whose behavior can be characterized as being pure. This would also apply to multi-component material where changes of the melting temperature and of the composition during the melting process can be ignored. General solutions for cases where these can not be ignored are much more difficult to obtain, and the readers are referred to the literature; some of the key citations are provided in the general reviews [1], [12], [28], [37], and [65], and are listed under the Multi-Component Systems and Crystals heading of Subsection IV.B.

Furthermore, the solutions presented here by closed-form equations are only for simple geometries, since no such

solutions are available for complex geometries. Simplified expressions, however, are presented for melting times also in arbitrary geometries.

II. PREDICTIVE EQUATIONS FOR MELTING

A. ONE-DIMENSIONAL MELTING OF PURE MATERIALS WITHOUT DENSITY CHANGE

Examination of the simplified one-dimensional case provides some important insights into the phenomena, identifies the key parameters, and allows analytical solutions and thus qualitative predictive capability for at least this class of problems. In this Section we deal with cases in which the densities of both phases is the same, and in which the melt does not flow, thus also ignoring, for simplification, the effects of buoyancy-driven convection which accompanies the melting process when a temperature gradient exists in the liquid phase. As stated in Subsection I.D., the effects of natural convection may sometimes be significant, and information about this topic can be found under the references quoted in that Subsection. Melting of non-opaque solids may also include internal radiative heat transfer, which is ignored in the equations presented below. Information about such problems is contained in references [11], [16], [25], [31], and [74]. The solutions presented below can be found in many books and reviews that deal with melting and freezing (cf. refs. [1], [4], [15], [32], [46], [74], [130], [153], and [168] and in textbooks dealing with heat conduction (cf. [50], and [52]).

1. Solutions for Materials that are Initially at the Melting Temperature

If the solid to be melted is initially at the melting temperature throughout its extent, as shown in Fig. 9-2, heat transfer occurs in the liquid phase only. This somewhat simplifies the solution and is presented first.

Consider a solid of infinite extent to the right ($x > 0$) of the infinite surface at $x = 0$ (i.e., semi-infinite), described in Fig. 9-2, initially at the fusion temperature T_f . For time $t > 0$ the temperature of the surface (at $x = 0$) is raised to $T_0 > T_f$ and the solid consequently starts to melt there. In this case the temperature in the solid remains constant, $T_s = T_f$, so the temperature distribution needs to be calculated only in the liquid phase. It is assumed that the liquid formed by melting remains motionless and in place, with the initial condition

$$T_L(x, t) = T_f \text{ in } x > 0, \text{ at } t = 0. \quad \text{Eq. (9-2)}$$

the boundary condition is

$$T_L(0, t) = T_0 \text{ for } t > 0. \quad \text{Eq. (9-3)}$$

and the liquid-solid interfacial temperature and heat flux continuity conditions

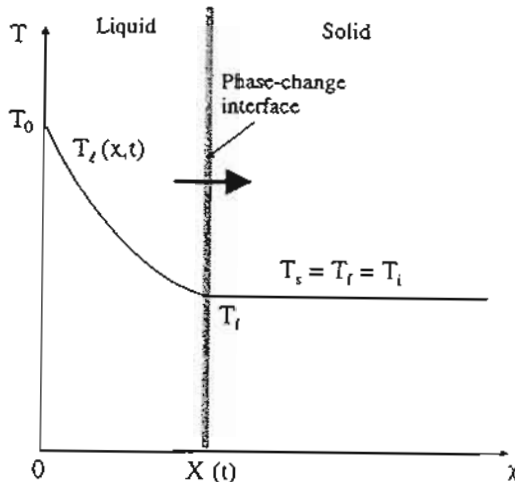


Figure 9-2. Melting of a semi-infinite liquid initially at the fusion temperature. Heat conduction takes place consequently in the liquid phase only.

$$T_l[X(t)] = T_f \text{ for } t > 0, \quad \text{Eq. (9-4)}$$

$$-k_l \left(\frac{\partial T_l}{\partial x} \right)_{[X(t)]} = \rho_l h_{sl} \frac{dX(t)}{dt} \text{ for } t > 0, \quad \text{Eq. (9-5)}$$

The analytical solution of this problem yields the temperature distribution in the liquid as

$$T_l(x,t) = T_0 - (T_0 - T_f) \frac{\operatorname{erf}\left(\frac{x}{2\sqrt{\alpha_l t}}\right)}{\operatorname{erf}\lambda'} \text{ for } t > 0, \quad \text{Eq. (9-6)}$$

where erf stands for the *error function* (described and tabulated in mathematical handbooks), and λ' is the solution of the equation

$$\lambda' e^{\lambda'^2} \operatorname{erf}(\lambda') = \frac{N_{Stel}}{\sqrt{\pi}}, \quad \text{Eq. (9-7)}$$

with N_{Stel} being the *Stefan Number*, here defined for the liquid as

$$N_{Stel} \equiv \frac{c_l(T_0 - T_f)}{h_{sl}}. \quad \text{Eq. (9-8)}$$

Equation (9-7) can be solved to find the value of λ' for the magnitude of N_{Stel} , which is calculated for the problem at hand by using Eq. (9-8). The solution of Eq. (9-7), yielding the values of λ' as a function of N_{Stel} for $0 \leq N_{Stel} \leq 5$, is given in Fig. 9-3.

The interface position (which is also the melting front progress) is defined by

$$X(t) = 2\lambda'(\alpha_l t)^{1/2}. \quad \text{Eq. (9-9)}$$

EXAMPLE

The temperature of the vertical surface of a large volume of solid paraffin wax used for heat storage, initially at the fusion temperature, $T_i = T_f = 28^\circ\text{C}$, is suddenly raised to 58°C . Any motion in the melt may be neglected. How long would it take for the paraffin to solidify to a depth of 0.1 m? Given properties: $\alpha_l = (1.09)10^{-7} \text{ m}^2/\text{s}$, $\rho_s = \rho_l = 814 \text{ kg/m}^3$, $h_{sl} = 241 \text{ kJ/kg}$, $c_l = 2.14 \text{ kJ/kg}^\circ\text{C}$. To find the required time we use Eq. (9-9), in which the value of λ' needs to be determined. λ' is calculated from Eq. (9-7), which requires the knowledge of N_{Stel} . From Eq. (9-8)

$$N_{Stel} = \frac{(2.14 \text{ kJ/kg}^\circ\text{C})(58^\circ\text{C} - 28^\circ\text{C})}{241.2 \text{ kJ/kg}} = 0.266.$$

The solution of Eq. (9-7) as a function of N_{Stel} is given in Fig. 9-3, yielding $\lambda' \approx 0.4$. Using Eq. (9-9), the time of interest is calculated by

$$t = \frac{[X(t)]^2}{4\lambda'^2\alpha_l} = \frac{(0.1\text{m})^2}{4(0.4)^2[(1.09)10^{-7} \text{ m}^2/\text{s}]} = (1.43)10^5 \text{ s} = 39.8\text{h}.$$

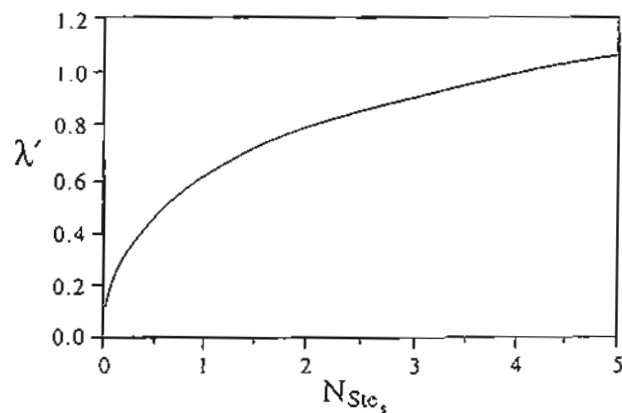


Figure 9-3. The Root λ' of Eq. (9-7)

2. Solutions for Materials that are Initially not at the Melting Temperature

a. Slab geometry

If, initially, the solid to be melted is below the melting temperature, conductive heat transfer takes place in both phases. Consider a semi-infinite solid initially at a temperature T_i lower than the melting temperature T_f (Fig. 9-4). At time $t = 0$ at the solid surface temperature at $x = 0$ is suddenly raised to a temperature $T_0 > T_f$, and maintained at that temperature for $t > 0$. Consequently, the solid starts to melt at $x = 0$, and the melting interface (separating in Fig. 9-4 the liquid to its left from the solid on its right) located at the position $x = X(t)$ moves gradually to the right (in the positive x direction).

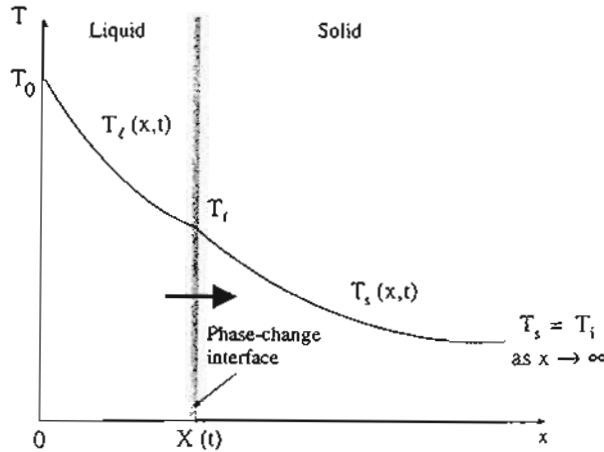


Figure 9-4. Melting of a semi-infinite solid initially at a below-melting temperature. Heat conduction takes place in both phases.

The analytical solution of this problem yields the temperature distributions in the solid and liquid phases, respectively, as

$$T_s(x,t) = T_f - (T_f - T_0) \frac{\operatorname{erfc}\left(\frac{x}{2\sqrt{\alpha_s t}}\right)}{\operatorname{erfc}\left(\lambda\sqrt{\alpha_l / \alpha_s}\right)}, \quad \text{Eq. (9-10)}$$

and

$$T_l(x,t) = T_0 - (T_0 - T_f) \frac{\operatorname{erf}\left(\frac{x}{2\sqrt{\alpha_l t}}\right)}{\operatorname{erf}\lambda}, \quad \text{Eq. (9-11)}$$

where erfc is the complementary error function, λ is a constant, obtained from the solution of the equation

$$\frac{e^{-\lambda^2}}{\operatorname{erf}\lambda} - \frac{k_s}{k_l} \sqrt{\frac{\alpha_l}{\alpha_s}} \left(\frac{T_f - T_0}{T_0 - T_f} \right) \frac{e^{-(\alpha_l/\alpha_s)\lambda^2}}{\operatorname{erfc}\left(\lambda\sqrt{\alpha_l / \alpha_s}\right)} = \frac{\lambda\sqrt{\pi}}{N_{St\ell}}, \quad \text{Eq. (9-12)}$$

where $N_{St\ell}$ is the Stefan number defined by Eq. (9-8).

Solutions of Eq. (9-12) are available for some specific cases in several of the references (cf. [1], [50], and can, in general, be obtained relatively easily by a variety of commonly-used software packages used for the solution of nonlinear algebraic equations.

The transient position of the melting interface is

$$X(t) = 2\lambda(\alpha_l t)^{1/2}, \quad \text{Eq. (9-13)}$$

where λ is the solution of Eq. (9-12), and thus the expression for the rate of melting, i.e. the velocity of the motion of the solid-liquid interface, is

$$\frac{dX(t)}{dt} = \lambda\alpha_l^{1/2} t^{-1/2}. \quad \text{Eq. (9-14)}$$

b. Cylinder

Very few exact solutions exist for melting of bodies that are shaped differently than slabs. This is an available exact solution for the somewhat practical problem of the melting of an axially-symmetric cylinder due to a line heat source in the center (Fig. 9-5), say an electric wire or heat-carrying pipe inside an insulator). It should be noted that natural convection of melt was found to have an important role in such problems (cf. Yao and Prusa [74]), and the solution shown can only be used for rough evaluation. Other phase-change problems in non-planar geometries are solved by approximate and numerical methods (Alexiades and Solomon [1], Cheung and Epstein [13], Yao and Prusa [74],

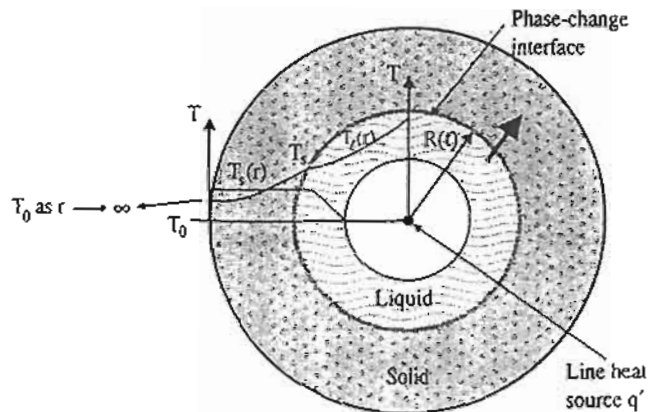


Figure 9-5. Outward Melting of an Initially Solid Cylinder at Temperature $T_0 < T_f$, Having a Line Heat Source of Intensity q' (W/m) Along its Axis

Caldwell and Chiu [115], Gupta and Kumar [134], Kern and Wells [143], Kim and Ro [145], Lazaridis [146], Li [149], Meyer [156-158], Rabin and Korin [164], Raw and Schneider [167], and Shamsundar and Sparrow [173]).

For an infinite cylinder having constant properties, initially solid at $T_0 < T_f$, with a line heat source of intensity q' (W/m) at $r = 0$, the initial conditions are

$$R(0) = 0, \quad T(r, 0) = T_0 < T_f, \quad \text{Eq. (9-15)}$$

and the far-field boundary condition is

$$\lim_{r \rightarrow \infty} T(r,t) = T_f. \quad \text{Eq. (9-16)}$$

The heat transfer energy balance at the line heat source is expressed by

$$\lim_{r \rightarrow 0} \left[-2\pi r k_l \frac{\partial T_l(r,t)}{\partial r} \right] = q' > 0. \quad \text{Eq. (9-17)}$$

The analytical solution of this problem (by similarity) gives the position of the outward-moving melt front as

$$R(t) = 2\lambda''(\alpha_t t)^{1/2}, \quad \text{Eq. (9-18)}$$

and the temperature distributions in the liquid and solid phases as,

$$T_t(r, t) = T_f + \frac{q'}{4\pi k_t} \left[\text{Ei} \left(-\frac{r^2}{4\alpha_t t} \right) - \text{Ei}(-\lambda''^2) \right] \quad \text{Eq. (9-19)}$$

in $0 < r < R(t)$ for $t > 0$,

$$T_s(r, t) = T_0 + (T_f - T_0) \frac{\text{Ei}(-r^2 / 4\alpha_s t)}{\text{Ei}(-\lambda''^2 \alpha_t / \alpha_s)} \quad \text{Eq. (9-20)}$$

in $R(t) \leq r$ for $t > 0$,

respectively, where Ei is a function defined by

$$\text{Ei}(x) \equiv \int_x^{\infty} \frac{e^{-s}}{s} ds, \quad x > 0, \quad \text{Eq. (9-21)}$$

and where λ'' is the root of the transcendental equation

$$\frac{q'}{4\pi} e^{-\lambda''^2} + \frac{k_s(T_f - T_0)}{\text{Ei}(-\lambda''^2 \alpha_t / \alpha_s)} e^{-\lambda''^2 \alpha_t / \alpha_s} = \lambda''^2 \alpha_t h_{st} \rho. \quad \text{Eq. (9-22)}$$

Solutions of Eq. (9-22) can, in general, be obtained relatively easily by a variety of commonly-used software packages for the solution of nonlinear algebraic equations.

B. ONE-DIMENSIONAL MELTING WITH DENSITY CHANGE

For most materials the density of the liquid and solid phases is somewhat different, usually by up to about 10% and in some cases up to 30%. Usually the density of the liquid phase is smaller than that of the solid one, causing volume expansion upon melting and shrinkage upon freezing. Solid metals and plastic materials that are confined would increase their volume during melting and may thus burst their enclosure. Water is one of the materials in which the density of the liquid phase is higher than that of the solid one, and thus the volume of the water is smaller than that of the ice from which it was formed by melting. If the densities of the liquid and solid phases differ, motion of the phase-change interface is not only due to the phase change process, but also due to the associated volume (density) change.

A reasonably good analytical solution for small ($\sim \pm 10\%$) solid-liquid density differences is available (Alexiades and Solomon [1]) for the semi-infinite slab at $x \geq 0$, initially solid at $T_i < T_f$, melted by imposing a constant temperature $T_0 > T_f$ at the surface $x = 0$. It is assumed that $\rho_t < \rho_s$, c_t , c_s , k_t , k_s , h_{st} , and T_f are constants and positive. The melt front $X(t)$ starts at $X(0) = 0$ and advances to the right (Fig. 9-4). Buoyancy-driven convection is ignored, but the liquid volume expansion upon melting is considered, in that it pushes the entire solid body also rightward without friction, at uniform speed $u(t)$ without motion in the liquid itself. The temperature distributions are, in the liquid and solid phases

$$T_t(x, t) = T_i - (T_i - T_f) \frac{\text{erf} \frac{x}{2\sqrt{\alpha_t t}}}{\text{erf}(\lambda''' \sqrt{\alpha_s / \alpha_t})} \quad \text{Eq. (9-23)}$$

in $0 \leq x \leq X(t)$ for $t > 0$,

$$T_s(x, t) = T_0 + (T_f - T_0) \frac{\text{erfc} \left[\frac{x}{2\sqrt{\alpha_s t}} - (1 - \mu) \kappa \lambda''' \right]}{\text{erfc}(\mu \kappa \lambda''')} \quad \text{Eq. (9-24)}$$

in $X(t) \leq x$ for $t > 0$.

The location of the melting front is

$$X(t) = 2\lambda'''(\alpha_t t)^{1/2}, \quad \text{Eq. (9-25)}$$

and the speed of the solid body motion due to the expansion is

$$u(t) = (1 - \mu) \lambda''' \sqrt{\alpha_t / t}. \quad \text{Eq. (9-26)}$$

In Eqs. (9-23) - (9-26) λ''' is the root of the equation

$$\frac{N_{St,t}}{\lambda''' e^{-\lambda'''^2} \text{erf} \lambda'''} - \frac{N_{St,s}}{(\mu \kappa \lambda''') e^{(\mu \kappa \lambda''')^2} \text{erfc}(\mu \kappa \lambda''')} = \sqrt{\pi}, \quad \text{Eq. (9-27)}$$

which can, for the specific problem parameters, be solved numerically or by using one of the many software packages for solving nonlinear algebraic equations. The remaining parameters are defined as

$$N_{St,t} \equiv \frac{c_t(T_0 - T_f)}{h_{st}}, \quad N_{St,s} \equiv \frac{c_s(T_f - T_i)}{h_{st}}$$

$$\kappa \equiv \sqrt{\frac{\alpha_t}{\alpha_s}}, \quad \mu \equiv \frac{\rho_t}{\rho_s} \quad \text{Eq. (9-28)}$$

Because of the approximate nature of this analytical solution it is expected that Eq. (9-25) slightly overestimates the position of the melt front. If $\rho_t > \rho_s$, melting will cause

the volume of formed liquid to become smaller than the volume formerly occupied by the solid, and may thus move the melting solid leftward, in a direction opposite to that of the melting interface, and the solution represented by Eqs. (9-15) - (9-20) would not be valid. Approximate, but less accurate solutions for this and other cases are described by Alexiades and Solomon [1] and Prusa and Yao [54], and additional results are given by Shamsundar and Sparrow [59].

C. THE QUASI-STATIC APPROXIMATION

To obtain rough estimates of melting processes quickly, in cases where heat transfer takes place in only one phase, it is assumed that effects of sensible heat are negligible relative to those of latent heat ($N_{Ste} \rightarrow 0$). This is a significant simplification, since the energy equation then becomes independent of time, and solutions to the steady-state heat conduction problem are much easier to obtain. At the same time, the transient phase-change interface condition [such as Eq. (9-5)] is retained, allowing the estimation of the transient interface position and velocity. This is hence a quasi-static approximation, and its use is shown below. The simplification allows solution of melting problems in more complicated geometries. Some solutions for the cylindrical geometry are presented below. More details can be found in refs. [1], [15], [32], [117], [130], and [177].

It is important to emphasize that these are just approximations, without full information on the effect of specific problem conditions on the magnitude of the error incurred when using them. In fact, in some cases, especially with a convective boundary condition, they may produce very wrong results. It is thus necessary to examine the physical viability of the results, such as overall energy balances, when using these approximations.

It is assumed here that the problems are one-dimensional, and that the material is initially at the freezing temperature T_f .

1. Examples of the Quasi-Static Approximation for Melting of a Slab

Given a semi-infinite solid (Fig. 9-2), on which a *time-dependent temperature* $T_0(t) > T_f$ is imposed at $x = 0$, the above-described quasi-static approximation yields the solution for the position of the phase-change front and of the temperature distribution in the liquid, respectively, as

$$X(t) = \left\{ 2 \frac{k_l}{\rho h_{sl}} \int_0^t [T_0(t) - T_f] dt \right\}^{1/2} \quad \text{for } t \geq 0, \quad \text{Eq. (9-29)}$$

$$T_l(x, t) = T_0(t) - [T_0(t) - T_f] \frac{x}{X(t)} \quad \text{Eq. (9-30)}$$

$$\text{in } 0 \leq x \leq X(t) \text{ for } t \geq 0.$$

The heat flux needed for melting, $q(x, t)$, can easily be determined from the temperature distribution in the liquid [Eq. (9-23)].

This approximate solution is equal to the exact one when $N_{Ste} \rightarrow 0$, and it otherwise overestimates the values of both $X(t)$ and $T(x, t)$. While the errors depend on the specific problem, they are confined to about 10% in the above-described case (Alexiades and Solomon [1].)

For the same melting problem but with the boundary condition of an imposed time-dependent heat flux $q_0(t)$,

$$-k_l \left(\frac{dT_l}{dx} \right)_{0,t} = q_0(t) \quad \text{for } t > 0, \quad \text{Eq. (9-31)}$$

the quasi-static approximate solution is

$$X(t) = \frac{1}{\rho h_{sl}} \int_0^t q_0(t) dt \quad \text{for } t > 0, \quad \text{Eq. (9-32)}$$

$$T_l(x, t) = T_f + \frac{q_0}{k_l} \left[\frac{q_0}{\rho h_{sl}} t - x \right] \quad \text{Eq. (9-33)}$$

$$\text{in } 0 \leq x \leq X(t) \text{ for } t > 0.$$

For the same case if the boundary condition is a convective heat flux from an ambient fluid at the transient temperature $T_a(t)$, characterized by a heat transfer coefficient \bar{h} ,

$$-k_l \left(\frac{dT_l}{dx} \right)_{0,t} = \bar{h} [T_a(t) - T_l(0, t)] \quad \text{for } t \geq 0, \quad \text{Eq. (9-34)}$$

the quasi-static approximate solution is

$$X(t) = -\frac{k_l}{\bar{h}} + \left\{ \left(\frac{k_l}{\bar{h}} \right)^2 + 2 \frac{k_l}{\rho h_{sl}} \int_0^t [T_a(t) - T_f] dt \right\}^{1/2} \quad \text{Eq. (9-35)}$$

for $t \geq 0$.

$$T_l(x, t) = T_f(t) + [T_a(t) - T_f] \frac{\bar{h} [X(t) - x]}{\bar{h} X(t) + k_l} \quad \text{Eq. (9-36)}$$

$$\text{in } 0 \leq x \leq X(t) \text{ for } t > 0,$$

2. Examples of the Quasi-Static Approximation for Melting of a Cylinder

It is assumed in these examples that the cylinders are very long and that the problems are axisymmetric. Just as in the Cartesian coordinate case, the energy equation is reduced by the approximation to its steady-state form.

Consider the outward-directed melting of a hollow cylinder with internal radius r_i and outer radius r_o (Fig. 9-6)

due to a temperature imposed at the internal radius r_i , i.e.

$$T_\ell(r_i, t) = T_0(t) > T_f \text{ for } t > 0. \quad \text{Eq. (9-37)} \quad 2R(t)^2 \ln \frac{R(t)}{r_i} =$$

The solution is

$$T_\ell(r, t) = T_f + [T_0(t) - T_f] \frac{\ln[r/R(t)]}{\ln[r_i/R(t)]} \quad \text{Eq. (9-38)}$$

$$\text{in } r_i \leq r \leq R(t) \text{ for } t > 0,$$

and the transient position of the phase front, $R(t)$, can be calculated from the transcendental-integral equation

$$2R(t)^2 \ln \frac{R(t)}{r_i} = R(t)^2 - r_i^2 + \frac{4k_\ell}{\rho h_{st}} \int_0^t [T_0(t) - T_f] dt. \quad \text{Eq. (9-39)}$$

If the melting for the same case occurs due to the imposition of a heat flux q_0 at r_i ,

$$-k_\ell \left(\frac{dT_\ell}{dr} \right)_{r_i, t} = q_0(t) > 0 \text{ for } t > 0. \quad \text{Eq. (9-40)}$$

the solution is

$$T_\ell(r, t) = T_f - \frac{q_0(t)r_i}{k_\ell} \ln \frac{r}{R(t)} \quad \text{Eq. (9-41)}$$

$$\text{in } r_i \leq r \leq R(t) \text{ for } t > 0,$$

$$R(t) = \left(r_i^2 + 2 \frac{r_i}{\rho h_{st}} \int_0^t q_0(t) dt \right)^{1/2} \text{ for } t > 0. \quad \text{Eq. (9-42)}$$

If the melting for the same case occurs due to the imposition of a convective heat flux from a fluid at the transient temperature $T_a(t)$, with a heat transfer coefficient \bar{h} , at r_b

$$-k_\ell \left(\frac{dT_\ell}{dr} \right)_{r_b, t} = \bar{h} [T_a(t) - T_\ell(r_b, t)] > 0 \text{ for } t > 0. \quad \text{Eq. (9-43)}$$

The solution is

$$T_\ell(r, t) = T_f + [T_a(t) - T_f] \frac{\ln[r/R(t)]}{\ln[r_b/R(t)] - k_\ell / \bar{h} r_i} \quad \text{Eq. (9-44)}$$

$$\text{in } r_i \leq r \leq R(t) \text{ at } t > 0,$$

with $R(t)$ calculated from the transcendental-integral equation

The solutions for inward melting of a cylinder, where heating is applied at the outer radius r_o , are the same as the above-described ones for the outward-melting cylinder, if the replacements $r_i \rightarrow r_o$, $q_0 \rightarrow -q_0$, and $\bar{h} \rightarrow -\bar{h}$ are made. If such a cylinder is not hollow then $r_i = 0$ is used.

D. ESTIMATION OF MELTING TIME

There are a number of approximate formulas for estimating the freezing and melting times of different materials having a variety of shapes. A brief introduction will be given here; other formulas, proposed by Alexiades and Solomon, [1], all applicable equally well to both freezing and melting times, can be found in Subsection II.D. of "Freezing" 507.8.

1. Melting Time of Foodstuff

The American Society of Heating, Refrigerating, and Air-Conditioning Engineers (ASHRAE) provides a number of approximations for estimating the freezing and thawing times of foods (ASHRAE, [3]). For example, if it can be assumed that the freezing or thawing occur at a single temperature, the time to freeze or thaw, t_f , for a body that has shape parameters \mathcal{P} and R (described below) and thermal conductivity k , initially at the fusion temperature T_f , and which is exchanging heat via heat transfer coefficient \bar{h} with an ambient at the constant temperature T_a , can be approximated by Plank's equation

$$t_f = \frac{h_\ell \rho}{|T_f - T_a|} \left(\frac{\mathcal{P}d}{\bar{h}} + \frac{Rd^2}{k} \right), \quad \text{Eq. (9-46)}$$

where d is the diameter of the body if it is a cylinder or a sphere, or the thickness when it is an infinite slab, and where the shape coefficients \mathcal{P} and R for a number of body forms are given in Table 9-1 below.

Table 9-1
Shape factors for Eq. (9-46) from ASHRAE [3]

Forms	\mathcal{P}	R
Slab	1/2	1/8
Cylinder	1/4	1/16
Sphere	1/6	1/24

Shape coefficients for other body forms are also available. To use Eq. (9-46) for melting, k and ρ should be the values for the food in its thawed state.

In fact, freezing or melting of food typically takes place over a range of temperatures, and approximate Plank-type formulas have been developed for various specific food-stuffs and shapes to represent reality more closely than Eq. (9-46) (ASHRAE [3], Cleland *et al* [118], Rubiolo [170], Tarnawski [178], and some of the references cited under Subsection IV.F. "Medical Applications and Food Preservation."

EXAMPLE

**USING PLANK'S EQUATION (9-46)
FOR ESTIMATING MELTING TIME**

Estimate the time needed to thaw a fish, the shape of which can be approximated by a cylinder 0.5 m long having a diameter of 0.1 m. The fish is initially at its freezing temperature, and during the thawing process it is surrounded by air at $T_a = 23^\circ\text{C}$, with the heating performed with a convective heat transfer coefficient $\bar{h} = 68 \text{ W/m}^2\text{K}$. For the fish, $T_f = -1^\circ\text{C}$, $h_{sf} = 200 \text{ kJ/kg}$, $\rho_s = 992 \text{ kg/m}^3$, and $k_s = 1.35 \text{ W/m K}$.

Using Table 9-1, the geometric coefficients for the cylindrical shape of the fish are $\mathcal{P} = 1/4$ and $R = 1/16$, while d is the cylinder diameter, = 0.1 m. Substituting these values into Eq. (9-46) gives

$$t_f = \frac{200000 \cdot 992}{[(-1) - 23]} \left(\frac{1/4(0.1)}{68} + \frac{1/16(0.1)^2}{1.35} \right) = 6866\text{s} = 1.9\text{h}.$$

2. Other Approximations for Melting Time

As stated above, the freezing chapter in the *Databook*, 507.8, Subsection II.D., shows a number of easily-computable approximate equations for estimating the time needed to freeze or melt a simple-shaped liquid volume initially at the the freezing temperature T_f .

Validating by comparison to the results of an experimentally-verified two-dimensional numerical model of melting of lake-shore water (with a mildly-sloped adiabatic lake bottom) initially at the freezing temperature, Dilley and Lior [121] have shown that for a constant heat flux q_0 from the ambient to the top surface of the ice, with relatively negligible heat fluxes in the water under the ice, the relationship between the depth of melting $X(t)$ and time is linear, viz.

$$X(t) = \frac{q_0}{\rho_s h_{sf}} t \quad \text{Eq. (9-47)}$$

with an error within only a few percent for the first 40 hours.

**III. SOME METHODS FOR
SIMPLIFYING SOLUTION**

**A. THE INTEGRAL METHOD, WITH SAMPLE
SOLUTION FOR MELTING OF A SLAB**

A simple approximate technique for solving melting and freezing problems is the *heat balance integral method* (Goodman [132]), which was found to give good results in many cases. The advantage of this method is that it reduces the second-order partial differential equations, describing the problem to ordinary differential equations which are much easier to solve. This is accomplished by guessing a temperature distribution shape inside the phase-change media, but making it satisfy the boundary conditions. These temperature distributions are then substituted into the partial differential energy equations in the liquid and solid, which are then integrated over the spatial parameter(s) (here just x) in the respective liquid and solid domains. This results in ordinary differential equations having time (t) as the independent variable. The disadvantage of the method is clearly the uncertainty in the temperature distribution within the media. This technique is introduced here by applying it to a useful case, and the reader can thus also learn to apply it to other cases.

Revisiting the above-analyzed melting case of a semi-infinite solid (described in Fig. 9-2), the equations describing this problem are the heat conduction equation

$$\frac{\partial T_\ell(x,t)}{\partial t} = \alpha_\ell \frac{\partial^2 T_\ell(x,t)}{\partial x^2} \quad \text{Eq. (9-48)}$$

$$\text{in } 0 < x < X(t), \text{ for } t > 0,$$

with the initial condition

$$T_\ell(x,t) = T_f \text{ in } x > 0, \text{ at } t = 0, \quad \text{Eq. (9-49)}$$

the boundary condition

$$T_\ell(0,t) = T_0 \text{ for } t > 0, \quad \text{Eq. (9-50)}$$

and the liquid-solid interfacial temperature and heat flux continuity conditions

$$T_\ell[X(t)] = T_f \text{ for } t > 0. \quad \text{Eq. (9-51)}$$

$$-k_\ell \left(\frac{\partial T_\ell}{\partial x} \right)_{x=X(t)} = \rho h_{sf} \frac{dX(t)}{dt} \text{ for } t > 0. \quad \text{Eq. (9-52)}$$

Equation (9-48) represents heat balance (energy conservation), and it is now integrated with respect to x over the entire liquid-phase domain, $0 \leq x \leq X(t)$, in combination with the initial and boundary conditions [Eqs. (9-49) - (9-52)], to obtain

$$\frac{1}{\alpha_\ell} \frac{d}{dt} \left[\int_0^{x(t)} T(x,t) dx - T_f X(t) \right] = -\frac{\rho h_{sf}}{k_\ell} \frac{dX(t)}{dt} - \frac{\partial T}{\partial x}(0,t). \quad \text{Eq. (9-53)}$$

This equation has two unknowns, $T(x, t)$ and $X(t)$. To solve it, some temperature profile is *guessed*, made to satisfy the boundary conditions, and substituted into the equation. Integrating the left-hand side of the equation with this temperature distribution, and taking the derivative of the distribution on the right side of the equation, reduces the original nonlinear partial differential equation system composed of Eqs. (9-48)-(9-52) to one ordinary differential equation in which the only unknown is then $X(t)$, and the independent variable is t . This equation is thus much easier to solve, but at the same time the adequacy of the solution depends on the quality of the chosen temperature distribution. Experience from previous successful solutions or experimental results naturally improves the choice of this temperature distribution.

Say that a quadratic polynomial is chosen as the temperature distribution,

$$T(x, t) = b_1 + b_2[x - X(t)] + b_3[x - X(t)]^2. \quad \text{Eq. (9-54)}$$

The coefficients b_1 , b_2 , and b_3 of this polynomial are calculated from the boundary conditions, and thus

$$b_0 = T_f, \quad b_1 = \frac{h_{sf}}{c_\ell X} \left[1 - (1 + 2N_{Ste_\ell})^{1/2} \right],$$

$$b_2 = \frac{b_1 X + (T_0 - T_f)}{X^2}. \quad \text{Eq. (9-55)}$$

Substitution of the polynomial [Eqs. (9-54), (9-55)] into Eq. (9-53), results in the ordinary differential equation

$$X \frac{dX}{dt} = \frac{6\alpha_\ell \left(1 - \sqrt{1 + N_{Ste_\ell}/2} + N_{Ste_\ell}/2 \right)}{5 + \sqrt{1 + N_{Ste_\ell}/2} + N_{Ste_\ell}/2}. \quad \text{Eq. (9-56)}$$

which, using the initial condition Eq. (2-49), gives the solution

$$X(t) = 2\delta(\alpha_\ell t)^{1/2}, \quad \text{Eq. (9-57)}$$

where

$$\delta \equiv \sqrt{3} \left(\frac{1 - \sqrt{1 + N_{Ste_\ell}/2} + N_{Ste_\ell}/2}{5 + \sqrt{1 + N_{Ste_\ell}/2} + N_{Ste_\ell}/2} \right)^{1/2}. \quad \text{Eq. (9-58)}$$

Comparison of the integral solution [Eq. (9-57)] with the exact one [Eq. (9-9)] has shown that they are nearly

identical for N_{Ste_ℓ} up to about 0.5, with the integral solution overestimating the value of δ by only about 4% when $N_{Ste_\ell} = 2.8$. An even better agreement is obtained if the temperature distribution is estimated as a third, instead of second, degree polynomial.

Many integral solutions yield good results, with errors within a few percent. The accuracy, as mentioned above, depends on the closeness of the chosen temperature distributions to the real ones. Experience from previous successful solutions or experimental results naturally improves this choice. Additional information about this method can be found in refs. [132], [50], [108], [154].

B. THE ENTHALPY METHOD

It is noteworthy that one of the biggest difficulties in numerical solution techniques for such problems is the need to track the location of the phase-change interface continuously during the solution process, so that the interfacial conditions can be applied there. One popular technique that alleviates this difficulty is the *enthalpy method* (refs. [1], [74], [109], [124-125], [130], [140], [173], and [183-184], in which a single partial differential equation, using the material enthalpy instead of the temperature, is used to represent the entire domain, including both phases and the interface. Based on the energy equation, the one-dimensional melting problem is thus described by

$$\rho \frac{\partial h}{\partial t} = k \frac{\partial^2 h}{\partial x^2} \quad \text{in } x \geq 0, \quad \text{for } t > 0, \quad \text{Eq. (9-59)}$$

where the temperature-enthalpy relationship is expressed by

$$T = \begin{cases} \frac{h}{c} & \text{for } h \leq cT_f \quad (\text{solid}) \\ T_f & \text{for } cT_f < h < cT_f + h_{sf} \quad (\text{interface}) \\ \frac{h - h_{sf}}{c} & \text{for } h \geq cT_f + h_{sf} \quad (\text{liquid}) \end{cases} \quad \text{Eq. (9-60)}$$

The numerical computation scheme is rather straightforward: knowing the temperature, enthalpy and thus from Eq. (9-60) the phase of a cell at time step j , the enthalpy at time step $(j+1)$ is computed from the discretized version of Eq. (9-59), and then Eq. (9-60) is used to determine the temperature and phase at that new time. If a computational cell i is in the mushy zone, the liquid fraction is simply h_i / h_{sf} .

Care must be exercised in the use of the enthalpy method when the phase change occurs over a very narrow range of temperatures. Oscillating non-realistic solutions were obtained in such cases, but several modifications to the numerical formulation were found to be reasonably successful (cf. [1], [109], [124-125], and [183-184]).

IV. APPLICATIONS BIBLIOGRAPHY

An extensive—yet by no means complete—bibliography identifying papers and books that treat melting in the main areas in which it takes place, is presented below. The classification is by application, and the internal order is alphabetical by author.

A. CASTING, MOLDING, SINTERING

Melting is a key process-component in casting, molding, and production of solid shapes from powders by processes such as sintering and combustion synthesis. The materials include metals, polymers, glass, ceramics, and superconductors. Flow of the molten material, the course of its solidification (including volume changes due to phase transition, and internal stress creation), and the evolving surface and interior quality, are all of significant industrial importance. In production of parts from powders, the conditions necessary for bonding of the particles by melting and resolidification are of importance. Such diverse processes as glass-making, spinning and wire-making are included. Much attention has lately been focused on the manufacturing of materials for superconductors.

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B. MULTI-COMPONENT SYSTEMS AND CRYSTAL-GROWTH

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One of the most prominent applications is alloy-making, and the last several decades have seen large and increasing involvement with crystal growth, primarily for the electronics and optical industries. Crystals are typically grown by melting the feedstock and letting it solidify in the form of a crystal. Crystals may be made of either pure or multi-component materials, but even when pure crystals are made, much research has been done on the effect of impurities introduced during the manufacturing process. This in effect renders even the pure crystal to be considered as a multi-component system. Crystal growth is accomplished by a variety of processes, including Czochralski, Bridgman, Float-Zone, and thin film deposition. A more extensive list of references on these subjects can be found in the Handbook chapter 507.8, "Freezing."

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C. WELDING, SOLDERING, AND LASER, ELECTRON-BEAM, AND ELECTRO- DISCHARGE PROCESSING

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D. COATING, AND VAPOR AND SPRAY DEPOSITION

The bibliography in this section is of various coating processes in which a solid is melted, and the liquid is then deposited on a surface at a temperature below the freezing point, on which it re-solidifies to form a coating. The process can also be continued to build up desired shapes. Some of the processes covered in the bibliography include plasma and thermal spray deposition, vapor deposition, dipping in a melt, and coating by liquid films.

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E. ABLATION AND SUBLIMATION

Melting sometimes occurs with *ablation*, a process in which the molten, or otherwise-separated material, is continuously removed from the solid substrate by aerodynamic shear forces. Ablation is of importance in applications such as atmospheric re-entry space vehicles, and in certain manufacturing process. When heated, some solid materials change phase directly into vapor, without going through the liquid phase. This process is called *sublimation*, and is mathematically treated in a manner similar to that of melting of solids with continuous removal of the melt, replacing the latent heat of melting with that of sublimation. A bibliography of publications dealing with ablation and sublimation is given below.

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- process following organ preservation, preservation of tissue cultures, cryosurgery, and freezing damage to live tissue, such as in frost-bite.
- Food preservation by freezing, an ancient practice, still attracts much R&D attention, in attempts to shorten freezing times, reduce energy consumption, prolong the life of foods, and minimize damage to their nutritional value, taste, odor and appearance. The thawing phase, which takes place when the food is to be used, is important both in the time it takes, and in the way it affects food quality.
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F. MEDICAL APPLICATIONS AND FOOD PRESERVATION

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G. NUCLEAR POWER SAFETY

Melting plays a role in several areas related to nuclear power plant safety and operation. The primary one is the risk of the reactor core meltdown upon failure of the cooling system to limit the temperature rise, such as the accident at the Three Mile Island reactor. Melting, and the subsequent solidification due to cooling, have been the subject of numerous publications, some of which are listed in the bibliography in this section. Further, interaction of the molten metal with coolant will also cause vapor explosions and chemical reactions that may generate large quantities of gas, some of which (such as hydrogen) are explosive.

Liquid-metal cooled reactors use coolants, such as potassium, sodium, and their alloys, which are solid at room temperature or somewhat above it. A number of publications on this topic are available. (MISSING?) and thus need to be melted for operation. Another nuclear application employing melting is vitrification of nuclear waste for longer term storage.

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H. THERMAL ENERGY STORAGE

Storage of heat in a material by melting, and subsequent release of the heat by re-freezing the material allows a large amount of heat to be stored in a relatively-small mass of material and at a theoretically constant temperature (that of fusion). An issue that has received much attention is the way to exchange with such a phase-change storage material at a high rate and low investment in heat transfer equipment. A very broad range of materials has been considered and used, including water, inorganic salts, hydrocarbons, polymers, and metals.

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