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Journal of Membrane Science 282 (2006) 362-369

www.elsevier.com/locate/memsci

## Heat and mass transfer resistance analysis of membrane distillation

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Received 29 October 2005; received in revised form 24 May 2006; accepted 24 May 2006

Available online 2 June 2006

### Abstract

Expressions for the mass transfer resistances of all the physical domains composing the air-gap membrane distillation (AGMD) and direct contact membrane distillation (DCMD) processes are developed and their absolute and relative effects are evaluated to improve the process understanding and identify promising ways for its improvement. The resistances are computed based on the authors' two-dimensional conjugate model in which a simultaneous numerical solution of the momentum, energy and diffusion equations of the feed and cold solutions have been carried out, and the results of which were validated in comparison with available experimental results. Some of the main conclusions are that: (1) the use and examination of process domain mass transfer resistances is indeed an effective method for understanding the process and identifying ways to improve it, (2) the air/vapor gap dominates the mass transfer resistances of the AGMD domains, and while increasing the air/vapor gap width reduces the parasitic heat transfer by conduction, increasing the width beyond 2 mm has thus not improved the process thermal efficiency, (3) the hot solution inlet temperature and the air gap width have by far the strongest effect on the domain mass transfer resistance, mainly as a consequence of their effect of the hot solution velocity is the higher one, (5) the concentration of the solution has a slight effect on the process, (6) the material used for the membrane should have a small thermal conductivity for a more efficient MD process and (7) efforts to minimize the mass transfer resistance of the cold solution will have a relatively small effect on the permeate flux. © 2006 Elsevier B.V. All rights reserved.

Keywords: Membrane distillation; Water desalination; Transport through membranes; Distillation; Air-gap membrane distillation; Direct contact membrane distillation

### 1. Introduction

Membrane distillation (MD) for water desalination is a technique for separating water vapor from a liquid saline aqueous solution by transport through the pores of hydrophobic membranes, where the driving force is the vapor pressure difference created by temperature difference across the membrane (cf. [1]). A recent state of the art review of MD and assessment of the process potential can be found in ref. [2].

The most common approach to modeling MD, as found in the literature, was by assuming the process as one-dimensional and applying empirical heat and mass transfer coefficients. In this approach (cf. [3-8]), a semi-empirical model is developed, in which the permeate flux is expressed in term of the bulk temperatures of the hot and the cold fluids, and the thermal and concentration effects are expressed in terms of simplified tem-

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0376-7388/\$ - see front matter © 2006 Elsevier B.V. All rights reserved. doi:10.1016/j.memsci.2006.05.040 perature and concentration "polarization" terms, determined, alongside with the heat and mass transfer coefficients, empirically.

A previous paper by the authors [9] presented a more advanced transport analysis based on a two-dimensional conjugate model, in which a simultaneous numerical solution of the momentum, energy and diffusion equations of the feed and cold solutions have been carried out. The results were validated in comparison with available experimental results. A sensitivity analysis to the major process parameters was conducted and provided useful basic detailed information about the nature of the process, and is helpful for process improvement and optimization. The model, its method of solution, validation and major results of both the velocity, temperature and concentration fields, and of the performance parameters, are shown and described in detail in ref. [9].

The analysis is extended here to a systematic evaluation of the individual mass transfer resistances in the different process domains, and their relative contributions to the total resistances, as well as their sensitivity to the major process parameters. This



Fig. 1. The AGMD model with the mass transfer resistances and interface labels.

method improves the understanding of the process, and is of great help in identifying ways to improve it, as shown below.

For AGMD, the distillation process takes place in the domains shown in Fig. 1, that are the hot solution (h), membrane (m), air/vapor gap (g), condensate film (f), cooling plate (p) and the cold fluid (c), and for DCMD the domains of g, f and p are eliminated, so that the cold fluid is in direct contact with the cold side of the membrane.

The major operating parameters include the inlet temperature of the hot solution,  $T_{\rm hi}$ , in the range 40–80 °C, the inlet temperature of the cold solution,  $T_{\rm ci}$ , in the range 5–45 °C, the concentration of the feed solution,  $w_{\rm si}$ , in the range 20,000–50,000 ppm, the inlet velocity of the hot ( $u_{\rm hi}$ ) and cold ( $u_{\rm ci}$ ) solutions in the range 0.1–0.3 m/s, the air gap width,  $\delta_{\rm g}$ , in the range 1–5 mm, and the thermal conductivity of the membrane,  $k_{\rm m}$ , in the range 0.05–0.3 W m<sup>-1</sup> K<sup>-1</sup>.

# 2. Definition and evaluation of the mass transfer resistances

Analogous to the heat flux resistances, we define here the mass transfer resistances for each domain *i* of the AGMD process. A mass transfer resistance,  $R'_{Mi}$ , can be defined for a domain *i* via a relationship of the type

$$R'_{\rm Mi} = \frac{\Delta P_i}{J_i},\tag{1}$$

where  $J_i$  is the vapor flux and  $\Delta P_i$  is the vapor pressure difference, across the *i*th domain, respectively.

Such a resistance for a domain *i*,  $R'_{Mi}$ , is seen thus to be proportional to the vapor pressure difference, across the *i*th domain,  $\Delta P_i$ 

$$R'_{\rm Mi} \propto \Delta P_i,$$
 (2)

The value of  $\Delta P_i$  depends on the hot and cold liquid inlet conditions, and to make the mass transfer resistance in the analysis independent of these conditions, we scaled each mass

transfer resistance by the vapor pressure difference between the flow-direction centerlines of the hot and cold solution channels,  $\Delta P_c \equiv (P_{ch} - P_{cc})$ , defining thereby a normalized (but not dimensionless) mass transfer resistance of domain *i* as

$$R_{\rm Mi} \propto \frac{\Delta P_i}{\Delta P_{\rm c}} = \frac{R'_{\rm Mi}}{\Delta P_{\rm c}},$$
(3)

To find the coefficient of proportionality in Eq. (3), Stefan's law is employed to model the permeate flux diffusion in the membrane at any location x along the membrane,

$$J_i \equiv J_{\rm m} = K \Delta P_{\rm m},\tag{4}$$

where  $\Delta P_{\rm m} = P_{\rm hm} - P_{\rm mg}$  is the water vapor pressure difference across the membrane (the vapor transfer driving force) and *K* is the permeability of the membrane, defined as (cf. [4])

$$K = \frac{\varepsilon D_{\rm v/a} M_{\rm v} P_{\rm T}}{\chi \delta_{\rm m} P_{\rm a,avg} R_{\rm u} T_{\rm avg,m}}$$
(5)

and where  $\varepsilon$  (porosity) and  $\chi$  (tortuosity), are membrane geometry parameters, and

$$T_{\rm avg,m} = \frac{T_{\rm hm} + T_{\rm mg}}{2}.$$
(6)

The driving force, alongside with Eq. (4) thus defines the mass transfer resistance of the membrane,  $R_{Mm}$ , as

$$R_{\rm Mm} \equiv \frac{\Delta P_{\rm m}}{J_{\rm m} \Delta P_{\rm c}} = \frac{1}{K \Delta P_{\rm c}},\tag{7}$$

leading to the following general definition of the mass transfer resistance for each of the domains:

$$R_{\rm Mi} = \frac{1}{K} \frac{\Delta P_i}{\Delta P_{\rm m} \Delta P_{\rm c}},\tag{8}$$

The vapor pressures (P) needed in these  $\Delta P_i$  expressions were calculated using the Antoine equation (cf. [10])

$$\ln P = A_1 - \frac{A_2}{T_{\rm hm} + A_3},\tag{9}$$

where  $A_1 = 16.2620$ ,  $A_2 = 3799.89$  and  $A_3 = 226.85$ ,  $T_{hm}$  is in °C. The validity of this equation was checked by comparison to the steam tables, and was found to be accurate to better than 0.4% within the 40–80 °C range studied in this paper. The effect of the presence of the salt in the solution on the vapor pressure at the hot surface of the membrane side was accounted for by using an empirical correlation for the boiling point elevation in ref. [11].

The definitions of the mass transfer resistance for each domain in light of Eq. (8), thus are:

The hot solution (domain h, Fig. 1)

$$R_{\rm Mh} = \frac{1}{K} \frac{P_{\rm ch} - P_{\rm hm}}{\Delta P_{\rm m} \Delta P_{\rm c}} \tag{10}$$

The membrane (m)

$$R_{\rm Mm} = \frac{1}{K} \frac{P_{\rm hm} - P_{\rm mg}}{\Delta P_{\rm m} \Delta P_{\rm c}} \tag{11}$$

The air/vapor gap (g)

$$R_{\rm Mg} = \frac{1}{K} \frac{P_{\rm mg} - P_{\rm gf}}{\Delta P_{\rm m} \Delta P_{\rm c}}$$
(12)

The condensate film (f)

$$R_{\rm Mf} = \frac{1}{K} \frac{P_{\rm gf} - P_{\rm fp}}{\Delta P_{\rm m} \Delta P_{\rm c}}$$
(13)

The cooling plate (p)

$$R_{\rm Mp} = \frac{1}{K} \frac{P_{\rm fp} - P_{\rm pc}}{\Delta P_{\rm m} \Delta P_{\rm c}} \tag{14}$$

The cold solution (c)

$$R_{\rm Mc} = \frac{1}{K} \frac{P_{\rm pc} - P_{\rm cc}}{\Delta P_{\rm m} \Delta P_{\rm c}}$$
(15)

The total mass transfer resistance between the hot and cold solution centerlines is the sum of all the domain resistances, viz.

$$R_{\rm MT} = R_{\rm Mh} + R_{\rm Mm} + R_{\rm Mg} + R_{\rm Mf} + R_{\rm Mp} + R_{\rm Mc}$$
(16)

It is more instructive to evaluate the relative magnitudes (and thus the importance) of each of the domain mass transfer resistances, when normalized by the total mass transfer resistance. We note, however, that in our sensitivity analysis, where we hold all of the parameters at the base-case value, and vary one parameter at a time around its base-case value, the total mass transfer resistances changes with the value of the variable parameter. To make the results and comparisons clearer, we keep the normalizing parameter constant during the variation of parameter *i* at the maximal value of the total resistance for that parameter,  $R_{\rm MT,max}$ , within the range that we vary it. A domain mass transfer resistance ratio is thus defined as

$$r_{\rm Mi} = \frac{R_{\rm Mi}}{R_{\rm MT,max}} \tag{17}$$

and provides not only an easy way to compare the different mass transfer resistances for a given set of conditions, but also a comparison for different conditions within the investigated range.

A comparison of the physical domains common to DCMD and AGMD is also made to quantify the importance of these domains in each process, by using the ratio of the domain mass transfer resistance to the total mass transfer resistance for that process ( $R_{Mi}/R_{MT}$ ). Since in Tables 1–4 below, we only display the values of the three resistance ratios  $R_{Mh}/R_{MT}$ ,  $R_{Mm}/R_{MT}$ and  $R_{Mc}/R_{MT}$  that are common to both processes, the sum  $\sum_i R_{Mi}/R_{MT}$  adds up to 1 for DCMD, but not for AGMD, as the domain of AGMD includes also the vapor/air gap, film condensate and cooling plate that are additional to the displayed domains.

### 3. Results and discussion

#### 3.1. Range and conditions

The analysis is made for the inlet temperature of the feed solution  $(T_{\rm hi})$  in the range 40–80 °C computed at 5 °C incre-

#### Table 1

Comparison of the mass transfer resistances ratio of the hot, cold and membrane domains of the AGMD and DCMD for different values of the hot solution inlet temperature  $(T_{hi})$ 

T <sub>hi</sub> (°C)	40		80	
	AGMD	DCMD	AGMD	DCMD
$R_{\rm Mh}/R_{\rm MT}$	0.08	0.39	0.31	0.64
$R_{\rm Mm}/R_{\rm MT}$	0.08	0.41	0.08	0.21
$R_{\rm Mc}/R_{\rm MT}$	0.04	0.20	0.04	0.15

 $\begin{array}{ll} T_{\rm ci} = 20 \ ^{\circ}{\rm C}, & w_{\rm si} = 0.025, & d_{\rm h} = 0.002 \ {\rm m}, & l_{\rm m} = 0.2 \ {\rm m}, & \delta_{\rm m} = 4(10)^{-4} \ {\rm m}, \\ \chi = 1.5, & k_{\rm m} = 0.2 \ {\rm W} \ {\rm m}^{-1} \ {\rm K}^{-1}, & \varepsilon = 0.78, & \delta_{\rm g} = 2 \ {\rm mm}, & k_{\rm p} = 60 \ {\rm W} \ {\rm m}^{-1} \ {\rm K}^{-1}, \\ \delta_{\rm p} = (1.5) 10^{-3} \ {\rm m}, & u_{\rm ci} = 0.1 \ {\rm m/s} \ (Re_{\rm c} = 193), & d_{\rm c} = 0.002 \ {\rm m}. \end{array}$ 

#### Table 2

Comparison of the mass transfer resistances ratio  $(R_{\rm Mi}/R_{\rm MT})$  of the hot, cold and membrane domains of the AGMD and DCMD for different values of the inlet temperature of the cold solution  $(T_{\rm ci})$ 

$T_{\rm ci}$ (°C)	5		45	
	AGMD	DCMD	AGMD	DCMD
$R_{\rm Mh}/R_{\rm MT}$	0.28	0.62	0.21	0.52
$R_{\rm Mm}/R_{\rm MT}$	0.06	0.28	0.10	0.18
$R_{\rm Mc}/R_{\rm MT}$	0.02	0.10	0.9	0.30

$$\begin{split} T_{\rm hi} &= 70~^{\circ}{\rm C}, \ u_{\rm hi} = 0.1~{\rm m/s} \ (Re_{\rm h} = 464), \ w_{\rm si} = 0.025, \ d_{\rm h} = 0.002~{\rm m}, \ l_{\rm m} = 0.2~{\rm m}, \\ \delta_{\rm m} &= (4)10^{-4}~{\rm m}, \ \chi = 1.5, \ k_{\rm m} = 0.2~{\rm W}~{\rm m}^{-1}~{\rm K}^{-1}, \ \varepsilon = 0.78, \ \delta_{\rm g} = 2~{\rm mm}, \\ k_{\rm p} &= 60~{\rm W}~{\rm m}^{-1}~{\rm K}^{-1}, \ \delta_{\rm p} = (1.5)10^{-3}~{\rm m}, \ u_{\rm ci} = 0.1~{\rm m/s} \ (Re_{\rm c} = 193), \ d_{\rm c} = 0.002~{\rm m}. \end{split}$$

Table 3 The ratio of the mass transfer resistance  $(R_{\rm Mi})$  to the total mass transfer resistance  $(R_{\rm MT})$  for the domains common to AGMD and DCMD for  $u_{\rm hi} = 0.1$  and 0.3 m/s

u <sub>hi</sub> (m/s)	0.1		0.3	
	AGMD	DCMD	AGMD	DCMD
$R_{\rm Mh}/R_{\rm MT}$	0.25	0.59	0.18	0.47
$R_{\rm Mm}/R_{\rm MT}$	0.9	0.25	0.95	0.31
$R_{\rm Mc}/R_{\rm MT}$	0.04	0.16	0.04	0.22

$$\begin{split} T_{\rm hi} &= 70~^{\circ}{\rm C}, \ T_{\rm ci} = 20~^{\circ}{\rm C}, \ u_{\rm ci} = 0.1~{\rm m/s} \ (Re_{\rm c} = 193), \ w_{\rm si} = 0.025, \ d_{\rm h} = 0.002~{\rm m}, \\ l_{\rm m} &= 0.2~{\rm m}, \ \delta_{\rm m} = (4)10^{-4}~{\rm m}, \ \chi = 1.5, \ k_{\rm m} = 0.2~{\rm W}~{\rm m}^{-1}~{\rm K}^{-1}, \ \varepsilon = 0.78, \ \delta_{\rm g} = 2~{\rm mm}, \\ k_{\rm p} &= 60~{\rm W}~{\rm m}^{-1}~{\rm K}^{-1}, \ \delta_{\rm p} = (1.5)10^{-3}~{\rm m}, \ d_{\rm c} = 0.002~{\rm m}. \end{split}$$

ments, hot solution inlet sodium chloride concentrations of 20,000–50,000 ppm at 5000 ppm increments, feed and cold solutions inlet velocities ( $u_{hi}$ ,  $u_{ci}$ ) of 0.1–0.3 m/s ( $Re_h = 464-1393$ ,  $Re_c = 193-583$ ), at 0.04 m/s increments, cooling solution inlet temperatures ( $T_{ci}$ ) of 5–45 °C at 5 °C increments, air/vapor gap

Table 4

The ratio of the mass transfer resistance  $(R_{\rm Mi})$  to the total mass transfer resistance  $(R_{\rm MT})$  for the domains common to AGMD and DCMD for  $u_{\rm ci} = 0.1$  and 0.3 m/s

<i>u</i> <sub>ci</sub> (m/s)	0.1		0.3	
	AGMD	DCMD	AGMD	DCMD
$R_{\rm Mh}/R_{\rm MT}$	0.25	0.59	0.25	0.63
$R_{\rm Mm}/R_{\rm MT}$	0.9	0.25	0.9	0.275
$R_{\rm Mc}/R_{\rm MT}$	0.06	0.16	0.025	0.095

$$\begin{split} T_{\rm hi} &= 70 \,\,^{\circ}{\rm C}, \ T_{\rm ci} = 20 \,\,^{\circ}{\rm C}, \ u_{\rm hi} = 0.1 \,\, {\rm m/s} \ (Re_{\rm c} = 193), \ w_{\rm si} = 0.025, \ d_{\rm h} = 0.002 \,\, {\rm m}, \\ l_{\rm m} &= 0.2 \,\, {\rm m}, \ \delta_{\rm m} = (4)10^{-4} \,\, {\rm m}, \ \chi = 1.5, \ k_{\rm m} = 0.2 \,\, {\rm W} \,\, {\rm m}^{-1} \,\, {\rm K}^{-1}, \ \varepsilon = 0.78, \ \delta_{\rm g} = 2 \,\, {\rm mm}, \\ k_{\rm p} &= 60 \,\, {\rm W} \,\, {\rm m}^{-1} \,\, {\rm K}^{-1}, \ \delta_{\rm p} = (1.5)10^{-3} \,\, {\rm m}, \ d_{\rm c} = 0.002 \,\, {\rm m}. \end{split}$$

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Fig. 2. The influence of the hot inlet temperature ( $T_{hi}$ ) on the mass transfer resistance ratios ( $r_{Mi}$ ) of (a) the AGMD domains and (b) DCMD domains;  $u_{hi} = 0.1 \text{ m/s}$  ( $Re_h = 464$ ),  $w_{si} = 0.025$ ,  $d_h = (2)10^{-3} \text{ m}$ ,  $l_m = 0.2 \text{ m}$ ,  $\delta_m = (4)10^{-4} \text{ m}$ ,  $\chi = 1.5$ ,  $k_m = 0.2 \text{ W} \text{ m}^{-1} \text{ K}^{-1}$ ,  $\varepsilon = 0.78$ ,  $\delta_g = (2)10^{-3} \text{ m}$ ,  $k_p = 60 \text{ W} \text{ m}^{-1} \text{ K}^{-1}$ ,  $\delta_p = (1.5)10^{-3} \text{ m}$ ,  $T_{ci} = 20 \text{ °C}$ ,  $u_{ci} = 0.1 \text{ m/s}$  ( $Re_c = 193$ ),  $d_c = (2)10^{-3} \text{ m}$ .

widths ( $\delta_g$ ) of 1–5 mm at 1 mm increments, membrane thermal conductivities ( $k_m$ ) of 0.05–0.3 W m<sup>-1</sup> K<sup>-1</sup> at 0.05 W m<sup>-1</sup> K<sup>-1</sup> increments, and membrane porosities ( $\varepsilon_m$ ) of 0.74, 0.78 and 0.84.

## 3.2. The influence of the hot solution inlet temperature on the individual mass transfer resistances

Fig. 2a shows the influence of  $T_{hi}$  on the mass transfer resistance ratios  $r_{Mi}$  of the different AGMD domains defined in Fig. 1. As  $T_{hi}$  is increased, the resistances of each domain decreases, but because the air gap constitutes 78% of the total resistance at  $T_{hi} = 40 \,^{\circ}$ C, the reduction of  $R_{Mg}$  dominates the reduction of the total resistance. With  $T_{ci}$  held at 20  $^{\circ}$ C, as  $T_{hi}$  was increased from 40 to 80  $^{\circ}$ C: all  $r_{Mi}$  have dropped, with the  $r_{M,g}$  dropping by 73%,  $r_{M,m}$  by 8.5%,  $r_{M,h}$  by 4.9%,  $r_{M,c}$  by 2.9% and the film and plate resistances had no tangible contribution to the total resistance and  $R_{Mh}$  constitutes 31% of the total resistance, with the other resistances constitute 5% or less each. So at high temperature the hot solution resistance becomes also important.

Fig. 2b shows the mass transfer resistance ratio  $(r_{Mi})$  of DCMD as a function of the inlet temperature of the hot solution  $(T_{hi})$ . Increasing the inlet temperature of the hot solution

from  $T_{\rm hi} = 40 \,^{\circ}$ C, to  $T_{\rm hi} = 80 \,^{\circ}$ C, reduces  $r_{\rm Mh}$  by 24%,  $r_{\rm Mm}$  by 36% and  $r_{\rm Mc}$  by 16%. Thus, increasing the inlet temperature of the hot solution reduces  $R_{\rm Mm}$  significantly.

Table 1 shows quantitative comparison of the resistances of the common domains of AGMD and DCMD process. At low  $T_{\rm hi}$ , for AGMD,  $R_{\rm Mh}$ ,  $R_{\rm Mm}$  and  $R_{\rm Mc}$  are relatively small and the dominant resistance is that of the air gap ( $R_{\rm Mg}$ ) as shown in Fig. 2a, but in DCMD,  $R_{\rm Mh}$  is important even at low  $T_{\rm hi}$ , and becomes even more important for high  $T_{\rm hi}$ .  $R_{\rm Mm}$  is roughly equal to  $R_{\rm Mh}$  at low  $T_{\rm hi}$ , but as  $T_{\rm hi}$  becomes high,  $R_{\rm Mh}$  becomes the largest mass transfer resistance.  $R_{\rm Mc}$  is the least affected by  $T_{\rm hi}$ .

One of the conclusions is that efforts to reduce  $R_{\rm Mh}$  in AGMD at low  $T_{\rm hi}$  have a very small effect on the process, but they are worthwhile at the higher values of  $T_{\rm hi}$  in both DCMD and AGMD, with more room for improvement in DCMD.

# 3.3. The influence of the cold solution inlet temperature on the individual mass transfer resistances

Fig. 3a shows the effect of  $T_{ci}$  on the mass transfer resistance ratios ( $r_{Mi}$ ) of the different AGMD domains. With  $T_{hi}$  held at 70 °C, as  $T_{ci}$  is decreased from 45 to 5 °C: the  $r_{Mg}$  dropped by 30%; the mass transfer resistances of the hot and cold solution ( $r_{M,h}$  and  $r_{M,c}$ , respectively) have a lower contribution



Fig. 3. The influence of the cold inlet temperature ( $T_{ci}$ ) on the mass transfer resistance ratios ( $r_{Mi}$ ) of (a) the AGMD domains and (b) DCMD domains,  $T_{hi} = 70 \,^{\circ}\text{C}$ ,  $u_{hi} = 0.1 \,\text{m/s}$  ( $Re_h = 464$ ),  $w_{si} = 0.025$ ,  $d_h = (2)10^{-3} \,\text{m}$ ,  $l_m = 0.2 \,\text{m}$ ,  $\delta_m = (4)10^{-4} \,\text{m}$ ,  $\chi = 1.5$ ,  $k_m = 0.2 \,\text{W} \,\text{m}^{-1} \,\text{K}^{-1}$ ,  $\varepsilon = 0.78$ ,  $\delta_g = (2)10^{-3} \,\text{m}$ ,  $k_p = 60 \,\text{W} \,\text{m}^{-1} \,\text{K}^{-1}$ ,  $\delta_p = (1.5)10^{-3} \,\text{m}$ ,  $u_{ci} = 0.1 \,\text{m/s}$  ( $Re_c = 193$ ),  $d_c = (2)10^{-3} \,\text{m}$ .



Fig. 4. The effect of the inlet concentration of the solution ( $w_{si}$ ) on the mass transfer resistance ratios ( $r_{Mi}$ ) of the (a) AGMD and (b) DCMD domains,  $T_{hi} = 70 \,^{\circ}$ C,  $u_{hi} = 0.1 \,\text{m/s}$  ( $Re_h = 464$ ),  $d_h = (2)10^{-3} \,\text{m}$ ,  $l_m = 0.2 \,\text{m}$ ,  $\delta_m = (4)10^{-4} \,\text{m}$ ,  $\chi = 1.5$ ,  $k_m = 0.2 \,\text{W} \,\text{m}^{-1} \,\text{K}^{-1}$ ,  $\varepsilon = 0.78$ ,  $\delta_g = (2)10^{-3} \,\text{m}$ ,  $k_p = 60 \,\text{W} \,\text{m}^{-1} \,\text{K}^{-1}$ ,  $\delta_p = (1.5)10^{-3} \,\text{m}$ ,  $T_{ci} = 20 \,^{\circ}$ C,  $u_{ci} = 0.1 \,\text{m/s}$  ( $Re_c = 193$ ),  $d_c = (2)10^{-3} \,\text{m}$ .

to  $R_{\text{MT}}$ , and they drop by about 6.5% and 7.9%, respectively;  $r_{\text{M,f}}$  drops by 3%,  $R_{\text{M,m}}$  by about 2% and  $r_{\text{M,p}}$  remains nearly unaffected.

While the effect of  $T_{ci}$  on the mass transfer resistance of the cold solution is very slight in AGMD, it is much more significant for DCMD. Fig. 3b shows the same effect but for DCMD domains. Reducing the inlet temperature of the cold solution  $(T_{ci})$  from 45 to 5 °C, reduces  $r_{Mc}$  by 24%,  $r_{Mh}$  by 17% and  $r_{Mm}$  by only 2%. Table 2 shows the ratio of the mass transfer resistance  $(R_{Mi})$  to the total mass transfer resistance  $(R_{MT})$  for AGMD and DCMD configuration at  $T_{ci}$  = 5 and 45 °C. It can be seen that  $R_{Mh}$  is important at low  $T_{ci}$  for both configurations. But  $R_{Mc}$  becomes unimportant at low  $T_{ci}$ , so for both processes it would be insensible to improve the process by minimizing  $R_{Mc}$ , by say increasing the cold fluid velocity, when  $T_{ci}$  is low.

### 3.4. The influence of inlet concentration of the hot solution on the domain mass transfer resistance ratios

Fig. 4a shows the effect of the hot solution inlet concentration  $(w_{si})$  on the mass transfer resistance ratios  $(r_{Mi})$  of the different AGMD domains. Increasing  $w_{si}$  from 20,000 to 50,000 ppm causes only a 12% increases in  $r_{M,g}$ , and 1.9% in  $r_{M,m}$ , with the other domain resistances practically unaffected.

Fig. 4b shows the same, for DCMD. The mass transfer resistances of the hot, cold and membrane domains are practically unaffected by the concentration of the hot solution ( $w_{si}$ ) and their share of the total resistance remain almost constant at about 57%, 28% and 15%. Therefore, it has no impact on *J*.

## 3.5. The influence of air/vapor gap width on the domain mass transfer resistance ratios

The influence of  $\delta_g$  on the mass transfer resistance ratios of the different AGMD domains is shown in Fig. 5. Consistent with other results shown above,  $R_{Mg}$  is the major resistance and practically the only one affected, where reduction of  $\delta_g$  from 5 to 1 mm reduces  $r_{Mg}$ , by 62%. When the air gap is made smaller, the thermal resistance and consequently the temperature difference across the air gap decrease, thus lowering the partial pressure across the air gap and consequently reducing the mass transfer resistance of the air gap.

## 3.6. The influence of the hot solutions inlet velocity on the domain mass transfer resistance ratios

The influence of the hot solution inlet velocity ( $u_{hi}$ ) on the mass transfer resistance ratios of the different AGMD domains and DCMD is shown in Fig. 6a and b, respectively. Increasing  $u_{hi}$  has more impact on  $R_{Mh}$  of DCMD than on that of AGMD. Increasing  $u_{hi}$  from 0.1 to 0.3 m/s reduces  $r_{M,h}$  by 9% for AGMD and by 20% for DCMD. That is because in DCMD the  $R_{Mh}$  fraction of  $R_{MT}$  is larger than that in AGMD, and as a result *J* in DCMD indeed increased more than that in AGMD.

The other mass transfer resistances are almost unaffected; the inlet velocity of the hot solution thus has an effect only on the feedwater channel domain.



Fig. 5. The domain mass transfer resistance ratios  $(r_{\rm Mi})$  as a function of the air gap width.  $T_{\rm hi} = 70$  °C,  $u_{\rm hi} = 0.1$  m/s  $(Re_{\rm h} = 464)$ ,  $w_{\rm si} = 0.025$ ,  $T_{\rm ci} = 20$  °C,  $d_{\rm h} = (2)10^{-3}$  m,  $l_{\rm m} = 0.2$  m,  $\delta_{\rm m} = (4)10^{-4}$  m,  $\chi = 1.5$ ,  $k_{\rm m} = 0.2$  W m<sup>-1</sup> K<sup>-1</sup>,  $\varepsilon = 0.78$ ,  $\delta_{\rm g} = (2)10^{-3}$  m,  $k_{\rm p} = 60$  W m<sup>-1</sup> K<sup>-1</sup>,  $\delta_{\rm p} = (1.5)10^{-3}$  m,  $u_{\rm ci} = 0.1$  m/s  $(Re_{\rm c} = 193)$ ,  $d_{\rm c} = (2)10^{-3}$  m.



Fig. 6. The effect of the inlet velocity of the hot solution ( $u_{hi}$ ) on the mass transfer resistance ratios ( $r_{Mi}$ ) of the (a) AGMD and (b) DCMD domains,  $T_{hi} = 70 \,^{\circ}$ C,  $w_{si} = 0.025$ ,  $T_{ci} = 20 \,^{\circ}$ C,  $d_h = (2)10^{-3}$  m,  $l_m = 0.2$  m,  $\delta_m = (4)10^{-4}$  m,  $\chi = 1.5$ ,  $k_m = 0.2$  W m<sup>-1</sup> K<sup>-1</sup>,  $\varepsilon = 0.78$ ,  $\delta_g = (2)10^{-3}$  m,  $k_p = 60$  W m<sup>-1</sup> K<sup>-1</sup>,  $\delta_p = (1.5)10^{-3}$  m,  $u_{ci} = 0.1$  m/s ( $Re_c = 193$ ),  $d_c = (2)10^{-3}$  m.



Fig. 7. The effect of the inlet velocity of the cold solution ( $u_{ci}$ ) on the mass transfer resistance ratios ( $r_{Mi}$ ) of the (a) AGMD and (b) DCMD domains.  $T_{hi} = 70 \,^{\circ}\text{C}$ ,  $u_{hi} = 0.08 \,\text{m/s}$  ( $Re_h = 464$ ),  $w_{si} = 0.025$ ,  $d_h = (2)10^{-3} \,\text{m}$ ,  $l_m = 0.2 \,\text{m}$ ,  $\delta_m = (4)10^{-4} \,\text{m}$ ,  $\chi = 1.5$ ,  $k_m = 0.2 \,\text{W} \,\text{m}^{-1} \,\text{K}^{-1}$ ,  $\varepsilon = 0.78$ ,  $\delta_g = (2)10^{-3} \,\text{m}$ ,  $k_p = 60 \,\text{W} \,\text{m}^{-1} \,\text{K}^{-1}$ ,  $\delta_p = (1.5)10^{-3} \,\text{m}$ ,  $T_{ci} = 20 \,^{\circ}\text{C}$ ,  $d_c = (2)10^{-3} \,\text{m}$ .

The mass transfer resistance ratio of the domains common to AGMD and DCMD (i.e. hot solution, membrane and cold solution) at  $u_{hi} = 0.1$  and 0.3 m/s is shown in Table 3. It can be seen that  $R_{Mh}$  constitutes the largest mass transfer resistance at both velocities for DCMD, but for AGMD it constitutes only 18% of  $R_{MT}$  and its fraction grows as  $u_{hi}$  decreases (25% at  $u_{hi} = 0.1$ ).

# 3.7. The influence of the cold solutions inlet velocity on the domain mass transfer resistance ratios

Fig. 7a shows the effect of the cold solution inlet velocity ( $u_{ci}$ ) on the mass transfer resistance ratios of the different AGMD domains: increasing  $u_{ci}$  from 0.1 to 0.3 m/s reduces the total resistance  $R_{TM}$  by 3.4%, stemming mainly from the reduc-



Fig. 8. The total mass transfer resistance ratio ( $r_{\rm MT}$ ) as a function of the membrane material conductivity ( $k_{\rm m}$ ).  $T_{\rm hi} = 70 \,^{\circ}\text{C}$ ,  $u_{\rm hi} = 0.1 \,\text{m/s}$  ( $Re_{\rm h} = 464$ ),  $w_{\rm si} = 0.025$ ,  $T_{\rm ci} = 20 \,^{\circ}\text{C}$ ,  $d_{\rm h} = (2)10^{-3} \,\text{m}$ ,  $l_{\rm m} = 0.2 \,\text{m}$ ,  $\chi = 1.5$ ,  $\delta_{\rm m} = (4)10^{-4} \,\text{m}$ ,  $\varepsilon = 0.78$ ,  $\delta_{\rm g} = (2)10^{-3} \,\text{m}$ ,  $k_{\rm p} = 60 \,\text{W} \,\text{m}^{-1} \,\text{K}^{-1}$ ,  $\delta_{\rm p} = (1.5)10^{-3} \,\text{m}$ ,  $u_{\rm ci} = 0.08 \,\text{m/s}$  ( $Re_{\rm c} = 193$ ),  $d_{\rm c} = (2)10^{-3} \,\text{m}$ .

tion of  $R_{Mg}$  and  $R_{Mc}$ , where  $r_{Mg}$  and  $r_{Mc}$  are reduced by 1.7% and 1.5%, respectively. The other resistances are not practically influenced by  $u_{ci}$ . It is important to note that  $u_{hi}$  and  $u_{ci}$  affect mostly the resistance of their domains by reducing the boundary layer thickness of their domains as shown in the fluid mechanics and heat and mass transfer analysis in ref. [8]. The effect of  $u_{hi}$  is larger than  $u_{ci}$  because the hot transfer resistance has a larger share of  $R_{MT}$  than that of the cold solution, for example, at  $u_{hi} = u_{ci} = 0.1$  m/s,  $R_{Mh}$  is about 25% of  $R_{MT}$  and  $R_{Mc}$  is only 4%.

Fig. 7b shows the mass transfer resistance ratio  $(r_{Mi})$  of the DCMD domains as a function of the inlet velocity of the cold solution  $(u_{ci})$ . Increasing the inlet velocity of the cold solution from 0.1 to 0.3 m/s, reduces  $r_{Mc}$  by 7%, with the other resistances being hardly affected for DCMD. Because the ratio of the cold solution mass transfer resistance  $(R_{Mc})$  to the total mass transfer resistance  $(R_{MT})$  is small, the increase of the fraction of  $R_{Mh}$  and  $R_{Mm}$  of the total is also minimal as shown in Tables 3 and 4, thus explaining the small impact of  $u_{ci}$  on J.

# 3.8. The influence of the membrane thermal conductivity and porosity on the mass transfer resistance ratios

Almost the entire increases of  $r_{\rm MT}$  is due to the increase of  $r_{\rm Mg}$  as shown in Fig. 8a. Increasing  $k_{\rm m}$  from 0.05 to 0.3 W m<sup>-1</sup> K<sup>-1</sup> causes  $r_{\rm Mg}$  to increase by about 48%, and  $r_{\rm Mh}$  only by 1%. This is because an increase in  $k_{\rm m}$  causes the effective thermal conductivity of the membrane (solid + pores) to increase, thus reducing the thermal resistance of the membrane, and consequently also the temperature drop across it, and consequently raising the temperature at the cold side of the membrane. This raises in turn the mass transfer resistance of the air gap ( $R_{\rm Mg}$ ) because, by definition,  $R_{\rm Mg}$  increases with the partial pressure at the cold side of the membrane.

Fig. 8b shows the mass transfer resistance ratio  $(r_{Mi})$  of the DCMD domains as a function of the thermal conductivity of the membrane material  $(k_m)$ . Lowering the thermal conductivity of the membrane material  $(k_m)$  from 0.3 to 0.05 W m<sup>-1</sup> K<sup>-1</sup>, causes  $r_{Mh}$  to increase from 57% to 65%, and the cold and membrane mass transfer resistances changes little with  $k_m$ . The thermal conductivity of the membrane material  $(k_m)$  has more significant affect on the total mass transfer resistance of AGMD than on that of DCMD. For example, as stated earlier, for  $\varepsilon = 0.74$ , as  $k_m$  is increased from 0.05 to 0.3 W m<sup>-1</sup> K<sup>-1</sup>,  $r_{MT}$  increases by 49% for AGMD. The respective increase in  $r_{MT}$  for DCMD is only about 10%. This is reflected in the higher impact of  $k_m$  on J for AGMD than for DCMD as shown in ref. [9].

Table 5 shows a comparison of the  $r_{Mi}$  for the domains common to AGMD and DCMD for  $k_m = 0.3$  and  $k_m = 0.05$ . It can be seen that the hot solution and membrane domains have the highest effect in AGMD.

### 4. Conclusions

The absolute and relative effects of all the physical domains composing the air-gap membrane distillation (AGMD) and the

#### Table 5

The ratio of the mass transfer resistance ( $R_{Mi}$ ) to the total mass transfer resistance ( $R_{MT}$ ) for the domains common to AGMD and DCMD for  $k_m = 0.05$  and 0.3 W m<sup>-1</sup> K<sup>-1</sup>

$k_{\rm m} ({\rm W}{\rm m}^{-1}{\rm K}^{-1})$	0.05		0.3	
	AGMD	DCMD	AGMD	DCMD
$R_{\rm Mh}/R_{\rm MT}$	0.33	0.56	0.19	0.60
$R_{\rm Mm}/R_{\rm MT}$	0.165	0.28	0.065	0.25
$R_{\rm Mc}/R_{\rm MT}$	0.055	0.16	0.027	0.15

$$\begin{split} T_{\rm hi} &= 70 \,^{\circ}{\rm C}, \ T_{\rm ci} = 20 \,^{\circ}{\rm C}, \ u_{\rm hi} = 0.1 \ {\rm m/s} \ (Re_{\rm h} = 464), \ w_{\rm si} = 0.025, \ d_{\rm h} = 0.002 \ {\rm m}, \\ l_{\rm m} &= 0.2 \ {\rm m}, \ \delta_{\rm m} = (4)10^{-4} \ {\rm m}, \ \chi = 1.5, \ \varepsilon = 0.78, \ \delta_{\rm g} = 2 \ {\rm mm}, \ k_{\rm p} = 60 \ {\rm W} \ {\rm m}^{-1} \ {\rm K}^{-1}, \\ \delta_{\rm p} &= (1.5)10^{-3} \ {\rm m}, \ u_{\rm ci} = 0.1 \ {\rm m/s} \ (Re_{\rm c} = 193), \ d_{\rm c} = 0.002 \ {\rm m}. \end{split}$$

direct contact membrane distillation processes, and of its dominant variables, are analyzed by constructing expressions of the domain mass transfer resistances and their evaluation. The resistances are computed based on the authors' two-dimensional conjugate model in which a simultaneous numerical solution of the momentum, energy and diffusion equations of the feed and cold solutions have been carried out, and the results of which were validated in comparison with available experimental results. The following can be concluded:

- The use and examination of process domain mass transfer resistances is an effective method for understanding the process and identifying ways to improve it.
- The air/vapor gap dominates the mass transfer resistances of the AGMD domains. The film condensate has nearly minimal relative contribution to the total mass transfer resistance, and the cooling plate mass transfer resistance is so small that it can typically be ignored.
- The hot solution inlet temperature and the air gap width have by far the strongest effect on the domain mass transfer resistance, mainly as a consequence of their effect on the air/vapor gap mass transfer resistance. Next in its effect is the cold solution inlet temperature.
- The inlet velocities of the hot and cold solutions have a "local" effect; they affect the mass transfer resistance of their domain, but because the mass transfer resistance of the hot solution in the AGMD process constitutes a smaller fraction of the total mass transfer resistance than in DCMD, increasing the inlet velocity of the hot solution  $(u_{hi})$  has more positive impact on improving the DCMD process (i.e. increasing the permeate flux and the process thermal efficiency).
- Efforts to reduce  $R_{\rm Mh}$  in AGMD at low  $T_{\rm hi}$  have a very small effect on the process, but they are worthwhile at the higher values of  $T_{\rm hi}$  in both DCMD and AGMD, with more room for improvement in DCMD. For example, reducing  $R_{\rm Mh}$  to zero at  $T_{\rm hi} = 80$  °C, increases the flux 2.77-fold for DCMD and 1.45-fold for AGMD, whereas it increases the flux by only 1.08-fold for AGMD at  $T_{\rm hi} = 40$  °C.
- In general, efforts to minimizing the mass transfer resistance of the cold solution will have relatively small effect on the permeate flux, because the mass transfer resistance of the cold stream is small compared with the resistances of the other domains. For example, at  $T_{\rm hi} = 80$  °C, the mass transfer

resistance of the cold solution constitutes only 15% of the total mass transfer resistance for DCMD, and just 4% for AGMD.

- While increasing the air/vapor gap width reduces the parasitic heat transfer by conduction, it also increases the mass transfer resistance of its domain and thus reduces the permeate flux. As shown in ref. [8], increasing the width beyond 2 mm has thus not improved the process thermal efficiency.
- The concentration of the solution has a slight effect on the process.
- The material used for the membrane should have low thermal conductivity for a more efficient MD process. Moreover, the membrane thermal conductivity affects the AGMD mainly by affecting its permeate flux, and affects DCMD mainly by affecting its thermal efficiency.

### Nomenclature

$A_1 - A_3$	see Eq. (9)
$D_{\rm v/a}$	diffusion coefficient of the vapor in the vapor/air
	mixture $(m^2 s^{-1})$
$d_{\mathrm{h}}$	half-width of the flow channel (m)
$d_{p}$	cooling plate thickness (m)
$\hat{J}$	membrane-length-averaged permeate flux at the
	hot side of the membrane $(\text{kg m}^{-1} \text{ h}^{-1})$
Κ	permeability of the membrane $(s m^{-1})$
k	thermal conductivity (W m <sup><math>-1</math></sup> K <sup><math>-1</math></sup> )
$l_{\rm m}$	membrane length (m)
М	molar mass (kg kmol <sup><math>-1</math></sup> )
m	membrane
Р	vapor pressure (Pa)
$R'_{Mi}$	mass transfer resistance of the <i>i</i> th domain, Eq. (2)
	((N h)/kg)
$R_{Mi}$	mass transfer resistance of the $i$ th domain, Eq. (2)
	((N h)/kg)
$R_{\rm MT}$	total mass transfer resistance ((N h)/kg)
R <sub>MT,max</sub>	maximal total mass transfer resistance for param-
	eter $i$ ((N h)/kg)
$r_{Mi}$	mass transfer resistance ratio of component <i>i</i> , Eq.
	(17)
$R_{\rm u}$	universal gas constant (J/kmol/K)
Т	temperature (°C)
u <sub>ci</sub>	the velocity at the inlet of the cold channel (m/s)
$u_{\rm hi}$	the velocity at the inlet of the hot channel (m/s)
$w_{ m si}$	mass fraction of NaCl at the inlet of the hot solu-
	tion
Greek le	etters
$\Delta P$	water vapor pressure difference across a domain
	(Pa)
δ	thickness or width (m)
ε	membrane porosity

Sub	corinte
Sub.	scripis

Subscr	Subscripts		
а	air		
avg	average		
c	cold solution		
сс	center of the cold channel		
ch	center of the hot channel		
ci	inlet of the cold channel		
co	outlet of the cold channel		
f	condensate film		
fp	condensate film/cooling plate interface		
g	vapor/air gap		
gf	air gap/condensate film interface		
h	hot channel		
hi	inlet of the hot channel		
hm	hot channel/membrane interface		
ho	outlet of the hot channel		
i	<i>i</i> th domain		
L	latent		
1	liquid water		
М	molar mass (kg kmol <sup>-1</sup> )		
m	membrane		
mg	membrane/air gap interface		
р	cooling plate		
pc	cooling plate/cold channel interface		
S	solution		
Т	total		
v	vapor		

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