Newton’s Second Law
Momentum
  - Linear momentum
  - Angular momentum

Work
Energy
  - Kinetic Energy
  - Potential Energy
Work

Work done by the force \( \mathbf{F} \) on the particle \( P \) over the path from \( Q \) to \( R \) is given by:

\[
W = \int_{Q}^{R} \mathbf{F} \cdot d\mathbf{r}
\]

Note \( O \) is a point fixed in an inertial frame

\[
dW = \mathbf{F} \cdot d\mathbf{r} = m \ddot{\mathbf{r}} \cdot d\mathbf{r} = \frac{1}{2} m \left( \dot{\mathbf{r}} \cdot \dot{\mathbf{r}} \right)
\]

The work done by \( \mathbf{F} \) is equal to the change in the kinetic energy of the particle

\[
W_{QR} = \int_{Q}^{R} \mathbf{F} \cdot d\mathbf{r} = \frac{1}{2} m \left( v_{R}^2 - v_{Q}^2 \right)
\]

Recall

\[
a = \frac{dv}{dt} \quad \text{ad}x = v \, dv
\]

\[
a = \frac{dv}{dx} \quad \text{ad}x = \frac{1}{2} d(v \cdot v)
\]

Only \( F_t \) does work!
F acts on the particle

- Work done by F
  \[ dW = F \cdot dr \]
- Power developed by F
  \[ \frac{dW}{dt} = F \cdot \frac{dr}{dt} = F \cdot v_p \]

- Units
  - **Metric**
    - Watts = Newton meter/second
  - **British**
    - Lb ft/second
    - Horsepower
      - 1 HP = 550 lb ft/sec
      - 1 HP = 746 W
Boeing 777-200

2 P&W turbofan engines providing ~ 74,000 lbs of thrust
Maximum take-off weight (MTOW) ~ 230,000 kg.
1 mile runway ~ 1600 m

What do you estimate the speed to be at the end of a 1 mile runway?

Estimated speed at the end of runway without drag = 96 m/s

But… take-off speed ~ 300 kmph (83.33 m/s)

What is the estimated drag force?

KE at the speed of 83.33 m/s = 798,610,000 Joules
Estimated average force through the length of the runway = 496 kN
Thrust = 2 × 329 kN

Drag force ~ 161 kN
Example

A particle of mass $m$ slides along a horizontal frictionless track which is shaped like a logarithmic spiral:

$$r = r_0 \exp(-a\theta)$$

If the initial speed is $v_0$ when $\theta=0$, find the speed of the particle and the magnitude of the track force acting on the particle as a function of $\theta$. 

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**FBD**

**IRD**
Example

Horizontal frictionless track which is shaped like a logarithmic spiral

\[ r = r_0 \exp(-a\theta) \]

The initial speed is \( v_0 \) when \( \theta = 0 \)

Find the speed of the particle and the magnitude of the track force acting on the particle.

\[ \mathbf{r} = r \mathbf{b}_1 \]

\[ \mathbf{v}_P = r\dot{\theta} (-a \mathbf{b}_1 + \mathbf{b}_2) = v \mathbf{e}_2 \]

Track force = \( N \mathbf{e}_1 \)

Force in the \( \mathbf{e}_2 \) direction?

\[ \mathbf{e}_1 = \frac{\mathbf{b}_1 + ab_2}{\sqrt{1 + a^2}} \] (unit normal)

\[ \mathbf{e}_2 = \frac{-ab_1 + b_2}{\sqrt{1 + a^2}} \] (Same as \( \mathbf{e}_t \))
Example

Since \( \mathbf{N} \) is normal to the track, \( d\mathbf{r} \) is tangential to the track.

\[
W = \int_{r_0}^{r} \mathbf{N} \cdot d\mathbf{r} = \]

Therefore the speed of the particle is…

\[
\mathbf{v}_P = \mathbf{v} \mathbf{e}_2 \quad \Rightarrow \quad \mathbf{a}_P = \]

Newton’s Laws

\[
\frac{d}{dt} \left( m \mathbf{v}_P \right) = \mathbf{N} \mathbf{e}_1 \\
\frac{d}{dt} \left( r\mathbf{b}_1 \times m \mathbf{v}_P \right) = r\mathbf{b}_1 \times \mathbf{N} \mathbf{e}_1
\]
Conservative Force Field

\( \mathbf{F} \) is conservative

- \( \mathbf{F} \) is a function only of the position of the particle and the work done by the force \( \mathbf{F} \) on the particle \( P \) to get it from \( Q \) to \( R \) is independent of the path

- \( \mathbf{F} \) is a function only of the position of the particle and the work done by the force \( \mathbf{F} \) on the particle \( P \) is zero along any closed path

There exists a scalar function \( \phi \) (\( PE \)) such that

\[
dW = \mathbf{F} \cdot d\mathbf{r} = -d\phi
\]

- There exists a scalar function \( \phi \) (\( PE \)) and a coordinate \( s \) such that*

\[
F = -\frac{d\phi}{ds}
\]

For multiple coordinates say \( x \) and \( y \):

\[
F_x = -\frac{\partial \phi}{\partial x}, \quad F_y = -\frac{\partial \phi}{\partial y}
\]
Conservation of Mechanical Energy

F is conservative

- There exists a scalar function \( \phi \) such that
  \[
dW = \mathbf{F} \cdot d\mathbf{r} = -d\phi
  \]

- Work done by \( \mathbf{F} \)
  \[
  W_{QR} = \int_{Q}^{R} \mathbf{F} \cdot d\mathbf{r} = \frac{1}{2} m \left( v_R^2 - v_Q^2 \right)
  \]
  \[
  = \phi(Q) - \phi(R)
  \]

- Total energy is constant
  \[
  \frac{1}{2} m \left( v_R^2 \right) + \phi(R) = \frac{1}{2} m \left( v_Q^2 \right) + \phi(Q)
  \]
Example

Find the magnitude of the force $T$ which acts on the midpoint of the crank, given the force $F$ acts on the piston. Neglect inertia, friction and gravity.

Velocity Equations (from before)

\[
\begin{align*}
\dot{r}_1 &= r_2 \frac{\sin(\theta_3 - \theta_2)}{\cos(\theta_3)} \dot{\theta}_2 \\
\dot{\theta}_3 &= -\frac{r_2 \cos(\theta_2)}{r_3 \cos(\theta_3)} \dot{\theta}_2
\end{align*}
\]