

# Definitions

- Degrees of freedom of a system

The number of independent variables (or coordinates) required to completely specify the configuration of the system.

- Point on a plane (2)                      ◆ 2 planar links connected by a pin joint (4)
- Point in 3-D space (3)                 ◆ Human shoulder
- Line on a plane (4)                      ◆ Car

- Kinematic chain

A system of rigid bodies connected together by joints. A chain is called closed if it forms a closed loop. A chain that is not closed is called an open chain.

- Serial chain

If each link of an open chain except the first and the last link is connected to two other links it is called a serial chain.

## Definitions (continued)

- Joints

Joints are connections between links

- Revolute, rotary, or pin joint ( $R$ )
- Prismatic, sliding, or telescoping joint ( $P$ )
- Helical or screw joint ( $H$ )
- Spherical or ball joint ( $S$ )

- Planar kinematic chain

All the links are constrained to move in or parallel to the same plane.

A planar chain can only allow prismatic and revolute joints. In fact, the axes of the revolute joints must be perpendicular to the plane of the chain while the axes of the prismatic joints must be parallel to or lie in the plane of the chain.

- Connectivity of a joint

The number of degrees of freedom of a rigid body connected to a fixed rigid body through the joint.

# Degrees of Freedom and Constraints

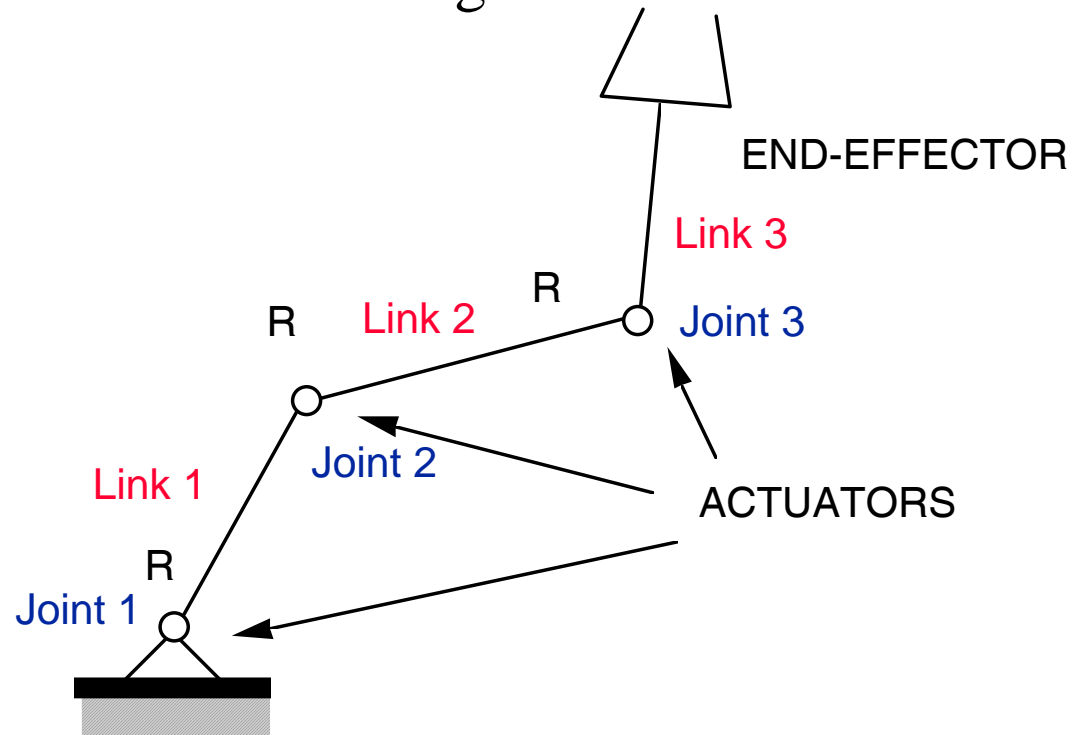
Number of Degrees of Freedom (for planar mechanisms)

= 3 (number of moving rigid bodies)

- Constraints due to joints/connections

# The Planar 3-R manipulator

- Planar kinematic chain
- All joints are revolute with connectivity = 1
- What is the number of degrees of freedom?



## Connectivity, Mobility, and Degrees of Freedom

### □ Connectivity of a joint

The number of degrees of freedom of a rigid body connected to a fixed rigid body through the joint

- prismatic joint (1)
- revolute joint (1)
- ◆ spherical joint (3)
- ◆ helical joint (1)

### □ Mobility of a chain

Number of degrees of freedom of the chain

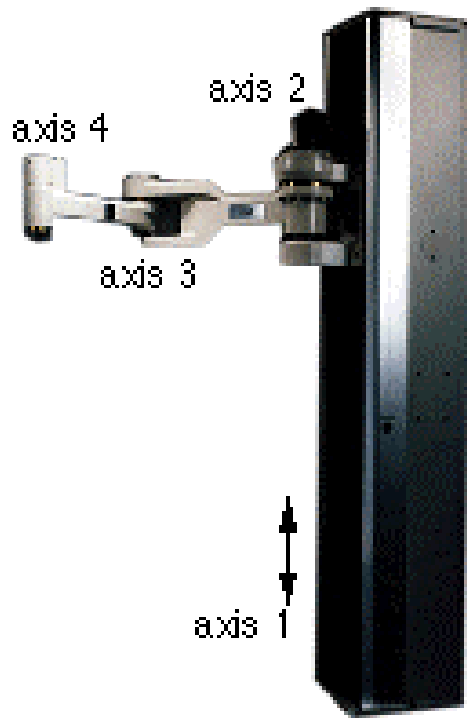
#### ● Serial chain

- Examples
  - ❖ 3-R chain
    - $M=?$

$$M = \sum_{i=1}^n f_i$$

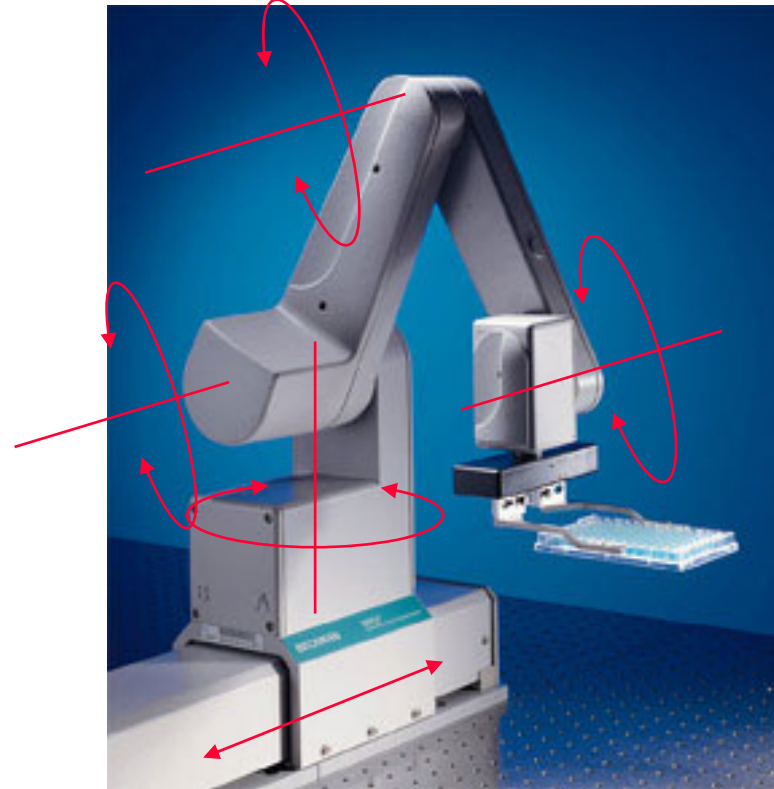
## Examples: Mobility (Degrees of Freedom)

### The Adept 1850 Palletizer



$$M = 4$$

### The Beckman Coulter ORCA



Five degrees of freedom

## Connectivity, Mobility, and Degrees of Freedom

### ◆ Mobility of a chain

Number of degrees of freedom of the chain

- General expression

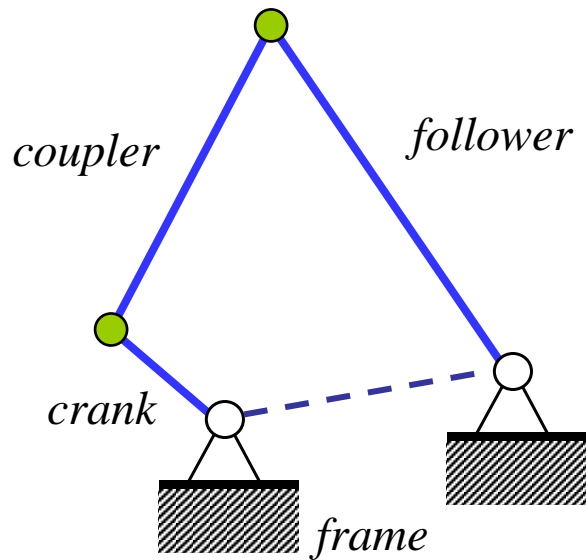
$$M = 3(n - j - 1) + \sum_{i=1}^j f_i$$

$n$  number of links

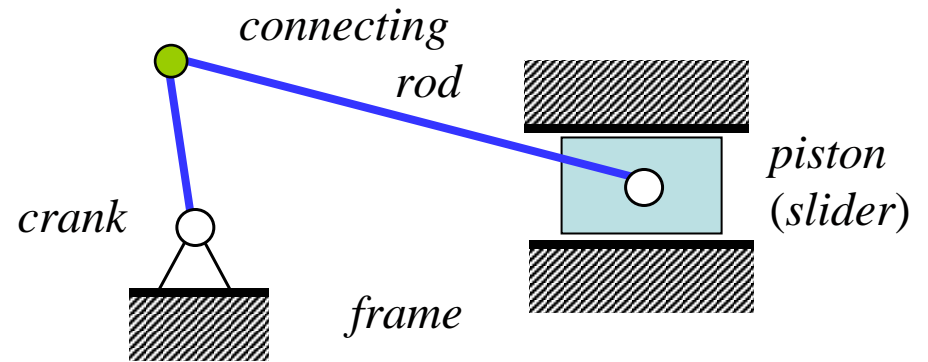
$j$  number of joints between the links

$f_i$  connectivity of joint  $i$

# Most Commonly Encountered Linkages

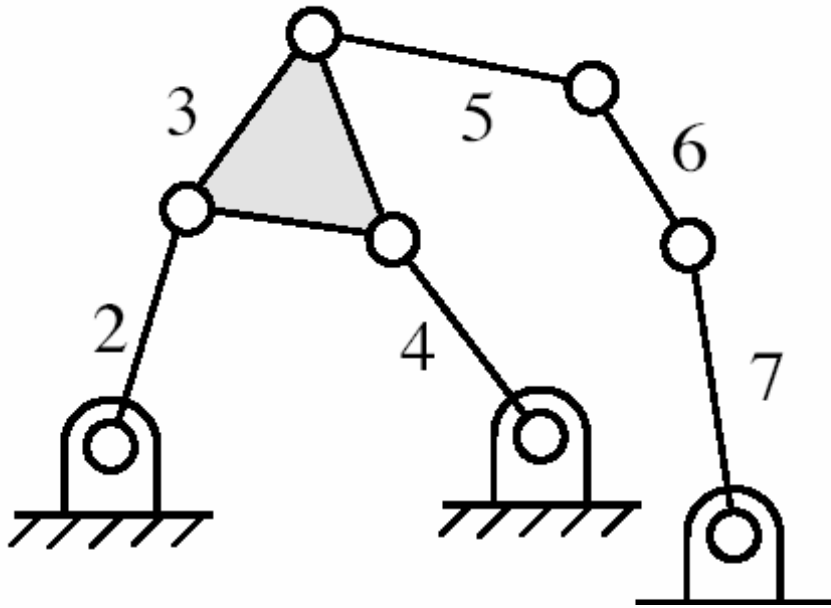


Four-bar linkage



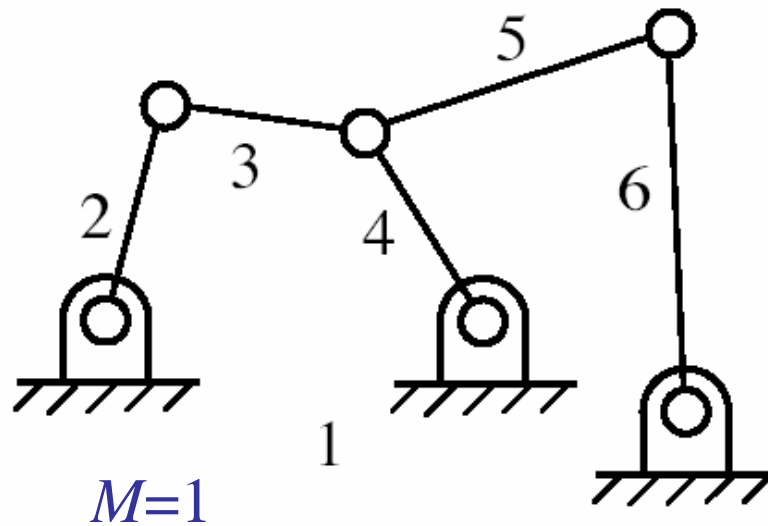
Slider-crank linkage

# Closed Chain Mechanisms

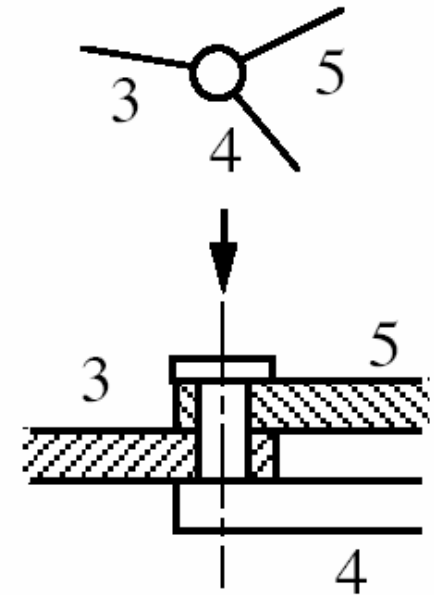


$$M = 3(n - j - 1) + \sum_{i=1}^j f_i$$

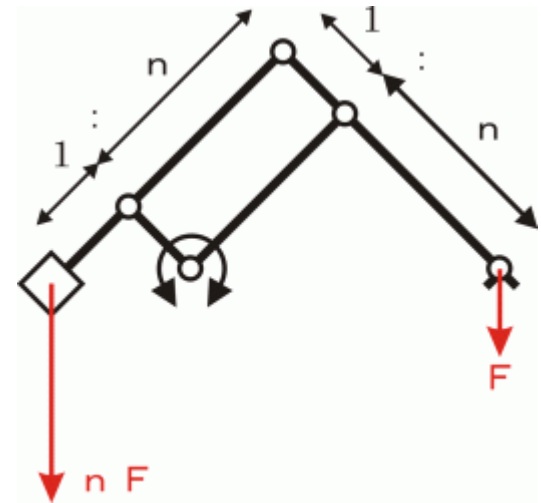
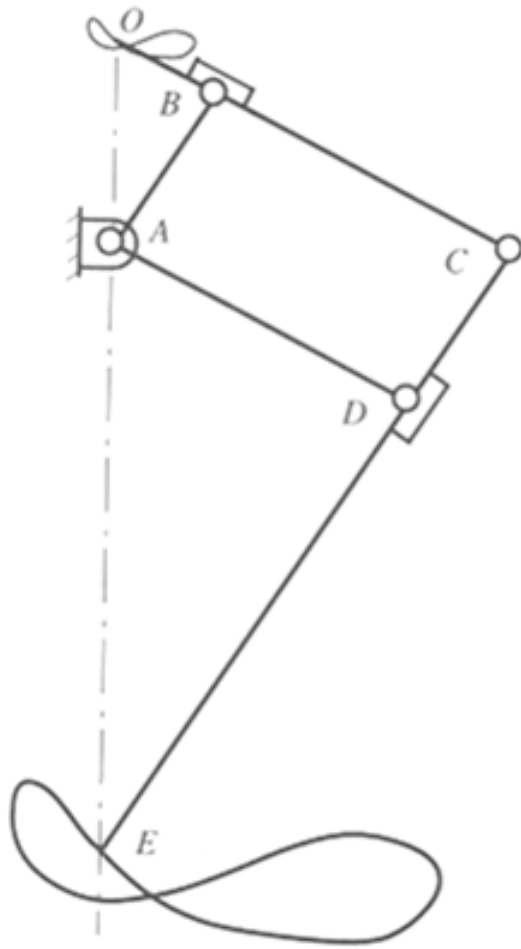
$M=2$



$M=1$



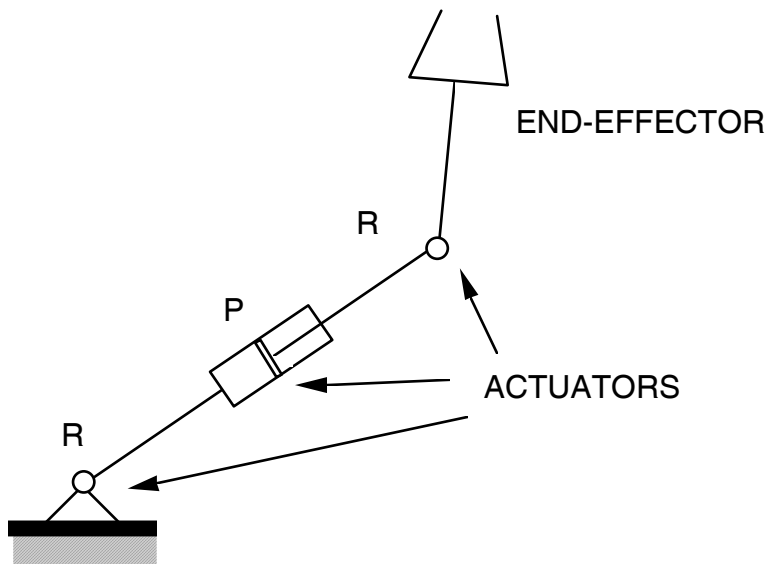
# The Pantograph Mechanism



## Examples: Mobility (Degrees of Freedom)

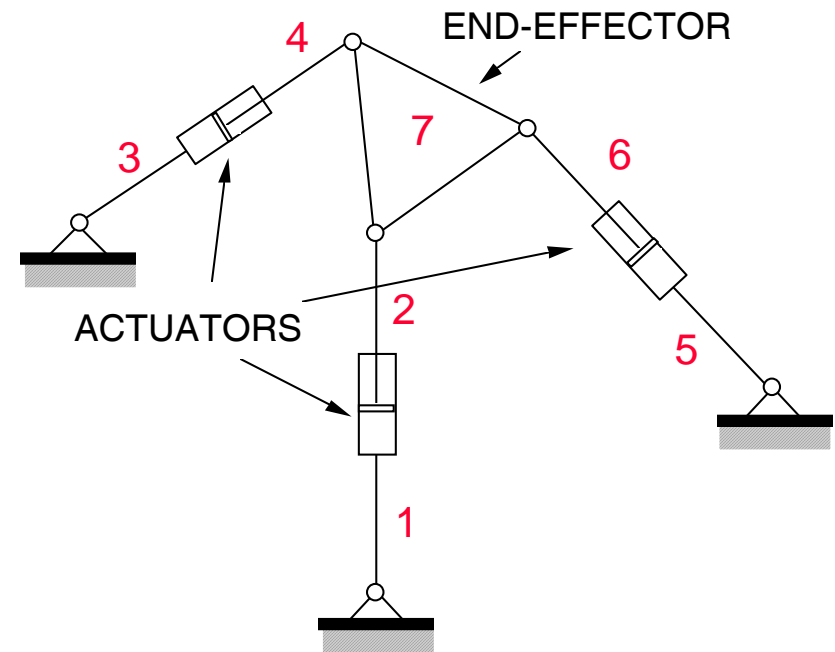
### □ Planar serial chain

- number of links,  $n = 4$
- number of joints,  $j = 3$
- connectivity,  $f_i = 1$



### □ Planar parallel manipulator

- number of links,  $n = 8$
- number of joints,  $j = 9$
- connectivity,  $f_i = 1$



$$M = 3(n - j - 1) + \sum_{i=1}^j f_i$$

# Four Bar Linkage

- $n=4$
- $j=4$
- $M=4$

*Types of four bar linkages*

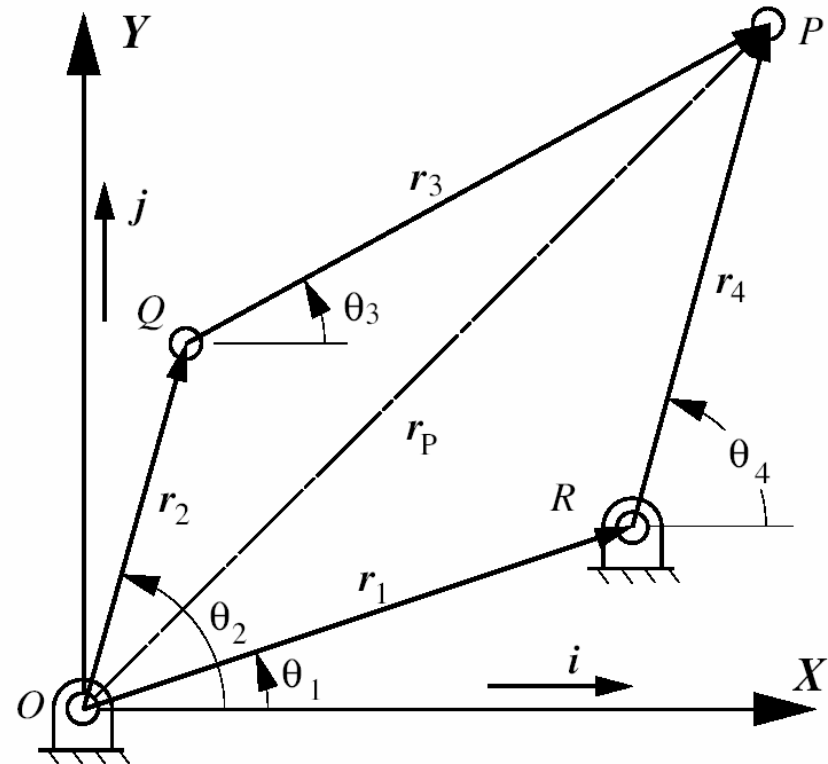
## Grashof Type I

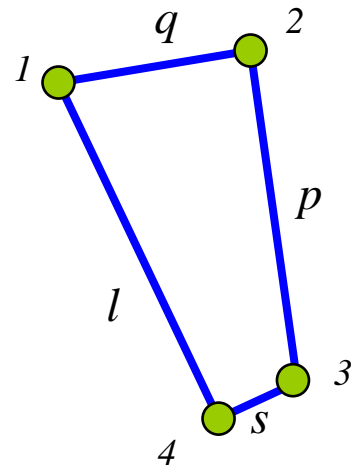
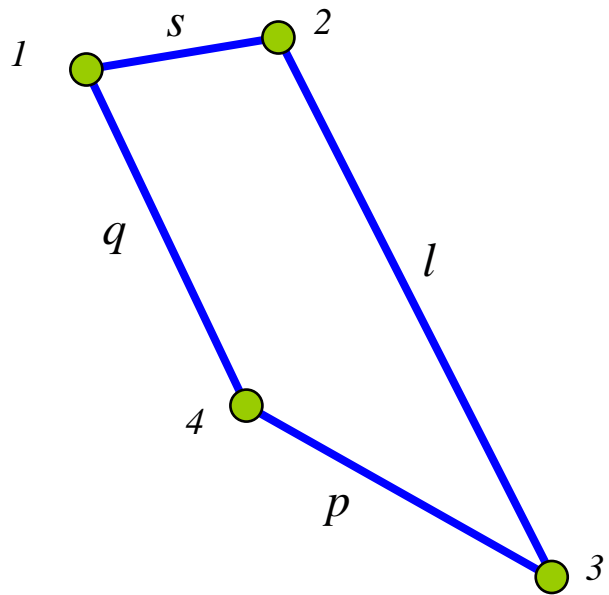
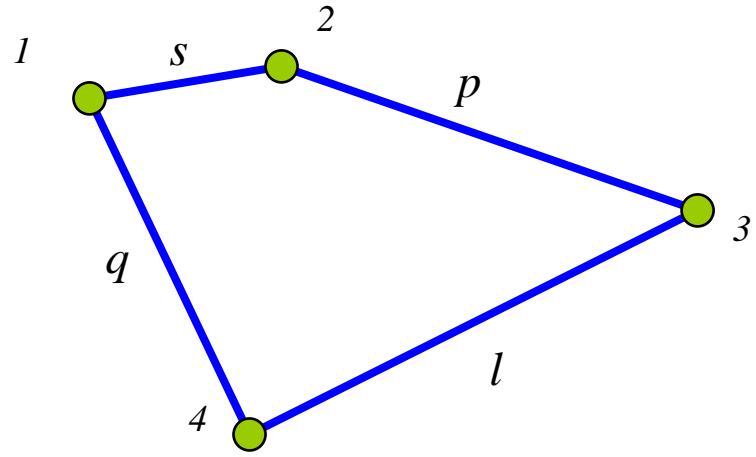
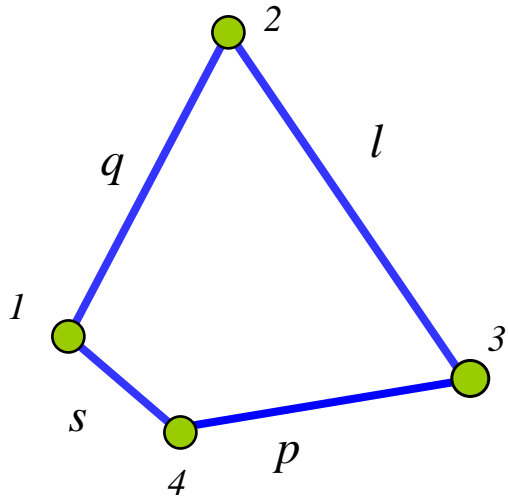
- Crank rocker
- Double-rocker
- Drag-link

$$s + l < p + q$$

## Non-Grashof or Grashof Type II

- Triple-rocker
- 





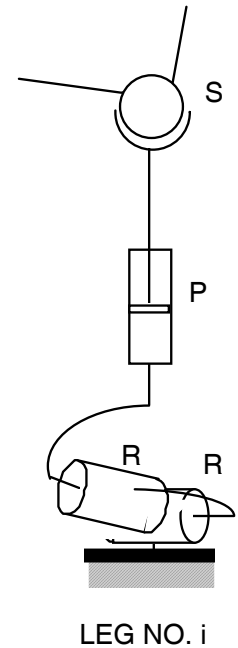
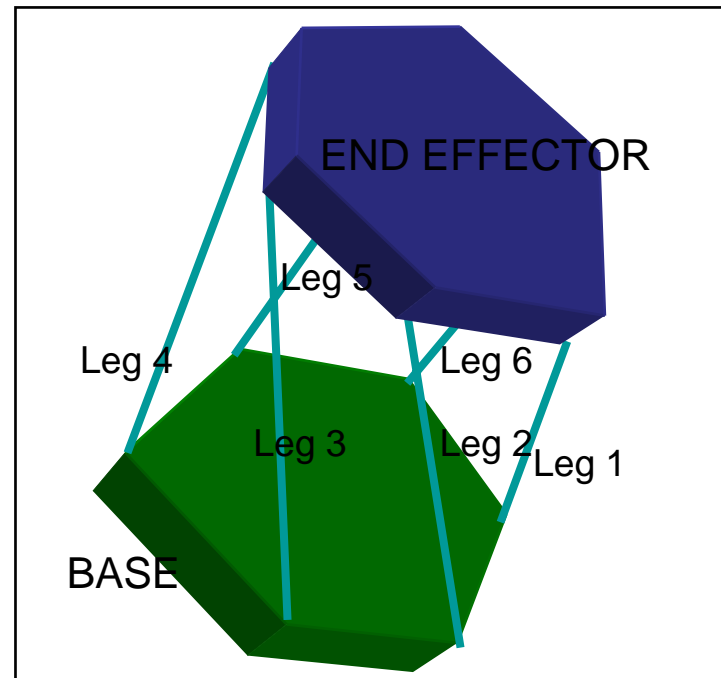
## Useful Websites

- ❑ <http://www.powertransmission.com/>
  - Gearing
- ❑ <http://www.howstuffworks.com>
  - General, particularly good for automotive mechanisms
- ❑ <http://www.flying-pig.co.uk/mechanisms/>
  - Tutorial on mechanisms
- ❑ <http://www.btinternet.com/~hognozesam/gcse/page2.html>
  - Tutorial on mechanisms

## Mobility of 3-dimensional mechanisms

### □ Ingersoll Rand machine tool (Stewart Platform)

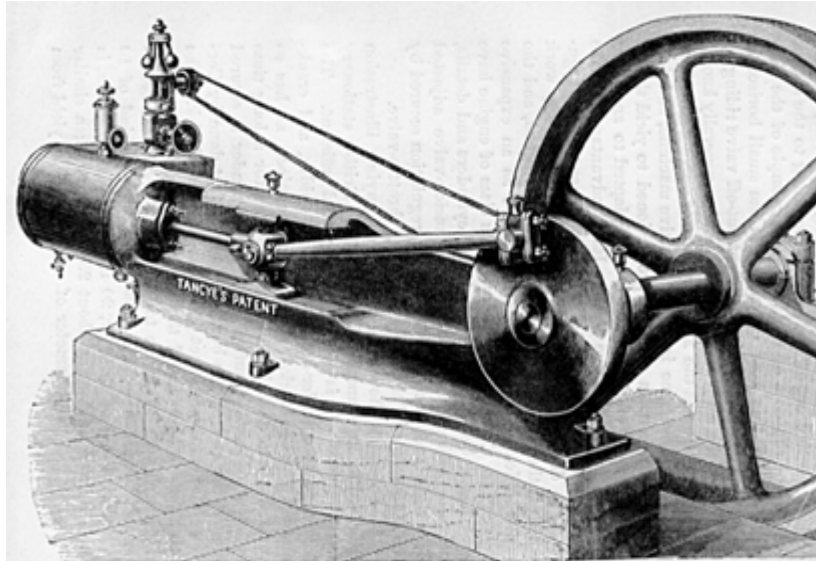
- number of links,  $n = 20$
- number of joints,  $j = 24$
- connectivity,  $f_i = 1$  or 3



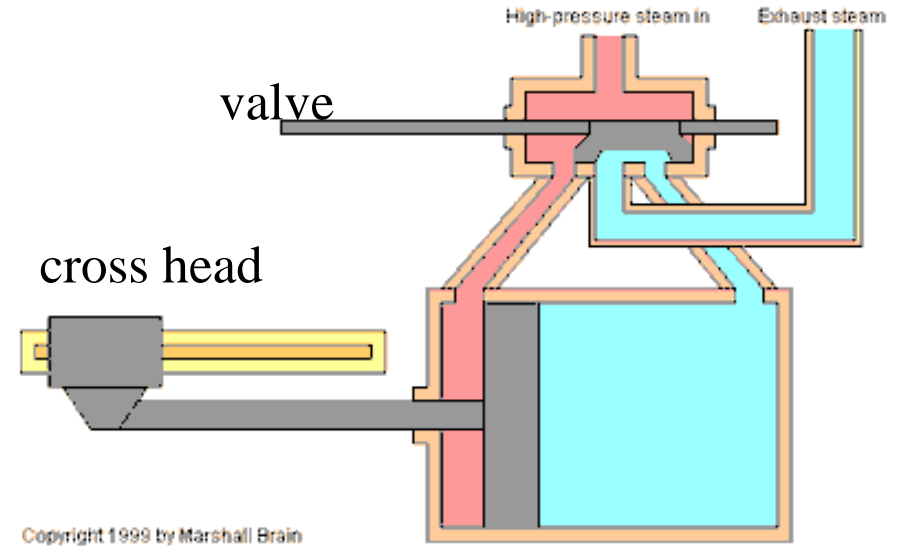
$$M = 6(n - j - 1) + \sum_{i=1}^j f_i$$

$$M = 6(19 - 24) + 36 = 6$$

# Double Acting Steam Engine



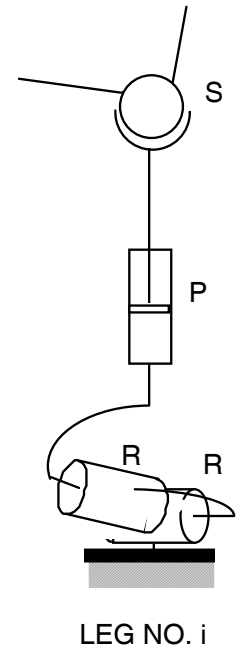
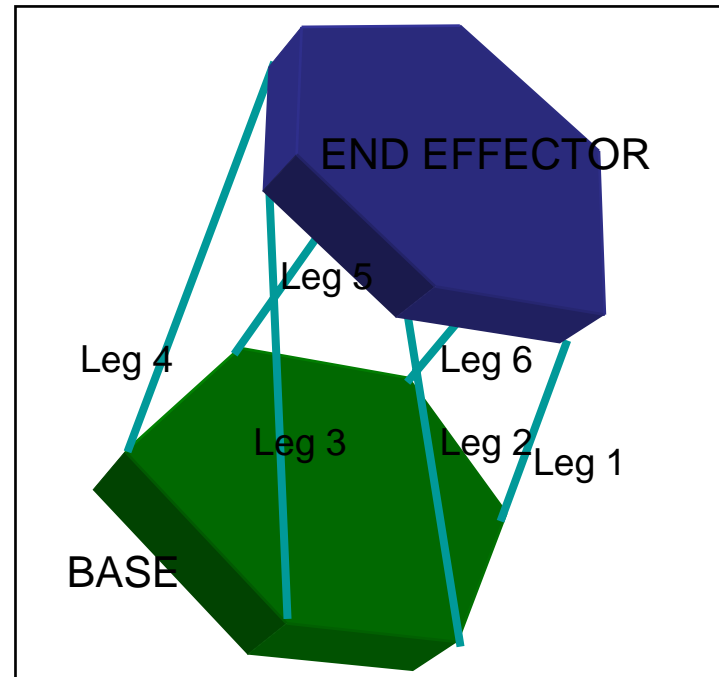
<http://www.history.rochester.edu/steam/thurston/1878/>



<http://travel.howstuffworks.com/steam1.htm>

# Six Degree-of-Freedom Manipulator

## Micromanipulator (PI Hexapod)



Six degrees of freedom