Mechanism

Any device that transforms motion

- Amplification
- Change type of motion
  - rotation ↔ translation
  - uniform ↔ reciprocating/oscillating

Cam follower

Crank connecting rod

Sprocket and chain

Slider-crank linkage

Piston (slider)

Frame
Goal

- Understand kinematics of mechanisms
  - Focus on motion
- Apply particle kinematics
  - Focus on motions of particles attached to parts of mechanisms
  - Interested in constrained motion

Note:
Technically machines are devices that transmit or modify energy/forces.
Mechanisms transmit/modify motion.
Position Vectors

Position Vector(s)

\[
\mathbf{r}_{A/O} = \mathbf{r}_{B/O} + \mathbf{r}_{A/B}
\]

Triangle law of vector addition

Note: No need for a x-y-z coordinate system
Velocity and Acceleration Vectors

What do these vectors really mean?

\[ \mathbf{r}_{A/O} = \mathbf{r}_{B/O} + \mathbf{r}_{A/B} \]

\[ \frac{d}{dt} \mathbf{r}_{A/O} = \frac{d}{dt} \mathbf{r}_{B/O} + \frac{d}{dt} \mathbf{r}_{A/B} \]

\[ \frac{d}{dt} \left( \frac{d}{dt} \mathbf{r}_{A/O} \right) = \frac{d}{dt} \left( \frac{d}{dt} \mathbf{r}_{B/O} \right) + \frac{d}{dt} \left( \frac{d}{dt} \mathbf{r}_{A/B} \right) \]
Differentiation of Vectors

Assume

- All coordinate systems are parallel (for now)
- \( O \) is fixed to an inertial frame

Justified because we are dealing with particles and not rigid bodies!
Velocity and Acceleration Vectors

\[ \frac{d}{dt} \mathbf{r}_{A/O} = \frac{d}{dt} \mathbf{r}_{B/O} + \frac{d}{dt} \mathbf{r}_{A/B} \]

\[ \mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B} \]

Why have dropped the subscript \( /O \) from \( \mathbf{v}_A /O \)? And from \( \mathbf{v}_{B/O} \)?

\[ \mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{A/B} \]
Constraints

Constraints on the position (configuration) of a particle (or system of particles).

- Particle in 3-D, but constrained to lie on a plane:
  \[ Ax + By + Cz + D = 0 \]

- A particle suspended from a string in three dimensional space.
  \[ (x - a)^2 + (y - b)^2 + (z - c)^2 - r^2 = 0 \]

- A particle on spinning platter (carousel)
  \[ x = a \cos(\omega t + \phi); \]
  \[ y = a \sin(\omega t + \phi) \]
Pulleys, cables, blocks and tackles

Block and Tackle assembly

- An assembly of blocks, pulleys, and cables that allow a single cable to hoist a large weight (or apply a large force)

American Heritage

http://www.jimloy.com/cindy/block.htm
Example

If she moves to the left at 2 m/sec, how fast does the mass $m$ move? I.e., what is the velocity of the mass?

Exercise 2.5.10 (p. 85)
Example

A is pulled down at 3 \textit{m/s}. What is the motion of \textit{B}?

Exercise 2.5.11 (p. 85)
Four Bar Linkages

crank

coupler

follower

crank

frame
\[ \mathbf{r}_{B/A} = r_i \cos \theta_i \mathbf{i} + r_i \sin \theta_i \mathbf{j} \]