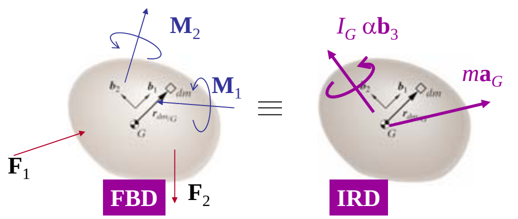
Summary: Newton's laws

- Force balance

$$\mathbf{F} = \sum_{i=1}^{N} \mathbf{F}_i = m\,\mathbf{a}_G$$

- Moment balance

$$\frac{d\,\mathbf{H}_G}{dt} = \sum \mathbf{M}_G$$



Beyond Force Balance

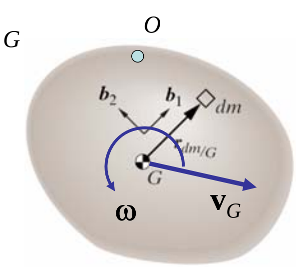
- Linear momentum
  = mass times velocity of center of mass
- Angular momentum
  recap
- Work
- Energy

Angular momentum about O and G

Angular momentum about *O*

= Angular momentum about *G*

\+ Angular momentum of particle of mass *m* at *G*



Kinetic Energy of Rigid Bodies undergoing Planar Motion

- Kinetic energy of infinitesimal element, *dm*

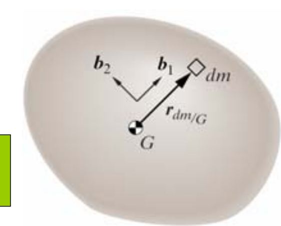
$$KE_{dm} = \frac{1}{2} dm\left(\mathbf{v}_{dm} \cdot \mathbf{v}_{dm}\right)$$

- Kinetic energy of the body

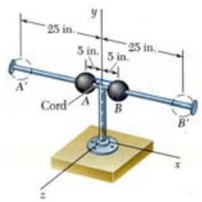
$$KE = \frac{1}{2} m\left(\mathbf{v}_G \cdot \mathbf{v}_G\right) + \frac{1}{2} I_G\left(\omega \cdot \omega\right)$$

Translational kinetic energy associated with a particle of mass *m* moving with the center of mass

Rotational kinetic energy associated with a rigid body of mass *m* and inertia I_G rotating about the center of mass



Example 1

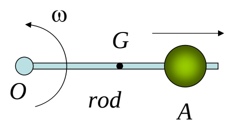


Determine the angular velocity of the rod after the balls have moved to *A'* and *B'*.

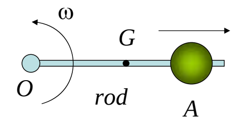
Two solid spheres (radius r = 3 in., mass m, W = 2 lb) are mounted on a spinning horizontal rod (R) with angular velocity ω = 6 rad/sec as shown. The mass moment of inertia of the rod is I_R. The balls are held together by a string which is suddenly cut.

$$\bar{I}_R = 0.25\,\text{lb}\cdot\text{ft}\cdot\text{s}^2$$

Let us first do a problem in calculating the angular momentum of a system



Example: Angular momentum of a system of rigid bodies



Angular momentum about O

= Angular momentum of rod about G

\+ Angular momentum of particle of mass of the rod at G

\+ Angular momentum of sphere about A

\+ Angular momentum of particle of mass of the sphere at A

Linear/Angular Impulse and Momentum

- Force balance

$$\mathbf{F} = \sum_{i=1}^{N} \mathbf{F}_i = m\frac{d\,\mathbf{v}_G}{dt}$$

- Moment balance

$$\frac{d\,\mathbf{H}_G}{dt} = \mathbf{M}_G$$

or, for a *fixed point, O*

$$\frac{d\,\mathbf{H}_O}{dt} = \mathbf{M}_O$$

- Linear momentum

$$LI_{1-2} = \int_{t_1}^{t_2} \mathbf{F}dt = m\mathbf{v}_G(t_2) - m\mathbf{v}_G(t_1)$$

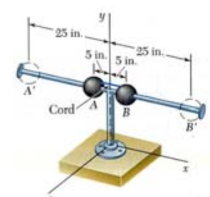
- Angular momentum

$$AI_{G1-2} = \int_{t_1}^{t_2} \mathbf{M}_G dt = \mathbf{H}_G(t_2) - \mathbf{H}_G(t_1)$$

or, for a *fixed point, O*

$$AI_{O1-2} = \int_{t_1}^{t_2} \mathbf{M}_O dt = \mathbf{H}_O(t_2) - \mathbf{H}_O(t_1)$$

Example 1



Two solid spheres (radius r = 3 in., mass m, W = 2 lb) are mounted on a spinning horizontal rod (R) with angular velocity ω = 6 rad/sec as shown. The mass moment of inertia of the rod is I_R. The balls are held together by a string which is suddenly cut.

$$\bar{I}_R = 0.25\,\text{lb}\cdot\text{ft}\cdot\text{s}^2$$

Determine the angular velocity of the rod after the balls have moved to *A'* and *B'*.

Key Observation:

- None of the external forces produce a moment about the y axis, the angular momentum is conserved.
- The angular momentum consists of the angular momenta of the spheres and the angular momentum of the rod

Work (from particle dyamics)

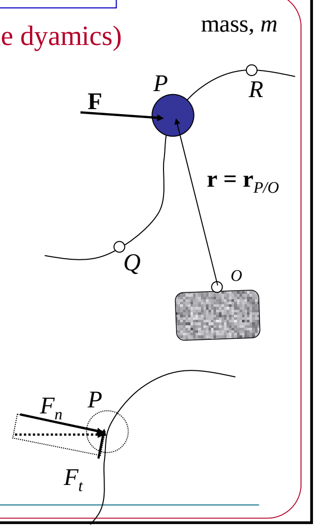
- Work done by the force **F** on the particle P over the path from Q to R is given by:

$$W = \int_Q^R \mathbf{F}\cdot d\mathbf{r}$$

- The work done by **F** is equal to the change in the *kinetic energy* of the particle

$$W_{QR} = \int_Q^R \mathbf{F}\cdot d\mathbf{r} = \frac{1}{2}m\left(v_R^2 - v_Q^2\right)$$

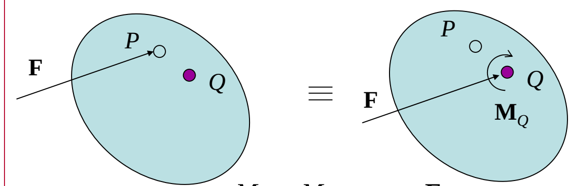
How do you calculate the work done for a rigid body?



Equivalent systems (1)

For any two points, P and Q

$$M_Q = M_P + \mathbf{r}_{Q/P} \times \mathbf{F}$$

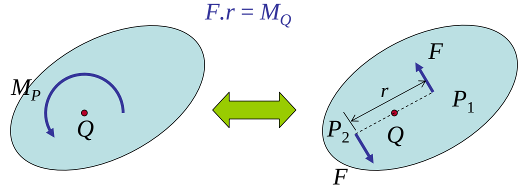


$$M_Q = M_P + \mathbf{r}_{Q/P} \times \mathbf{F}$$
$$= \mathbf{r}_{Q/P} \times \mathbf{F}$$

Equivalent systems (2)

A (pure) moment is equivalent (equipollent) to a set of equal and opposite forces of magnitude F separated by a distance r such that

$$F.r = M_Q$$



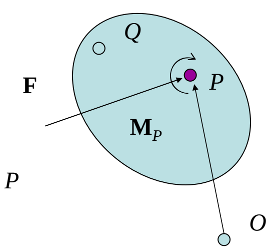
Work done by forces and moments

Work done by a force **F** acting at P

$$dW = \mathbf{F}\cdot d\mathbf{r}_{P/O}$$

or

$$dW = \mathbf{F}\cdot v_P dt$$



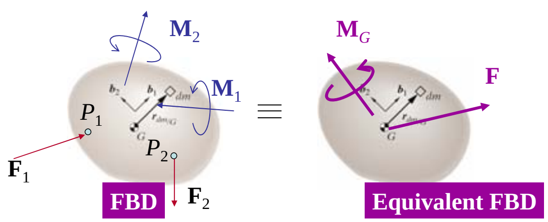
Work done by $\mathbf{M}_P$, a moment about P

$$dW = \mathbf{F}\cdot M d\theta$$

or

$$dW = \mathbf{F}\cdot M\omega dt$$

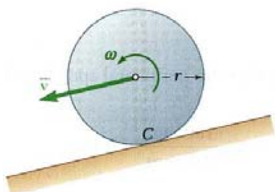
Work done for a rigid body



$$\mathbf{F} = \sum_{i=1}^{N} \mathbf{F}_i$$

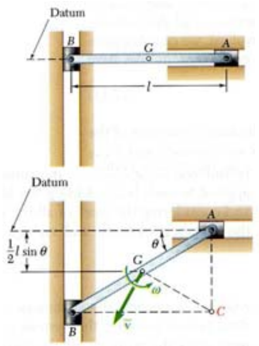
$$\mathbf{M}_G = \sum \mathbf{M}_j + \sum \mathbf{r}_{P_i/G} \times \mathbf{F}_i$$

Example 2



A sphere, cylinder, and hoop, each having the same mass and radius, are released from rest on an incline. Determine the velocity of each body after it has rolled through a distance corresponding to a change of elevation *h*.

Example 3



- mass *m*
- released with zero velocity
- determine ω at θ