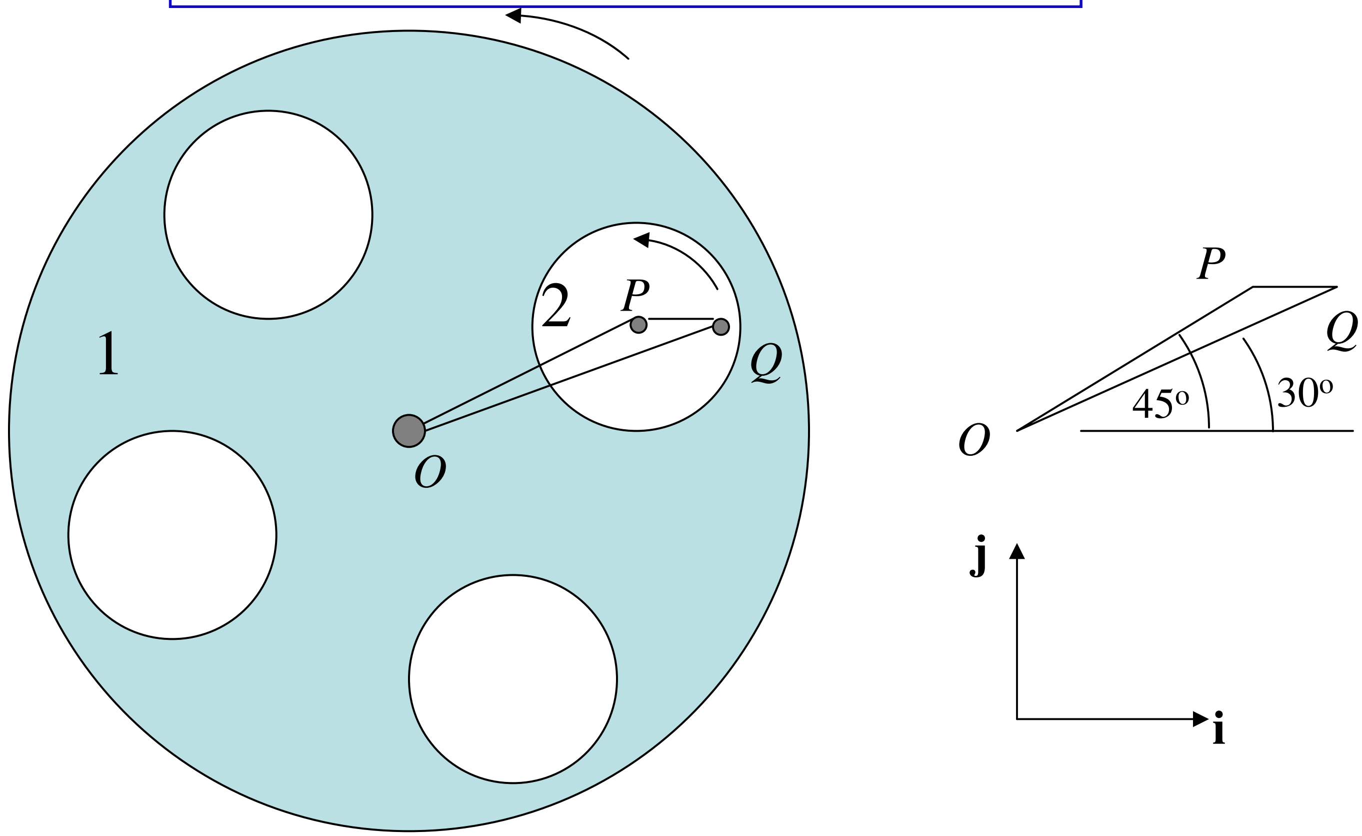
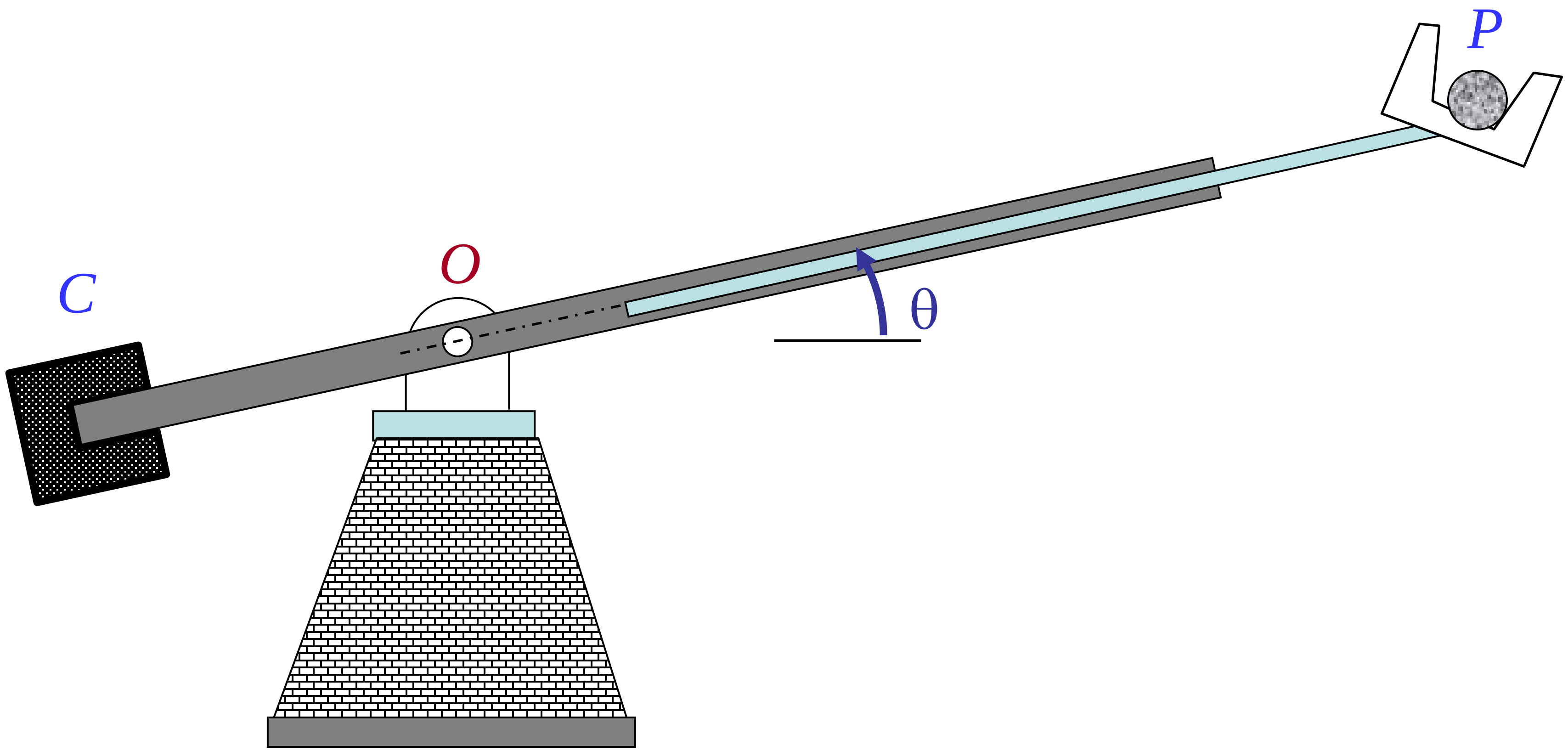


The amusement park ride shown in Figure 2 consists of two rotating platforms 1 and 2 with the riders on seats attached to 2. The rider in question is sitting at point P. The carousel (body 1 in the figure) rotates at the uniform angular velocity $\omega_1 = 1 rad/sec$ $\mathbf{k}$ with the axis of rotation through point O. The smaller platform (body 2 in the figure) is pinned to carousel 1 at point P, and it rotates at the uniform angular velocity $\omega_2 = 2 rad/sec$ $\mathbf{k}$. (If you want to think about relative angular velocities, the angular velocity of the platform 2 is 1 rad/second relative to an observer on platform 1). The length OP is 2 meters while PQ is 1 meter. Find the acceleration of the rider Q.



A trebuchet with a telescopic link (one of the many designs exhibited on April 17) has the projectile secured in a cup as shown in Figure 1. At the instant shown, the length OP is 0.5 meters but is extending outwards at a uniform rate of 1 $meter/sec$. The angle $\theta = 30°$, $\dot{\theta} = 1 rad/s$, and $\ddot{\theta} = 0.1 rad/s^2$. What is the acceleration vector, $\mathbf{a}_P$, for the projectile? Clearly draw the unit vectors used in your solution procedure.

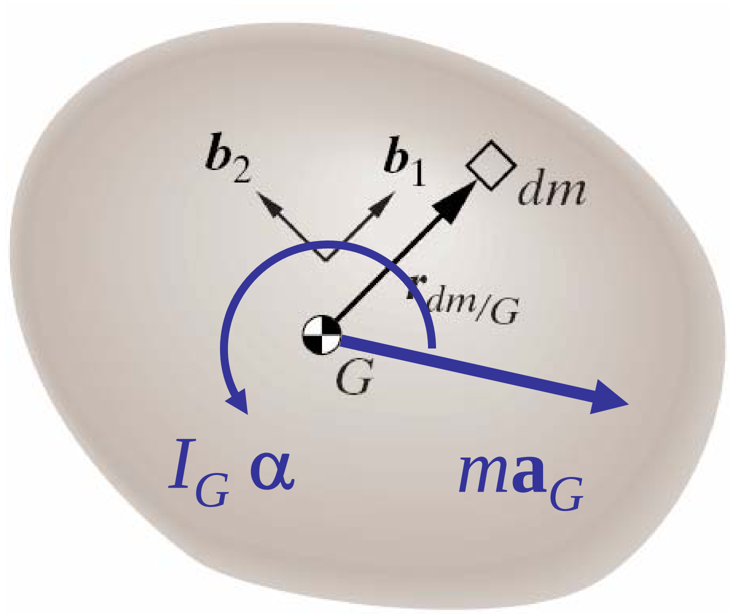
Kinetics of planar rigid bodies

Impulse and Momentum

- Linear momentum
- Angular momentum

$$\mathbf{F} = \sum_{i=1}^{N} \mathbf{F}_i = m \frac{d\,\mathbf{v}_G}{dt}$$

$$\mathbf{M}_G = \sum_i \mathbf{M}_{i,G} = \frac{d\,\mathbf{H}_G}{dt} = I_G \frac{d\omega}{dt}$$



Inertia Response Diagram

Linear/Angular Impulse and Momentum

- Force balance

$$\mathbf{F} = \sum_{i=1}^{N} \mathbf{F}_i = m \frac{d\,\mathbf{v}_G}{dt}$$

- Moment balance

$$\frac{d\,\mathbf{H}_G}{dt} = \mathbf{M}_G$$

or, for a *fixed point*, *O*

$$\frac{d\,\mathbf{H}_O}{dt} = \mathbf{M}_O$$

- Linear momentum

$$LI_{1-2} = \int_{t_1}^{t_2} \mathbf{F}\,dt = m\mathbf{v}_G(t_2) - m\mathbf{v}_G(t_1)$$

- Angular momentum

$$AI_{G_{1-2}} = \int_{t_1}^{t_2} \mathbf{M}_G\,dt = \mathbf{H}_G(t_2) - \mathbf{H}_G(t_1)$$

or, for a *fixed point*, *O*

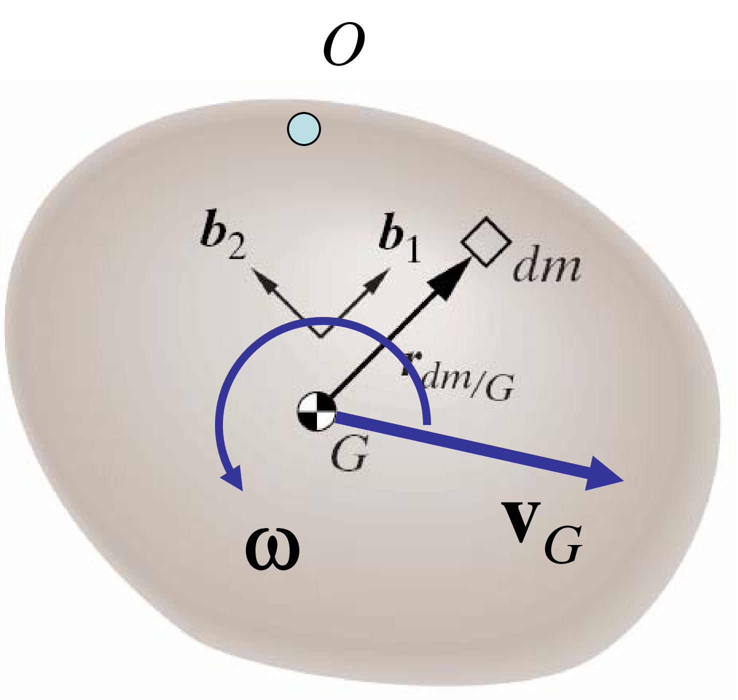
$$AI_{O_{1-2}} = \int_{t_1}^{t_2} \mathbf{M}_O\,dt = \mathbf{H}_O(t_2) - \mathbf{H}_O(t_1)$$

Angular momentum about O and G

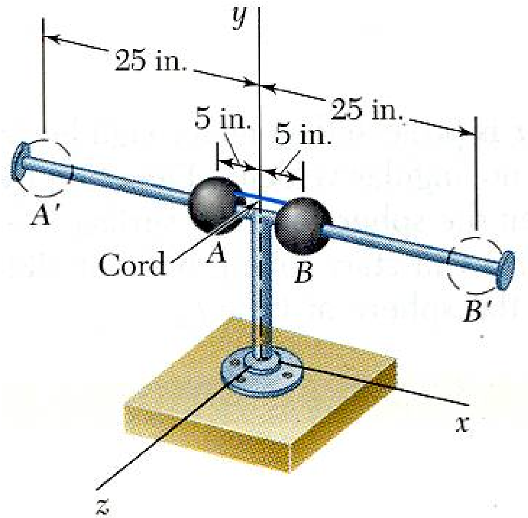
Angular momentum about *O*

= Angular momentum about *G*

+ Angular momentum of particle of mass *m* at *G*



Example 1

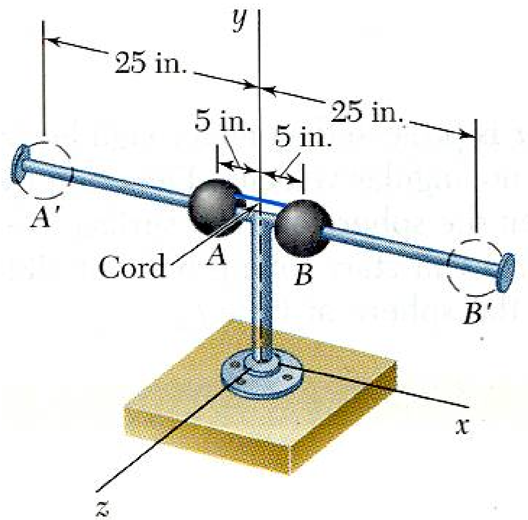


Determine the angular velocity of the rod after the balls have moved to *A'* and *B'*.

Two solid spheres (radius r = 3 in., mass m, W = 2 lb) are mounted on a spinning horizontal rod (R) with angular velocity ω = 6 rad/sec as shown. The mass moment of inertia of the rod is I_R. The balls are held together by a string which is suddenly cut.

$$\bar{I}_R = 0.25\,\mathrm{lb}\cdot\mathrm{ft}\cdot\mathrm{s}^2$$

Example 1



Two solid spheres (radius r = 3 in., mass m, W = 2 lb) are mounted on a spinning horizontal rod (R) with angular velocity ω = 6 rad/sec as shown. The mass moment of inertia of the rod is I_R. The balls are held together by a string which is suddenly cut.

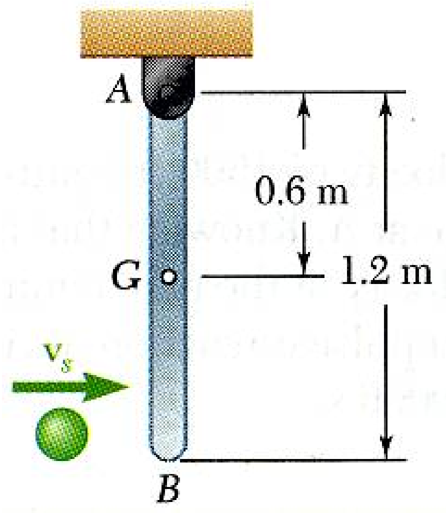
$$\bar{I}_R = 0.25\,\mathrm{lb \cdot ft \cdot s^2}$$

Determine the angular velocity of the rod after the balls have moved to *A'* and *B'*.

Key Observation:

- None of the external forces produce a moment about the *y* axis, the angular momentum is conserved.
- The angular momentum consists of the angular momenta of the spheres and the angular momentum of the rod

Example 2



A 2-kg sphere with an initial velocity of 5 m/s strikes the lower end of an 8-kg rod *AB*. The rod is hinged at *A* and initially at rest. The coefficient of restitution between the rod and sphere is 0.8.

Determine the angular velocity of the rod and the velocity of the sphere immediately after impact.

FBD/IRD for the system

FBD/IRD for the system