MEAM 211

Project 1

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University of Pennsylvania

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1 Introduction

The goal of this project is to investigate the dynamics of projectiles (balls) used in sports and study their trajectories using the numerical integration capability built into MATLAB.

2 Background

Soccer players are able to find the most incredible ways to kick the soccer ball past defenders into the goal making the ball curve and drop in ways that are hard to predict. Go to `http://www.youtube.com/` and search for "David Beckham Freekick" or "Ronaldinho Freekick" to see some truly amazing goals. Golf players know how to hit draw or hooks which start out one way (right) and then curve the other way (left) for targets that do not permit direct shots. Baseball pitchers know how to impart spin to the ball to throw curveballs and sliders. In all these cases, the trajectory deviates from the typical parabolic trajectory that one sees in ideal projectiles because of two reasons. First, balls are subject to *drag* forces which increases with its velocity. Second, ball are subject to *lift forces* that depend on the velocity and the spin imparted to the ball. In both cases, the shape and texture of the ball also play an important role [1]. In the next section, we develop a model for the dynamics of a projectile subject to these drag and lift forces.

3 Dynamics

The dynamics of the ball can be represented as a particle (point mass) subject to three distinct forces. The first force is the drag force which is due to the fact that the ball is moving through the air. Second, the ball is subject to lift forces. Finally, acceleration due to gravity impacts the dynamics of the projectile. Newton's second law gives us:

$$\mathbf{F}_L + \mathbf{F}_D + \mathbf{F}_G = m\mathbf{a}, \tag{1}$$

[1] The texture has a pronounced effect on the ball's trajectory. A dimpled surface or a surface with seams can alter the mechanics of drag and lift forces.

where m is the mass of the missile, and the three forces are

$$\begin{aligned}\mathbf{F}_L &= F_{L,x}\mathbf{i} + F_{L,y}\mathbf{j} + F_{L,z}\mathbf{k},\\ \mathbf{F}_D &= F_{D,x}\mathbf{i} + F_{D,y}\mathbf{j} + F_{D,z}\mathbf{k},\\ \mathbf{F}_G &= F_{G,z}\mathbf{k},\end{aligned}$$

where the subscripts L, D, and G denote lift, drag, and the force due to gravity, respectively.

In order to calculate the lift and drag forces, we will need to define another set of unit vectors. We denote by $\mathbf{t}$ a unit vector that is tangential to the trajectory (along the velocity vector) of the ball. Thus, the velocity of the ball, $\mathbf{v}$ is given by:

$$\mathbf{v} = v\mathbf{t} \tag{2}$$

There are two other unit vectors that can be defined so that they are perpendicular to $\mathbf{t}$ and to each other.

If the ball is spun, it has an *angular velocity.* While you will learn about angular velocity later in the course, here is a quick introduction which builds on what you know from physics. If a rigid body rotates in a plane, it is easy to identify a line on the rigid body (say AB in Figure 1) and measure the angle the line makes with a suitable reference line (e.g., the x axis). The rate of change of this angle is the angular speed, and the angular velocity is obtained by attaching to this speed a direction that is obtained by the right hand thumb rule — curl the fingers of your right hand in the direction of rotation and the direction vector points in the direction of the thumb.

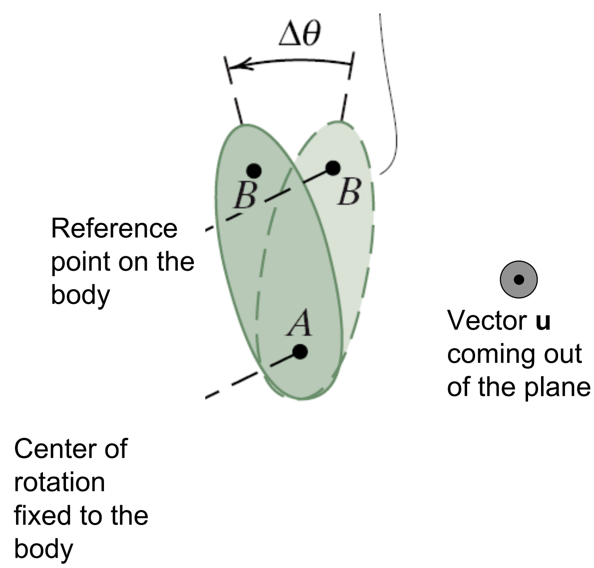

Figure 1: The angular velocity vector $\underline{\omega}$ is the limit of $\frac{\Delta\theta}{\Delta t}\mathbf{u}$ as $\Delta t \to 0$.

In three dimensions, the angular velocity of the ball (particle) can point in any direction. There is, in general, no special plane associated with the motion. Let us assume the player can impart a desired

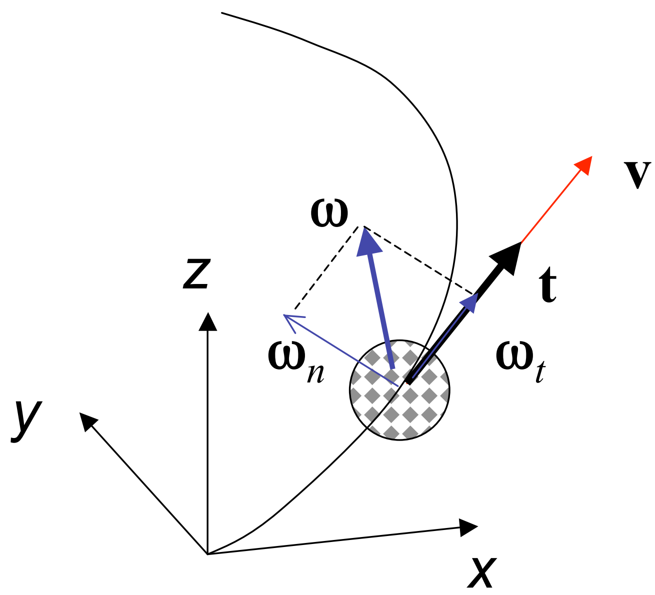

Figure 2: The angular velocity vector, $\underline{\omega}$, can be written as the vector sum of two vectors, $\underline{\omega}_t$ and $\underline{\omega}_n$. $\underline{\omega}_t = (\underline{\omega}.\mathbf{t})\mathbf{t}$ and $\underline{\omega}_n = \underline{\omega} - \underline{\omega}_t$.

angular velocity vector and this angular velocity vector is constant in an inertial frame [2]. As shown in Figure 2, the angular velocity vector and the velocity of the ball are pointed in different directions.

Drag forces The drag force on a projectile is proportional to the square of the speed relative to the air and in a direction that is opposite to the velocity:

$$\mathbf{F}_D = -\frac{1}{2}\rho C_D A(\mathbf{v}.\mathbf{v})\,\mathbf{t} \tag{3}$$

where ρ is the density of air, A is the cross-sectional area of the projectile and C_D is the non dimensional drag coefficient which depends on the shape and surface texture of the projectile, the kinematic viscosity of the air, and the relative speed. It can be shown to depend on two non dimensional quantities. The first quantity is the *Reynolds number*, a non dimensional quantity that is defined by:

$$Re = \frac{\rho v D}{\mu} \tag{4}$$

where ρ and μ are the density and viscosity of air respectively, v is the relative speed, and D is the diameter of the projectile. We will discuss how this coefficient needs to be determined later. The second quantity is

[2] This may not be a realistic assumption and you should think about this carefully. Note that this is different from having the angular velocity constant in a body-fixed frame. For example, think about a football thrown in a tight spiral where one may assume the angular velocity components in a frame attached to the football are constant.

a non dimensional number that characterizes the surface roughness and texture.

$$\epsilon = \frac{k}{D} \tag{5}$$

where k characterizes the roughness of the surface (dimensions of length). C_D is a function of these two numbers:

$$C_D = \hat{f}(Re, \epsilon) \tag{6}$$

Note that this function $\hat{f}$ is not obtained through a scientific law but is instead empirically determined through wind tunnel tests.

Lift forces The lift force on a projectile is proportional to the square of the speed relative to the air and in a direction that is perpendicular to the plane formed by the angular velocity vector and the linear velocity vector. Specifically, define $\mathbf{n}$ to be the unit vector along $\underline{\omega} \times \mathbf{v}$:

$$\mathbf{n} = \frac{\underline{\omega} \times \mathbf{v}}{|\underline{\omega} \times \mathbf{v}|} \tag{7}$$

The lift force is given by:

$$\mathbf{F}_L = \frac{1}{2}\rho C_L A(\mathbf{v}.\mathbf{v})\ \mathbf{n} \tag{8}$$

where C_L is the non dimensional lift coefficient which again depends on the shape and surface texture of the projectile (which are fixed), the Reynolds number which depends on the speed and the spin or the angular velocity, ω of the ball.

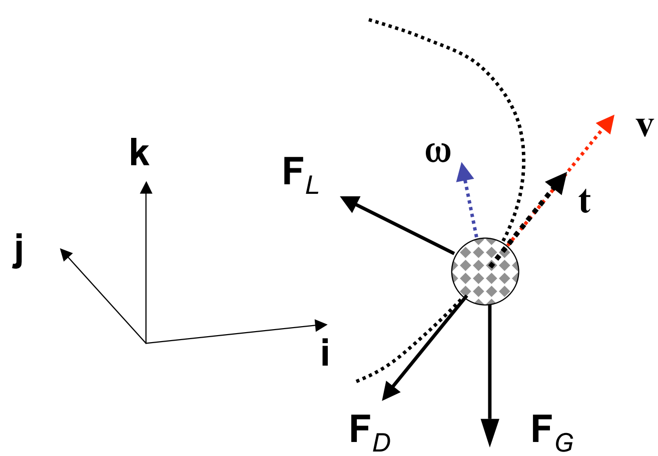

Figure 3: The angular velocity vector, $\underline{\omega}$, can be written as the vector sum of two vectors, $\underline{\omega}_t$ and $\underline{\omega}_n$. $\underline{\omega}_t = (\underline{\omega}.\mathbf{t})\mathbf{t}$ and $\underline{\omega}_n = \underline{\omega} - \underline{\omega}_t$.

The curving of the trajectory of the ball is due to the lift force and is called the *Magnus effect.* This lift force always acts normal to both $\mathbf{v}$ and $\underline{\omega}$, in the direction of $\mathbf{n}$ as shown in Figure 4. Furthermore, we

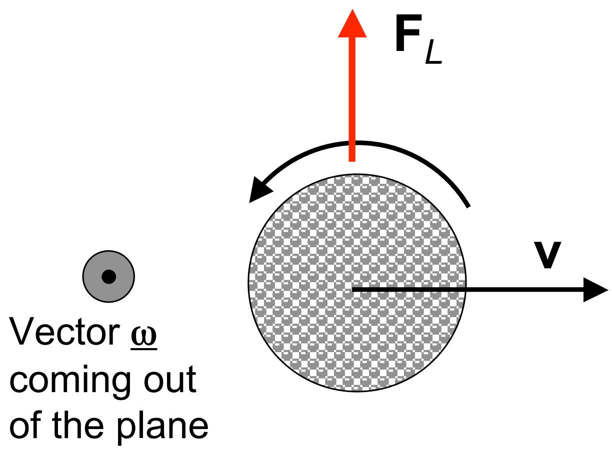

Figure 4: The *Magnus* effect predicts a lift force that is along the cross product of $\underline{\omega}$ and $\mathbf{v}$.

will assume that C_L depends only on the non dimensional spin in the direction of $\mathbf{n}$. Write

$$\underline{\omega}_t = (\underline{\omega}.\mathbf{t})\mathbf{t}$$

and define the normal component of $\underline{\omega}$ as:

$$\underline{\omega}_n = \underline{\omega} - \underline{\omega}_t \tag{9}$$

The non dimensional spin in the direction of $\mathbf{n}$ is defined as:

$$s = \frac{|\omega_n|\ D}{2v} \tag{10}$$

The lift coefficient, C_L depends on both non dimensional numbers, Re and s. Unfortunately, the mutual dependence of C_L on Re and s is very complicated and not well known. However, we will make a suitable approximation for our calculations:

$$C_L = \hat{g}(s) \tag{11}$$

where the function $\hat{g}$ is estimated from experimental data.

4 Projectile trajectories

Three special cases The following special cases do not really depend on the models for lift and drag and can be used to verify the dynamic model.

1. $C_L = 0,\ \mathbf{v}(t_0) = v_0\mathbf{k}$: If the lift coefficient is zero and the projectile is launched vertically, we know that the trajectory of the projectile will follow a vertical line and can be calculated analytically.

2. $C_L = 0,\ C_D = 0$: If the lift and drag forces are zero, the equations reduce to those for an ideal projectile resulting in parabolic trajectories.

3. $g = 0,\ C_D = 0,\ C_L \neq 0$: In this special case, if C_L is a constant, the only force is the constant lift force and since it is always perpendicular to the direction of the velocity the projectile will trace a circular path.

The general case The parameters and initial conditions that you will need to test your simulation are presented in Table 1. The values for C_L and C_D are presented in a table along with the lecture material.

5 Project

The project consists of the following steps.

1. Write the equations of motion in state space notation to get it into the standard form:

$$\dot{X} = f(X, t),$$

   where $X = [x, y, z, \dot{x}, \dot{y}, \dot{z}]^T$ is the state of the system. Note there will not be any explicit dependence on time.

2. Develop a MATLAB simulator to simulate $\dot{X} = f(X, t)$ from a given initial condition:

$$t = 0, \quad X_0 = [x(0), y(0), z(0), \dot{x}(0), \dot{y}(0), \dot{z}(0)]^T,$$

   for a specified

$$\omega = [\omega_x, \omega_y, \omega_z]^T.$$

3. Test your simulator using the three special cases Section 4.

4. Once you are confident the simulator is working, run your simulator with realistic C_L and C_D values to find the best trajectories for a shot and sport of your choice. If you need help in modeling a particular style of projectile talk to the instructor or teaching assistants. Dimensions of a regulation soccer field are available for your use in Figure 5. Attempts to model exact trajectories that you or a friend can produce with any sports ball (soccer, cricket, baseball, ping-pong, golf) or a trajectory associated with any athlete's free kick or slice or curve ball that you can find on the internet will be considered for extra credit.

6 Report

The report should follow the outline below (with the section numbers below).

Table 1: The parameters and initial conditions required for the simulation. You can use $C_L = 0.23$, $C_D = 0.1$, $m = 0.43kg$, $D = 0.7m$, an initial speed of $\dot{x}(0) = 0$, $\dot{y}(0) = 18m/s$, $\dot{z}(0) = 18m/s$, and a spin of $70rad/s$ in any direction perpendicular to the velocity vector to test the simulation.

Parameter	**Value**	**Units**
C_D	(look up from table)	non dimensional
C_L	(look up from table)	non dimensional
g	9.81	m/s^2
ρ	1.229	kg/m^3
μ	1.73×10^{-5}	$N\ s/m^2$
D	look up	m
m	look up	kg
Initial Condition	**Value**	**Units**
$x(0)$	choose	m
$y(0)$	choose	m
$z(0)$	0	m
$\dot{x}(0)$	choose	m/s
$\dot{y}(0)$	choose	m/s
$\dot{z}(0)$	choose (> 0)	m/s
ω_x	choose	rad/s
ω_y	choose	rad/s
ω_z	choose	rad/s

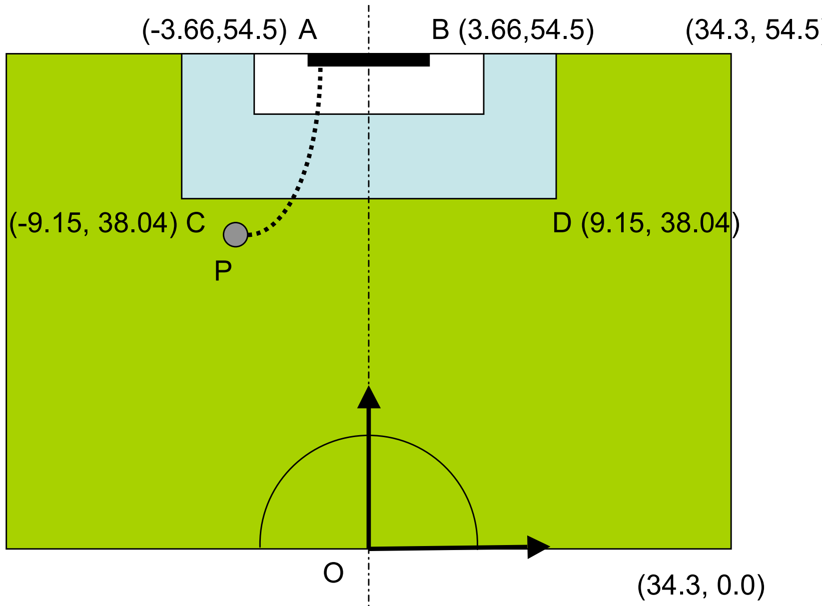

Figure 5: A freekick launched from point P must curve past the defenders into the corner of the goal. All coordinates reflect dimensions in meters.

1. **Goal**: The goal of the project (1-2 sentences);

2. **Dynamics**: The dynamics and control of the projectile, citing appropriate references (1 page maximum with equations);

3. **Simulator**: The simulator (1 page maximum, including a description of what each module does and the rationale underlying your code);

4. **Analysis**:

   - Include sample plots for all three special cases explaining why you think your simulator works.
   - Include sample plots showing a hook, curve, drop or slice shot from the sport of your choice (*e.g.*, soccer free kick): (a) The $x - y - z$ trajectory; (b) The speed as a function of time; (c) The magnitude of the lift and drag force as a function of time. You may wish to show the soccer field with defenders or the golf course with the location of the hole and blocking trees in plot (a).
   - A brief description (1-2 paragraphs maximum) of the effects of changing the shape, texture, the initial speed and angular velocity of the shot; and

5. **Discussion** (1 page maximum, what did you learn from the project, what are the shortcomings of the modeling approach, what enhancements can you think of?)

6. **References** List references you used in this project.

7. **Appendix**: Source code with comments

You are encouraged to include any background research on the dynamics of projectiles or aerodynamics in your write up. Any insight you can provide from your knowledge of the sport will also be useful.

References

[1] Mehta, Rabindra D., "Aerodynamics of Sports Balls," *Annual Review of Fluid Mechanics*, 17, 1985:151 189 (Use `google scholar` and `PennText` to see this article).

[2] Ireson, Gren, "Beckham as Physicist?" *J. Physics Education.* `http://www.physics.umn.edu/classes/getfile.html?id=6025&name=beckham.pdf` .