Dynamics of Systems of Particles (Toward Rigid Bodies)

General Methodology

- For each particle
  - Free-Body Diagram (FBD) and Inertia Response Diagram (IRD)
  - Force balance
- As many equations as there are particles
  - potential problem as you scale up to a rigid body!
- But…Sum up equations for all the particles
  - simplification for the whole system

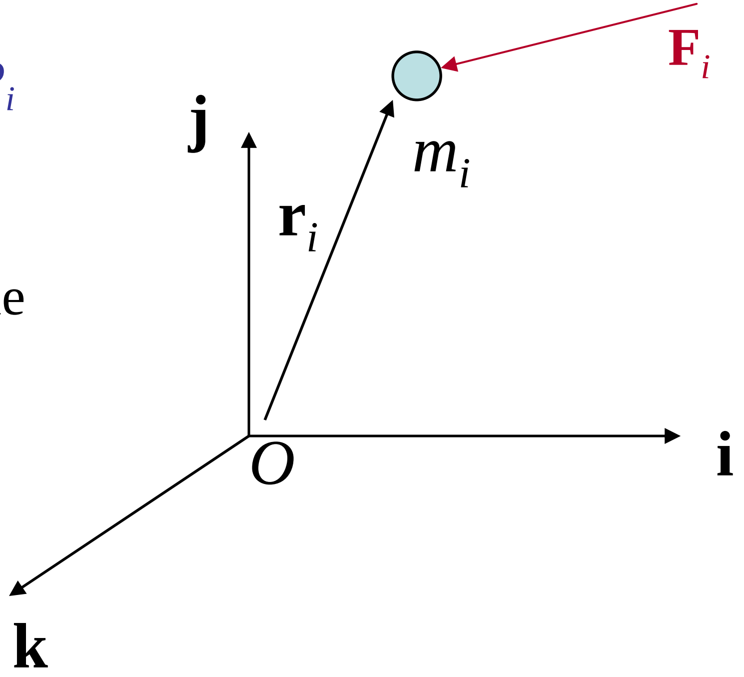
[TS, Chapter 5]

“Newton’s Amplifier”

Newton's Second Law for a System of Particles

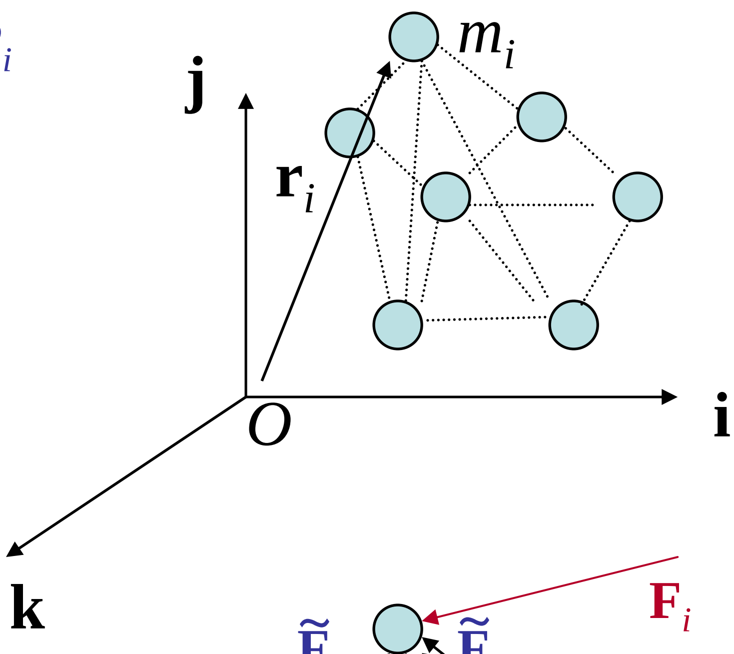
- Newton's Second Law for a particle P_i
  - Position vector $\mathbf{r}_i$ in an inertial frame A
  - $\mathbf{F}_i$ is the force acting on the particle P_i, mass m_i

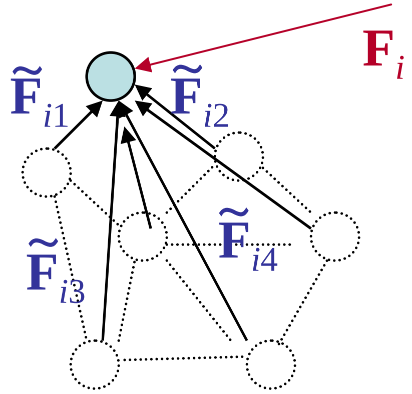
$$\mathbf{F}_i = m\frac{d\ \mathbf{v}_{P_i}}{dt} \qquad \mathbf{v}_{P_i} = \frac{d\mathbf{r}_i}{dt}$$



Newton's Second Law for a System of Particles (cont'd)

- Newton's Second Law for a particle P_i in a **system** of particles
  - Position vector $\mathbf{r}_i$ in an inertial frame A
  - *Two types of forces*
    - External forces
    - Internal forces
  - $\mathbf{F}_i$ is the *resultant external* force acting on P_i
  - $\tilde{\mathbf{F}}_{ij}$ is the force exerted by P_j on P_i

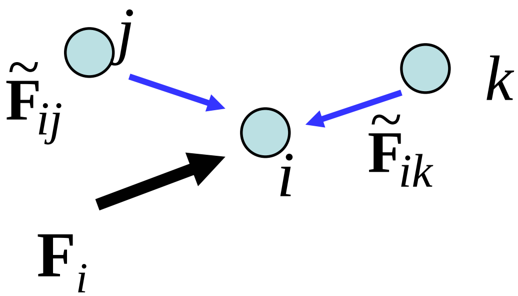


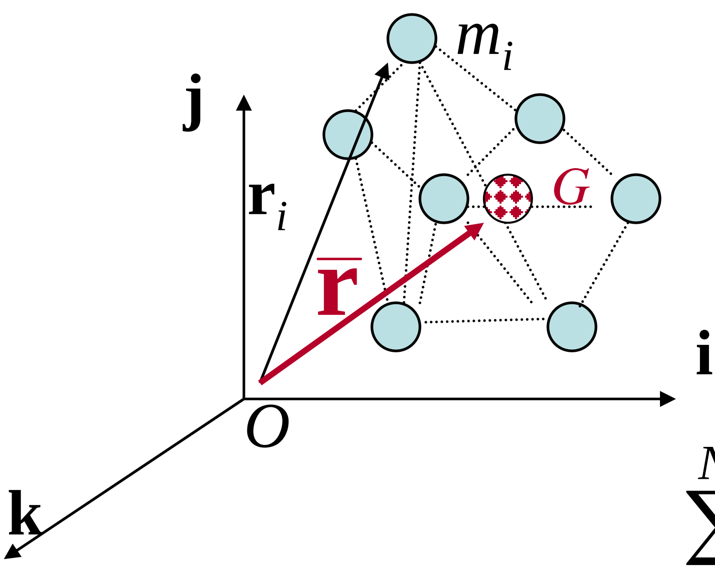


Center of Mass

Define

$$\bar{\mathbf{r}} = \frac{1}{m}\sum_i m_i \mathbf{r}_i$$





Newton's Second Law

$$\mathbf{F}_i + \sum_{j=1,\, j\neq i}^{N} \tilde{\mathbf{F}}_{ij} = m_i \ddot{\mathbf{r}}_i$$

Add *all* equations of force balance

$$\sum_{i=1}^{N}\left[\mathbf{F}_i + \sum_{j=1,\, j\neq i}^{N} \tilde{\mathbf{F}}_{ij}\right] = \sum_{i=1}^{N}\left[m_i \ddot{\mathbf{r}}_i\right]$$

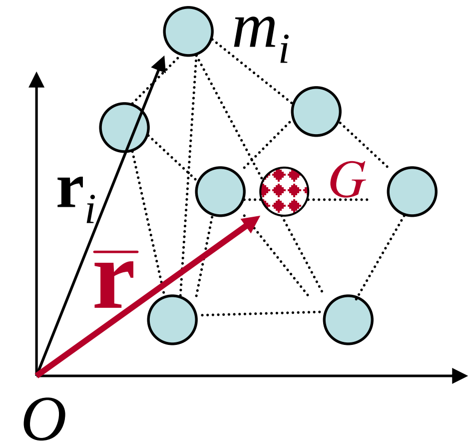
$$\frac{d\bar{\mathbf{r}}}{dt} = \left[\quad\right] \qquad \frac{d^2\bar{\mathbf{r}}}{dt^2} = \left[\quad\right]$$

$$\sum_{i=1}^{N}\left[\sum_{j=1,\, j\neq i}^{N} \tilde{\mathbf{F}}_{ij}\right] = \left[\quad\right]$$

$$\boxed{\sum_{i=1}^{N} F_i = m\ddot{\bar{\mathbf{r}}}}$$

Newton's Second Law for a System of Particles

The center of mass for a system of particles accelerates in an inertial frame as if it were a single particle with mass *m* (equal to the total mass of the system) acted upon by a force equal to the net external force.



Center of mass

$$\bar{\mathbf{r}} = \tfrac{1}{m}\sum_{i=1}^{N} m_i \mathbf{r}_i$$

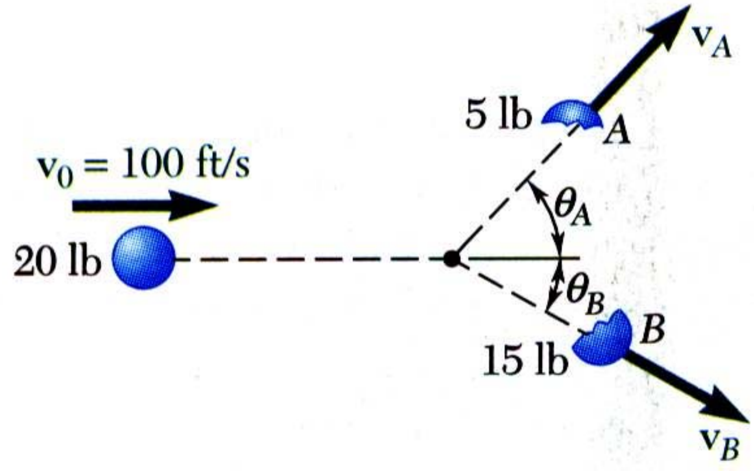
$$\mathbf{F} = \sum_{i=1}^{N} \mathbf{F}_i = m\,\frac{d\,\mathbf{v}_G}{dt}$$

Conservation of Linear Momentum

- The linear momentum of a system of particles stays constant in an inertial frame if the net external force equals zero

The center of mass *G* exhibits uniform motion (constant velocity) if the net external force equals zero

Example

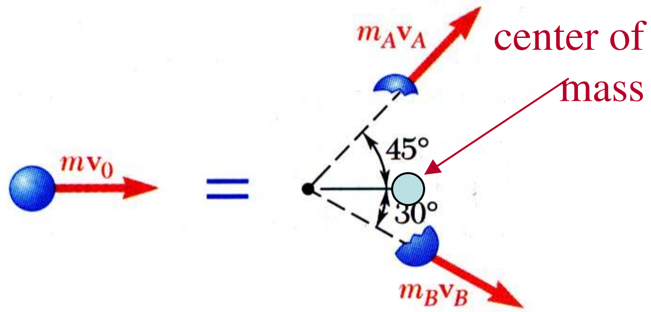


A 20-lb projectile is moving with a velocity of 100 ft/s when it explodes into 5 and 15-lb fragments. Immediately after the explosion, the fragments travel in the directions $\theta_A = 45^o$ and $\theta_B = 30^o$.

Determine the velocity of each fragment and the velocity of the center of mass after the explosion.

Key:

- Since there are no external forces, the linear momentum of the system is conserved.



- The center of mass velocity is unchanged.

$$\mathbf{v}_G = 100 \text{ ft/s } \mathbf{i}$$

$$v_A = 207 \text{ ft/s} \qquad v_B = 97.6 \text{ ft/s}$$