MEAM 333: Heat and Mass Transfer
Assignment 4
Due Date: 1/31/2012

Homework Guidelines:

- All papers must be stapled (No dog eared pages!).
- Solutions must proceed in a step-by-step fashion so that it is easy for the grader to follow and the method is well explained.
- Handwriting should be neat and presentation should be professional.
- Final answers for each part should be boxed or double underlined.
- Failure to do the above will result in point deductions.

Problem 1 [Solution]:

A simple, though inefficient, flat-plate solar collector used to heat hot water is designed by embedding water tubes within a steel absorber plate. Only one surface is exposed to the sun and all other plate surfaces are well insulated. Water flows through the tube passages within the absorber plate and is heated from an inlet temperature T_i to an outlet temperature T_o. The convection coefficient between the absorber plate and the ambient air at 27°C is 10 W/m2·K. A solar flux of G = 850 W/m^2 is incident on the exposed surface and the plate experiences radiation exchange with the sky at 0°C. Assume that the operating temperature of the plate is 65°C, the steel is gray with emissivity 0.95. (Gray means that absorptivity α is equal to emissivity).

A. Perform an overall energy balance on the absorber plate to obtain an expression for the rate at which useful heat (heat added to the water) is collected per unit area of the collector, q_u". Determine the value of q_u".
B. Assuming that average household daily hot water consumption is 100 gallons and that ground water is heated from 15°C (T_i) to a storage temperature of 50°C (T_o), what minimum solar collector area would meet the average household's hot water needs? Assume that the given solar flux is available for 8 hours a day and that all of the water usage occurs at the end of the day.
C. The collector efficiency, η, is defined as the ratio of the useful heat collected to the rate at which solar energy is incident on the collector. What is the value of η?

Problem 2 [Solution]:

Derive the heat equation for cylindrical coordinates by following these steps:

1. Draw a picture of a differential cylindrical control volume.
2. Write an overall energy balance for the control volume.
3. Find expressions for the total heat moving through each differential surface using Fourier's law and a Taylor series expansion (neglecting higher order terms).
4. Find an expression for the total heat generated in the differential volume. Assume that q''' is the rate at which energy is generated per unit volume of the medium (W/m^3). Note that both q''' and $\dot{q}$ (from the book's notation) are equivalent.
5. Find an expression for the rate of energy storage in the control volume in terms of the time derivative of temperature.

Combine steps 3-6 with the overall energy balance found in step 2. Cancel terms and check equation against that given in the book (Eq. 2.26).