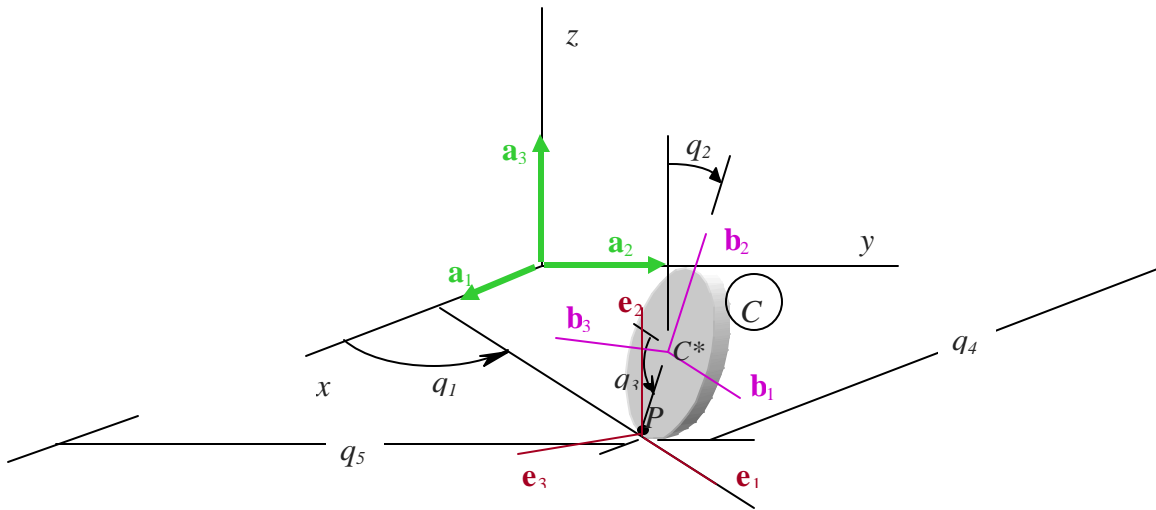


# MEAM 535 Class Project

## Analysis of a rolling/sliding disk



A circular disk  $C$  of radius  $R$  is in contact with a horizontal plane (not shown in the figure) at the point  $P$ . The point  $P$  is attached to the disk. The plane is the  $x$ - $y$  plane. It is rigidly attached to the earth. The standard reference triad  $\mathbf{b}_i$  is chosen so that  $\mathbf{b}_1$  is along the direction of progression of the disk (parallel to the tangent to the disk at  $P$ ),  $\mathbf{b}_2$  is parallel to the plane of the disk, and  $\mathbf{b}_3$  is normal to the disk. Note that this triad is *not* fixed to the disk. Call the earth-fixed reference frame  $A$  and choose the standard reference triad  $\mathbf{a}_x, \mathbf{a}_y,$  and  $\mathbf{a}_z$  in an obvious fashion along the  $x, y,$  and  $z$  axes shown in the figure.

1. Derive the equations of motion for (a) the rolling disk; and (b) the sliding disk. Use Newton-Euler equations and Lagrange's equations to make sure the results are consistent. Note that for the rolling disk you will get three equations of motion, and for the sliding disk you will get five equations of motion. Also note that you will need to solve for the contact forces.
2. Develop a simulation (Matlab is preferable) to simulate the motion of the disk from an arbitrary initial position and velocity (nearly vertical with an initial rolling velocity might be a good test condition) for (a) the rolling disk; and (b) the sliding disk.
3. In reality, there will be instants in time when the disk is rolling and others when the disk is sliding. Start with an initial condition (nearly vertical position) at which the velocities are consistent with the conditions required for rolling. Use the following algorithm to transition between rolling and sliding.
  - (a) Assume rolling conditions are satisfied.
  - (b) Solve the equations of motion assuming rolling for the accelerations and the contact forces
  - (c) Check that the contact forces satisfy Coulomb's law (tangential force is less than or equal to  $\mu$  times the normal force).

- (d) If this check is not satisfied, assume disk is sliding. Set the tangential force equal to  $\mu$  times the normal force with a direction that is opposite to  ${}^A\mathbf{v}^P$ . Solve for the accelerations.
- (e) Integrate from the given position and velocity, using the obtained acceleration to get a new position and new velocity.

Your program should be able to display:

- (i) a three-dimensional plot of the trajectory of the center of the disk;
- (ii) the history of the generalized coordinates and speeds as a function of time; and