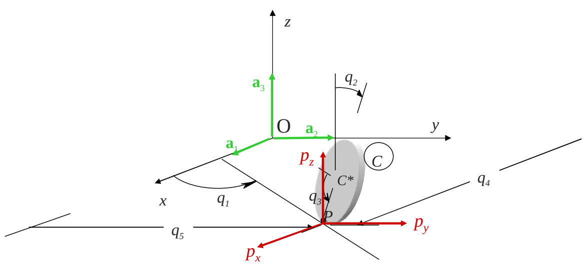
MEAM 535



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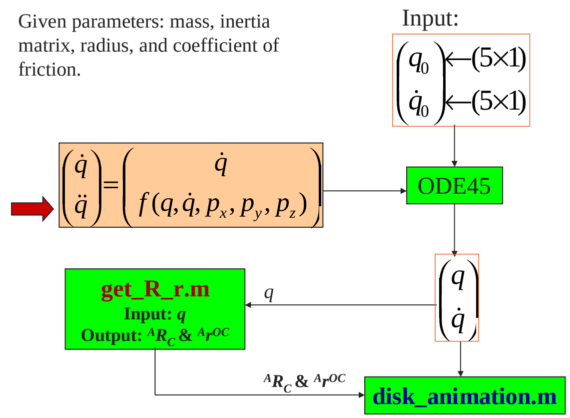
Given parameters: mass, inertia matrix, radius, and coefficient of friction.



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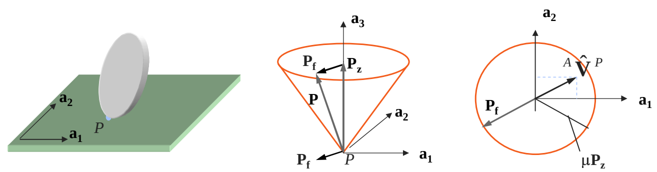
Coulomb's Friction Law in 3-D Space



$$\mathbf{P} = P_x \mathbf{a}_1 + P_y \mathbf{a}_2 + P_z \mathbf{a}_3$$

$$\mathbf{P}_\mathrm{f} = P_x \mathbf{a}_1 + P_y \mathbf{a}_2$$

Rolling: $\left\| {}^A\mathbf{V}^P \right\| = 0 \qquad \left\| \mathbf{P}_\mathrm{f} \right\| \le \mu P_z$

Sliding: $\left\| {}^A\mathbf{V}^P \right\| \ne 0 \qquad \mathbf{P}_\mathrm{f} = -{}^A\hat{\mathbf{V}}^P \cdot \mu P_z$

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Given: q, $\dot{q}$, compose $\ddot{q} = f(q, \dot{q}, p_x, p_y, p_z)$

1. Check if ${}^A\mathbf{V}^P = 0$: if $\left\| {}^A\mathrm{V}^P \right\| < \varepsilon$, the contact is rolling; if $\left\| {}^A\mathrm{V}^P \right\| > \varepsilon$, the contact is sliding.

2.1 If rolling, solve for $\ddot{q}_i$ $(i = 1...3)$ using the Lagrange Equations (3 eqs.) and $\ddot{q}_i$ $(i = 4, 5)$ using the rolling constraint (2 eqs.). Plug $\ddot{q}$ into the Newton - Euler Equations to solve for contact forces $P_{x,y,z}$.

2.2 If sliding, solve for $\ddot{q}_i$ $(i = 1...5)$ and $P_{x,y,z}$ with the Newton - Euler Equations (6 eqs.) and the Coulomb's friction model (2 eqs.):

$$P_x \mathbf{a}_1 + P_y \mathbf{a}_2 = -\mu\, P_z \frac{{}^A\mathbf{V}^P}{\left\| {}^A\mathbf{V}^P \right\|}.$$

3. If $\left\| P_x \mathbf{a}_1 + P_y \mathbf{a}_2 \right\| > \mu\, P_z$, goto 2.1.