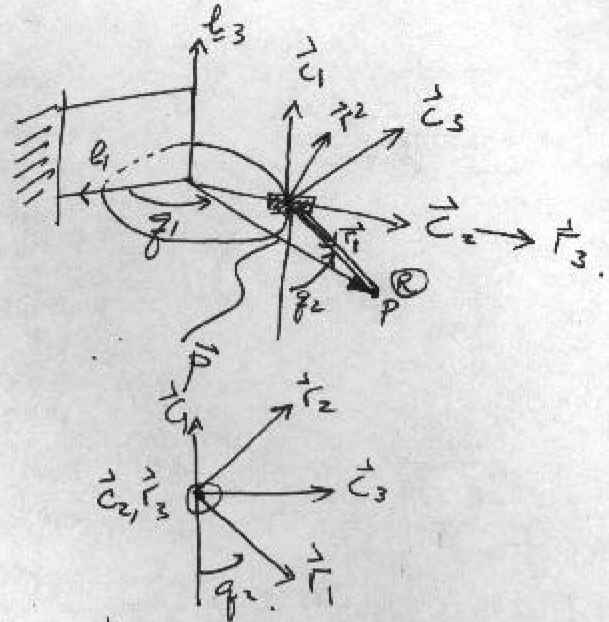
HW # 1 Solution

PROBLEM 1



(a) $\vec{r}^{OP} = 3r\vec{r}_1 + r\vec{c}_2$

$${}^L\vec{v}^{OP} = \frac{{}^Ld}{dt}({}^L\vec{r}^{OP})$$

$$= 3r\cdot\frac{{}^Ld}{dt}(\vec{r}_1) + r\cdot\frac{{}^Ld}{dt}(\vec{c}_2)$$

$$= 3r({}^L\omega^R \times \vec{r}_1) + r({}^L\omega^C \times \vec{c}_2)$$

Note: ${}^L\omega^R \times \vec{r}_1 = \dot{q}_2 s_2 \vec{c}_1 - \dot{q}_1 s_2 \vec{c}_2 + \dot{q}_2 c_2 \vec{c}_3$

$${}^L\omega^C = \dot{q}_1 \vec{c}_1$$

$${}^C\omega^R = \dot{q}_2 \vec{r}_3 = \dot{q}_2 \vec{c}_2$$

$${}^L\omega^R = {}^L\omega^C + {}^C\omega^R = \dot{q}_1 \vec{c}_1 + \dot{q}_2 \vec{c}_2$$

$$\vec{r}_1 = -c_2 \vec{c}_1 + s_2 \vec{c}_3$$

$${}^L\omega^C \times \vec{c}_2 = \dot{q}_1 \vec{c}_3.$$

$$\boxed{\begin{aligned} S_1 &= \sin q_1 \\ S_2 &= \sin q_2 \\ C_1 &= \cos q_1 \\ C_2 &= \cos q_2 \end{aligned}}$$

Thus, ${}^L\vec{v}^{OP} = 3r(\dot{q}_2 s_2 \vec{c}_1 - \dot{q}_1 s_2 \vec{c}_2 + \dot{q}_2 c_2 \vec{c}_3) + r\dot{q}_1 \vec{c}_3$

$$\boxed{{}^L\vec{v}^{OP} = 3r\dot{q}_2 s_2 \vec{c}_1 - 3r\dot{q}_1 s_2 \vec{c}_2 + (3r\dot{q}_2 \cos q_2 + r\dot{q}_1)\vec{c}_3}$$

$${}^L\vec{a}^{OP} = \frac{{}^Ld}{dt}({}^L\vec{v}^{OP}) = (3r\ddot{q}_2 s_2 + 3r\dot{q}_2^2 c_2)\vec{c}_1 + 3r\dot{q}_2 s_2 \cancel{({}^L\omega^C \times \vec{c}_1)}^{0} +$$
$$(-3r\ddot{q}_1 s_2 - 3r\dot{q}_1\dot{q}_2 c_2)\vec{c}_2 - 3r\dot{q}_1 s_2 ({}^L\omega^C \times \vec{c}_2) +$$
$$(3r\ddot{q}_2 c_2 - 3r\dot{q}_2^2 s_2 + r\ddot{q}_1)\vec{c}_3 + (3r\dot{q}_2 \cos q_2 + r\dot{q}_1)({}^L\omega^C \times \vec{c}_3)$$

Note: ${}^L\omega^C \times \vec{c}_3 = -\dot{q}_1 \vec{c}_2$

Thus ${}^L\vec{a}^{OP} = (3r\ddot{q}_2 s_2 + 3r\dot{q}_2^2 c_2)\vec{c}_1 + 3r\dot{q}_2 s_2 (0) - (3r\ddot{q}_1 s_2 - 3r\dot{q}_1\dot{q}_2 c_2)\vec{c}_2$
$- 3r\dot{q}_1^2 s_2 \vec{c}_3 + (3r\ddot{q}_2 c_2 - 3r\dot{q}_2^2 s_2 + r\ddot{q}_1)\vec{c}_3 - (3r\dot{q}_1\dot{q}_2 c_2 + r\dot{q}$

$$\boxed{\begin{aligned} {}^L\vec{a}^{OP} = {} & 3r(\ddot{q}_2 s_2 + \dot{q}_2^2 c_2)\vec{c}_1 - (3r\ddot{q}_1 s_2 + 6r\dot{q}_1\dot{q}_2 c_2 + r\dot{q}_1^2)\vec{c}_2 + \\ & 3r(\ddot{q}_2 c_2 - \dot{q}_2^2 s_2 - \dot{q}_1^2 s_2 + \tfrac{1}{3}\ddot{q}_1)\vec{c}_3 \end{aligned}}$$

(b) $\vec{r}^{\,OP} = 3r\vec{r}_1 + r\vec{c}_2$

Note: ${}^C\vec{\omega}^R = \dot{q}_2\vec{c}_2$, $\vec{r}_1 = -c_2\vec{c}_1 + s_2\vec{c}_3$ (from (a))

$${}^C\vec{v}^{\,OP} = \frac{{}^C d}{dt}({}^C\vec{r}^{\,OP}) = 3r({}^C\vec{\omega}^R \times \vec{r}_1) + \cancelto{0}{\frac{{}^C d}{dt}(r\vec{c}_2)}$$

$$= 3r(\dot{q}_2 s_2\vec{c}_1 + \dot{q}_2 c_2\vec{c}_3)$$

$$\boxed{{}^C\vec{v}^{\,OP} = 3r\dot{q}_2 s_2\vec{c}_1 + 3r\dot{q}_2 c_2\vec{c}_3}$$

$$\boxed{{}^C\vec{a}^{\,OP} = \frac{{}^C d}{dt}({}^C\vec{v}^{\,OP}) = (3r\ddot{q}_2 s_2 + 3r\dot{q}_2^2 c_2)\vec{c}_1 + (3r\ddot{q}_2 c_2 - 3r\dot{q}_2^2 s_2)\vec{c}_3}$$

(c) $$\frac{{}^L d}{dt}\vec{p} = 3r\dot{q}_2 s_2\vec{c}_1 - 3r\dot{q}_1 s_2\vec{c}_2 + (3r\dot{q}_2 c_2 + r\dot{q}_1)\vec{c}_3$$

$$\frac{{}^C d}{dt}\left(\frac{{}^L d}{dt}\vec{p}\right) = (3r\ddot{q}_2 s_2 + 3r\dot{q}_2^2 c_2)\vec{c}_1 - 3r(\ddot{q}_1 s_2 + \dot{q}_1\dot{q}_2 c_2)\vec{c}_2 + (3r\ddot{q}_2 c_2 - 3r\dot{q}_2^2 s_2 + r\ddot{q}_1)\vec{c}_3$$

$$\frac{{}^C d}{dt}\vec{p} = 3r(\dot{q}_2 s_2\vec{c}_1 + \dot{q}_2 c_2\vec{c}_3)$$

$$\frac{{}^L d}{dt}\left(\frac{{}^C d}{dt}\vec{p}\right) = 3r(\ddot{q}_2 s_2 + \dot{q}_2^2 c_2)\vec{c}_1 + 3r\dot{q}_2 s_2\cancelto{0}{({}^L\vec{\omega}^C \times \vec{c}_1)} + 3r(\ddot{q}_2 c_2 - \dot{q}_2^2 s_2)\vec{c}_3$$
$$+ 3r\dot{q}_2 c_2({}^L\vec{\omega}^C \times \vec{c}_3)$$
$$= 3r(\ddot{q}_2 s_2 + \dot{q}_2^2 c_2)\vec{c}_1 - 3r(\dot{q}_1\dot{q}_2 c_2)\vec{c}_2 + 3r(\ddot{q}_2 c_2 - \dot{q}_2^2 s_2)\vec{c}_3$$

Comparing, we see that $\frac{{}^C d}{dt}\left(\frac{{}^L d\vec{p}}{dt}\right) \neq \frac{{}^L d}{dt}\left(\frac{{}^C d}{dt}\vec{p}\right)$

This is b/c derivatives are frame dependent, rate of change ~~describe~~ as seen from one frame that is moving w/ respect to another frame is different. ~~Furthermore~~
In other words

$$\frac{{}^L d}{dt}\vec{c}_i \neq \frac{{}^C d}{dt}\vec{c}_i$$