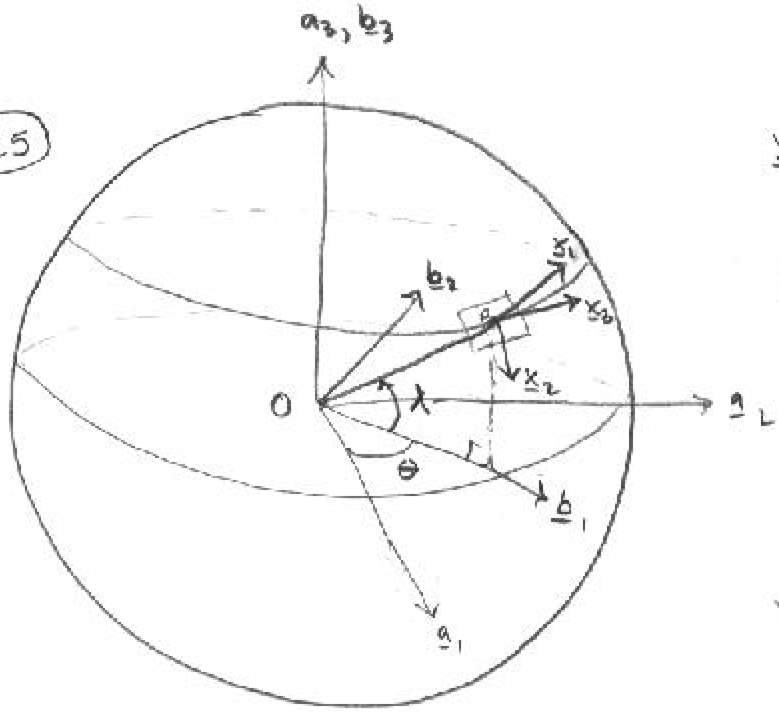
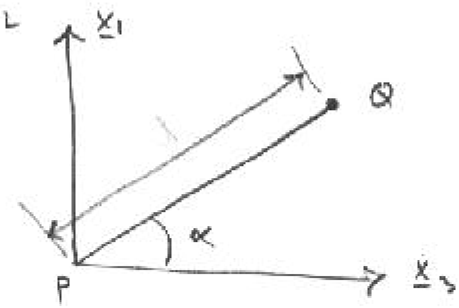
25



$\underline{x}_1 \equiv$ normal to earth (direction of R)

Q ≡ location of monorail

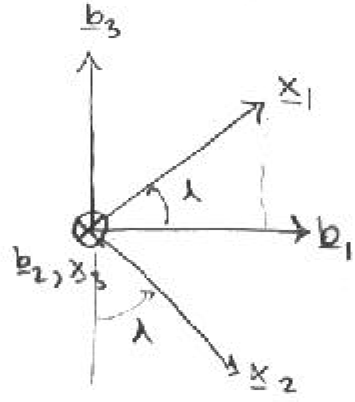


$$\overline{OQ} = \overline{OP} + \overline{PQ}$$

$$= R(\cos\lambda\cos\theta\, \underline{g}_1 + \cos\lambda \sin\theta\, \underline{g}_2 + \sin\lambda\, \underline{g}_3) + \overline{PQ}$$

where $\overline{PQ} = d(\cos\alpha\, \underline{x}_3 + \sin\alpha\, \underline{x}_1)$

$$\overline{OQ} = R(\cos\lambda\cos\theta\, \underline{g}_1 + \cos\lambda\sin\theta\, \underline{g}_2 + \sin\lambda\, \underline{g}_3) + d(\cos\alpha\, \underline{x}_3 + \sin$$

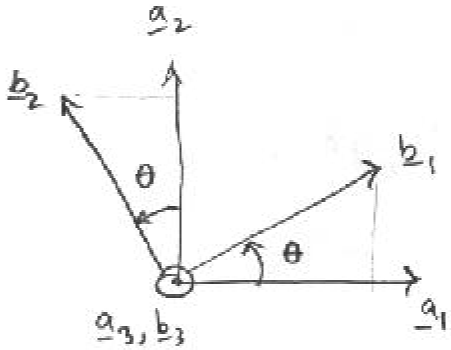


$$\underline{x}_1 = \underline{b}_1 \cos\lambda + 0 + \underline{b}_3 \sin\lambda$$

$$\underline{x}_2 = \underline{b}_1 \sin\lambda + 0 - \underline{b}_3 \cos\lambda$$

$$\underline{x}_3 = 0 + \underline{b}_2 + 0$$

$$^{x}R_{B} = \begin{bmatrix} \cos\lambda & 0 & \sin\lambda \\ \sin\lambda & 0 & -\cos\lambda \\ 0 & 1 & 0 \end{bmatrix}$$



$$\underline{b}_1 = \underline{a}_1 \cos\theta + \underline{a}_2 \sin\theta + 0$$

$$\underline{b}_2 = -\underline{a}_1 \sin\theta + \underline{a}_2 \cos\theta + 0$$

$$\underline{b}_3 = 0 + 0 + \underline{a}_3$$

$$^{B}R_{A} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

<u>now examine $\overline{OQ}$ equation:</u>

$$\overline{OQ} = \underset{\substack{\uparrow \\ \text{ref. frame "A"}}}{\overline{OP}} + \overline{PQ} \leftarrow \text{ref. frame "X"}$$

Let's put everything in terms of reference frame "B"

$$^{B}[\overline{OP}] = {}^{B}R_{A}\, {}^{A}[\overline{OP}]$$

$$= \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} R\cos\lambda\cos\theta \\ R\cos\lambda\sin\theta \\ R\sin\lambda \end{bmatrix}$$

$$= \begin{bmatrix} R\cos^2\theta\cos\lambda + R\sin^2\theta\cos\lambda \\ \cancel{-R\sin\theta\cos\theta\cos\lambda + R\sin\theta\cos\theta\cos\lambda} \\ R\sin\lambda \end{bmatrix}$$

$$= \begin{bmatrix} R(\underbrace{\cos^2\theta + \sin^2\theta}_{1})\cos\lambda \\ 0 \\ R\sin\lambda \end{bmatrix}$$

$$= \begin{bmatrix} R\cos\lambda \\ 0 \\ \ldots \end{bmatrix}$$

$${}^{B}\overline{OP} = R\cos\lambda\,\underline{b}_1 + R\sin\lambda\,\underline{b}_3$$

$${}^{B}[\overline{PQ}] = {}^{B}R_X\,{}^{X}[\overline{PQ}] = {}^{X}R_B^{-1}\,{}^{X}[\overline{PQ}]$$

$$= \begin{bmatrix} \cos\lambda & \sin\lambda & 0 \\ 0 & 0 & 1 \\ \sin\lambda & -\cos\lambda & 0 \end{bmatrix} \begin{bmatrix} d\sin\alpha \\ 0 \\ d\cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} d\cos\lambda\sin\alpha \\ d\cos\alpha \\ d\sin\lambda\sin\alpha \end{bmatrix}$$

$${}^{B}\overline{PQ} = d\cos\lambda\sin\alpha\,\underline{b}_1 + d\cos\alpha\,\underline{b}_2 + d\sin\lambda\sin\alpha\,\underline{b}_3$$

so, we have:

$$\overline{OQ} = \overline{OP} + \overline{PQ}$$

$$\overline{OQ} = [R\cos\lambda + d\cos\lambda\sin\alpha]\,\underline{b}_1 + d\cos\alpha\,\underline{b}_2 + [R\sin\lambda + d\sin\lambda\sin\alpha]$$

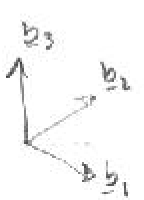
$${}^{A}\underline{v}^{Q} = \frac{{}^{A}d}{dt}(\overline{OQ}) = \frac{{}^{B}d}{dt}(\overline{OQ}) + {}^{A}\omega^{B} \times (\overline{OQ})$$

$${}^{A}\omega^{B} = \dot\theta\,\underline{b}_3$$

also, keep in mind that $v = v(t)$ [function of time]

$${}^{A}\underline{v}^{Q} = \frac{{}^{B}d}{dt}\left[(R\cos\lambda + d\cos\lambda\sin\alpha)\underline{b}_1 + (d\cos\alpha)\underline{b}_2 + (R\sin\lambda + d\sin\lambda\sin\alpha)\underline{b}_3\right]$$

$$\dot\theta\,\underline{b}_3 \times \left[(R\cos\lambda + d\cos\lambda\sin\alpha)\underline{b}_1 + (d\cos\alpha)\underline{b}_2 + \cancel{(R\sin\lambda + d\sin\lambda\sin\alpha)\underline{b}_3}\right]$$



$$= \left(R\overset{\to 0}{\cancel{\frac{{}^{B}d}{dt}}}(\cos\lambda) + \dot d\cos\lambda\sin\alpha\right)\underline{b}_1 + \dot d\cos\alpha\,\underline{b}_2 + (0 + \dot d\sin\lambda\sin\alpha)\underline{b}_3$$

$$+ \dot\theta(R\cos\lambda + d\cos\lambda\sin\alpha)\underline{b}_2 - \dot\theta d\cos\alpha\,\underline{b}_1$$

$${}^{A}\underline{v}^{Q} = [\dot d\cos\lambda\sin\alpha - d\dot\theta\cos\alpha]\,\underline{b}_1 + [\dot d\cos\alpha + R\dot\theta\cos\lambda + d\dot\theta\cos\lambda\sin\alpha$$

$$+ [\dot d\sin\lambda\sin\alpha]\,\underline{b}_3$$

And we know the velocity of the monorail car is "v"

$$\dot{d} = v$$

$${}^A\underline{v}^Q = [v\cos\lambda\sin\alpha - d\dot{\theta}\cos\alpha]\underline{b}_1 + [v\cos\alpha + R\dot{\theta}\cos\lambda + d\dot{\theta}\cos\lambda\sin$$

$$+ [v\sin\lambda\sin\alpha]\underline{b}_3$$

$${}^A\underline{a}^Q = \frac{{}^Ad}{dt}({}^A\underline{v}^Q) = \frac{{}^Bd}{dt}({}^A\underline{v}^Q) + {}^A\omega^B \times {}^A\underline{v}^Q$$

$$= \frac{{}^Bd}{dt}\Big[(v\overset{C}{\cancel{\cos\lambda}}\sin\alpha - d\dot{\theta}\cos\alpha)\underline{b}_1 + (v\overset{C}{\cancel{\cos\alpha}} + \overset{C}{\cancel{R\dot{\theta}}}\cos\lambda + d\dot{\theta}\cos\lambda\sin\alpha)$$

$$+ (v\overset{C}{\cancel{\sin\lambda}}\sin\alpha)\underline{b}_3\Big] + \dot{\theta}\underline{b}_3 \times \Big[(v\cos\lambda\sin\alpha - d\dot{\theta}\cos\alpha)\underline{b}_1$$

$$+ (v\cos\alpha + R\dot{\theta}\cos\lambda + d\dot{\theta}\cos\lambda\sin\alpha)\underline{b}_2 + \cancel{(v\sin\lambda\sin\alpha)\underline{b}_3}\Big]$$

Remember: $v = \text{const}$

$\dot{\theta} = \text{const}$

$$= -(v\dot{\theta}\cos\alpha)\underline{b}_1 + (v\dot{\theta}\cos\lambda\sin\alpha)\underline{b}_2$$

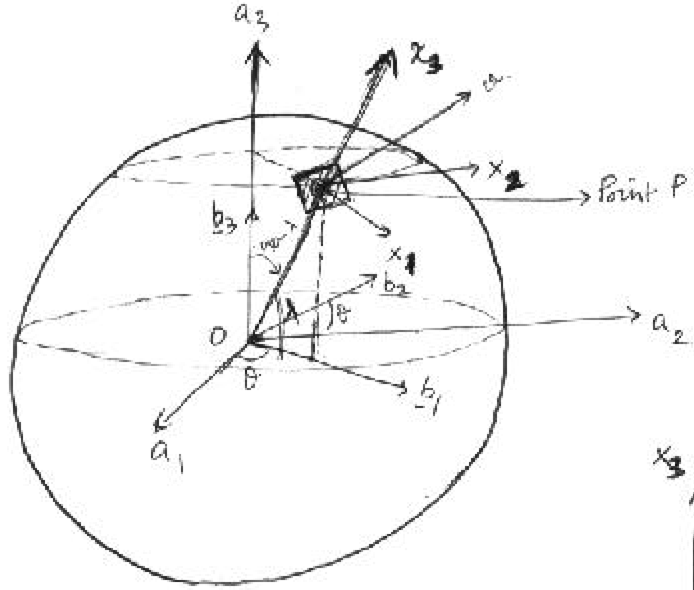
$$+ (v\dot{\theta}\cos\lambda\sin\alpha - d\dot{\theta}^2\cos\alpha)\underline{b}_2 - (v\dot{\theta}\cos\alpha + R\dot{\theta}^2\cos\lambda + d\dot{\theta}^2\cos\lambda\sin\alpha$$

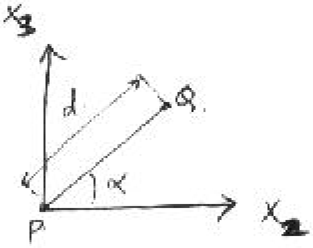
$$= -(v\dot{\theta}\cos\alpha + v\dot{\theta}\cos\alpha + R\dot{\theta}^2\cos\lambda + d\dot{\theta}^2\cos\lambda\sin\alpha)\underline{b}_1$$

$$+ (v\dot{\theta}\cos\lambda\sin\alpha + v\dot{\theta}\cos\lambda\sin\alpha - d\dot{\theta}^2\cos\alpha)\underline{b}_2$$

$$\boxed{\begin{aligned} {}^A\underline{a}^Q &= -[2v\dot{\theta}\cos\alpha + \dot{\theta}^2\cos\lambda(R + d\sin\alpha)]\underline{b}_1 \\ &\quad + [2v\dot{\theta}\cos\lambda\sin\alpha - d\dot{\theta}^2\cos\alpha]\underline{b}_2 \end{aligned}}$$

5.





$$\bar{OQ} = \bar{OP} + \bar{PQ}$$

$$\bar{OP} = R\sin\lambda\,\bar{a}_3 + R\cos\lambda\cos\theta\,\bar{a}_1 + R\cos\lambda\sin\theta\,\bar{a}_2$$

$$\bar{PQ} = d\cos\alpha\,\bar{x}_2 + d\sin\alpha\,\bar{x}_3$$

Rotation of x frame to a frame is

$${}^{A}R_{X} = {}^{A}R_{B}\,{}^{B}R_{X}$$

$${}^{A}R_{X} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \sin\lambda & 0 & \cos\lambda \\ 0 & 1 & 0 \\ -\cos\lambda & 0 & \sin\lambda \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta\sin\lambda & -\sin\theta & \cos\theta\cos\lambda \\ \sin\theta\sin\lambda & \cos\theta & \sin\theta\cos\lambda \\ -\cos\lambda & 0 & \sin\lambda \end{bmatrix}$$

$\bar{PQ}$ in terms of A frame is

$$\left(-d\cos\alpha\sin\theta + d\sin\alpha\cos\theta.\cos\lambda\right)\underline{a}_1 + \left(d\cos\alpha\cos\theta + d\sin\alpha\sin\theta\cos\lambda\right)\underline{a}_2$$

$$+ \quad d\sin\alpha\sin\lambda\,\underline{a}_3$$

$\therefore$ $\bar{OQ}$ (i.e) distance of monorail from O is

$$\bar{OQ} = \left(R\cos\theta\cos\lambda - d\cos\alpha\sin\theta + d\sin\alpha\cos\theta\cos\lambda\right)\underline{a}_1$$

$$+$$

$$\left(R\cos\lambda\sin\theta\right) + d\cos\alpha\cos\theta + d\sin\alpha\sin\theta\cos\lambda)\,\underline{a}_2$$

$$+$$

$$\left(R\sin\lambda + d\sin\lambda\sin\alpha\right)\underline{a}_3$$

$$^{A}v^{Q} = \left(-R\cos\lambda\sin\theta\,\dot{\theta} - d\cos\theta.\dot{\theta}\cos\alpha - \dot{d}\sin\theta\cos\alpha - d\sin\alpha\cos\lambda\sin\theta.\dot{\theta} + \dot{d}\sin\alpha\cos\lambda\cos\theta\right)\underline{a}_1$$

$$+$$

$$\left(R\cos\lambda\cos\theta\,\dot{\theta} - d\cos\alpha\sin\theta\,\dot{\theta} + \dot{d}\cos\alpha\cos\theta + d\sin\alpha\cos\lambda\cos\theta.\dot{\theta} + \dot{d}\sin\alpha\sin\theta c\right.$$

$$+$$

$$\left(\dot{d}\sin\lambda\sin\alpha\right)a_3$$

$\dot{d} = v.$

$\dot{\theta} = \omega$

$$\therefore\ ^{A}v^{Q} = \left(-R\cos\lambda\sin\theta.\omega - d\cos\theta\,\omega\cos\alpha - v\sin\theta\cos\alpha - d\sin\alpha\cos\lambda\sin\theta\,\omega + v\sin\alpha\cos\theta\right.$$

$$+$$

$$\left(R\cos\lambda\cos\theta\,\omega - d\cos\alpha\sin\theta\,\omega + v\cos\alpha\cos\theta + d\sin\alpha\cos\lambda\cos\theta\,\omega + v\sin\alpha\sin\theta\cos\right.$$

$$+$$

$$ {}^{A}a^{Q} = \Big(-R\cos\lambda\,\omega\cos\theta\cdot\omega + d\omega\cos\alpha\sin\theta\,\omega - V\omega\cos\alpha\cos\theta - V\cos\alpha\cos\theta\cdot\omega $$

$$ - d\sin\alpha\cos\lambda\,\omega\cos\theta\cdot\omega - v\sin\alpha\cos\lambda\sin\theta\cdot\omega - V\sin\alpha\cos\lambda\sin\theta\cdot\omega\Big)\,\underline{a}_1 $$

$$ + $$

$$ \Big(-R\cos\lambda\,\omega\sin\theta\cdot\omega - d\cos\alpha\,\omega\cos\theta\cdot\omega - v\cos\alpha\sin\theta\,\omega - v\cos\alpha\sin\theta\cdot\omega $$

$$ - d\sin\alpha\cos\lambda\,\omega\sin\theta\cdot\omega + v\sin\alpha\cos\lambda\cos\theta\cdot\omega + V\sin\alpha\cos\lambda\cos\theta\cdot\omega\Big)\,\underline{a}_2 $$

Since d is negligible, ignore d terms!

$$ \boxed{\begin{aligned} {}^{A}a^{Q} = {} & \big(-\omega^2 R\cos\lambda\cos\theta - 2V\omega\cos\alpha\cos\theta - 2V\omega\sin\alpha\cos\lambda\sin\theta\big)\,\underline{a}_1 \\ & + \\ & \big(-R\omega^2\cos\lambda\sin\theta - 2V\omega\cos\alpha\sin\theta + 2V\omega\sin\alpha\cos\lambda\cos\theta\big)\,\underline{a}_2 \end{aligned}} $$

Total Force $= m\,{}^{A}a^{Q}$ $\quad m =$ mass of mono rail.

$$ m\,{}^{A}a^{Q} = \big(-mR\omega^2\cos\lambda\cos\theta - 2mV\omega\cos\alpha\cos\theta - 2mV\omega\sin\alpha\cos\lambda\sin\theta\big)\,\underline{a}_1 $$

$$ + $$

$$ \big(-mR\omega^2\cos\lambda\sin\theta - 2mV\omega\cos\alpha\sin\theta + 2mV\omega\sin\alpha\cos\lambda\cos\theta\big)\,\underline{a}_2 $$

Gravity force: $= mg\,\bar{x}_1$

$$ = mg(\cos\theta\cos\lambda)\,\underline{a}_1 + mg\sin\theta\cos\lambda\,\underline{a}_2 + mg\sin\lambda\,\underline{a}_3 $$

Force on the track by mono rail =

$$m\hat{a}^{a} - m\hat{g}^{a}$$

$$= \left(-mR\omega^2\cos\lambda\cos\theta - 2mv\omega\cos\alpha\cos\theta - 2mv\omega\sin\alpha\cos\lambda\sin\theta - mg\cos\theta\cos\lambda\right)\underline{a}_1$$

$$+$$

$$\left(-mR\omega^2\cos\lambda\sin\theta - 2mv\omega\cos\alpha\sin\theta + 2mv\omega\sin\alpha\cos\lambda\cos\theta - mg\sin\theta\cos\lambda\right)\underline{a}_2$$

$$+$$

$$\left(-mg\sin\lambda\right)\underline{a}_3$$

Force on mono rail by track is negative of the above!

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