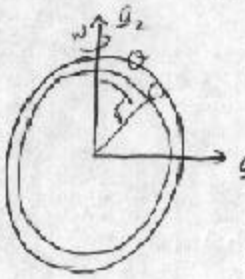
HW #7 Solutions

30



$$A\underline{\omega}^a = \omega \underline{a}_2 + \dot{\theta}\underline{a}_2$$

$$\underline{r} = r\sin\theta\,\underline{a}_1 + r\cos\theta\,\underline{a}_2$$

$$A\underline{v}^c = \frac{A d}{dt}\underline{r} = A\underline{\omega}^a \times \underline{r} = -\dot{\theta} r\cos\theta\,\underline{a}_1 + \dot{\theta} r\sin\theta\,\underline{a}_2 - \omega r\sin\theta\,\underline{a}_3$$

$$T = \frac{1}{2} m \left({}^I\underline{v}^c \cdot {}^A\underline{v}^c \right)$$

$$= \frac{1}{2} m \left[(-\dot{\theta} r\cos\theta)^2 + (\dot{\theta} r\sin\theta)^2 + (\omega r\sin\theta)^2 \right]$$

$$= \frac{1}{2} \left(\dot{\theta}^2 r^2 + \omega^2 r^2 \sin^2\theta \right) m$$

$$V = mgr\cos\theta$$

$$L = T - V = \frac{1}{2} m \left(\dot{\theta}^2 r^2 + \omega^2 r^2 \sin^2\theta \right) - mgr\cos\theta$$

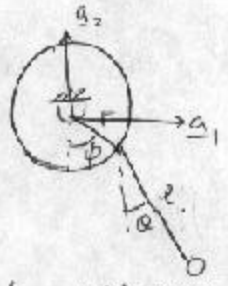
$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = 0$$

$$\frac{d}{dt}\left(m\dot{\theta} r^2 \right) - \left(m\omega^2 r^2 \sin\theta\cos\theta + mgr\sin\theta \right) = 0$$

$$m\ddot{\theta} r^2 - m\omega^2 r^2 \sin\theta\cos\theta - mgr\sin\theta = 0$$

$$\boxed{\ddot{\theta} - \omega^2 \sin\theta\cos\theta - \frac{g}{r}\sin\theta = 0.}$$

33.



$$\underline{r}^m = (r\sin\phi + l\sin\theta)\,\underline{a}_1 + (r\cos\phi + l\cos\theta)\,\underline{a}_2$$

$$\underline{v}^m = (\dot\phi r\cos\phi + \dot\theta l\cos\theta)\,\underline{a}_1 - (\dot\phi r\sin\phi + \dot\theta l\sin\theta)\,\underline{a}_2$$

$$T = \tfrac{1}{2}m\,\underline{v}\cdot\underline{v}^m$$

$$= \tfrac{1}{2}m\left[(\dot\phi r\cos\phi + \dot\theta l\cos\theta)^2 + (\dot\phi r\sin\phi + \dot\theta l\sin\theta)^2\right]$$

$$= \tfrac{1}{2}m\left(\dot\phi^2 r^2 + \dot\theta^2 l^2 + 2rl\,\dot\phi\dot\theta\cos(\phi-\theta)\right)$$

$$V = -mg(r\cos\phi + l\cos\theta)$$

$$L = T - V$$

$$= \tfrac{1}{2}m\left(\dot\phi^2 r^2 + \dot\theta^2 l^2 + 2rl\,\dot\phi\dot\theta\cos(\phi-\theta)\right) + mg(r\cos\phi + l\cos\theta)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot\phi}\right) - \frac{\partial L}{\partial \phi} = 0$$

$$mr^2\ddot\phi + mrl\ddot\theta\cos(\phi-\theta) - mrl\,\dot\theta\sin(\phi-\theta)(\dot\phi-\dot\theta) + mrl\dot\phi\dot\theta\sin(\phi-\theta) + mgr\sin\phi = 0$$

$$r^2\ddot\phi + rl\ddot\theta\cos(\phi-\theta) + rl\dot\theta^2\sin(\phi-\theta) + gr\sin\phi = 0$$

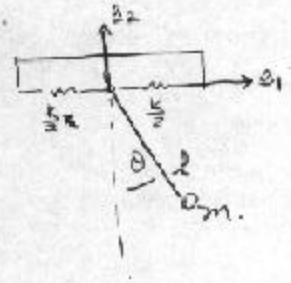
$$\boxed{\ddot\phi + \frac{l}{r}\ddot\theta\cos(\phi-\theta) + \frac{l}{r}\dot\theta^2\sin(\phi-\theta) + g\sin\phi = 0}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot\theta}\right) - \frac{\partial L}{\partial \theta} = 0$$

$$ml^2\ddot\theta + mrl\ddot\phi\cos(\phi-\theta) - mrl\dot\phi\sin(\phi-\theta)(\dot\phi-\dot\theta) + mrl\dot\phi\dot\theta\sin(\phi-\theta) + mgl\sin\theta = 0$$

$$\boxed{\ddot\theta + \frac{r}{l}\ddot\phi\cos(\phi-\theta) - \frac{r}{l}\dot\phi^2\sin(\phi-\theta) + \frac{g}{l}\sin\theta = 0}$$

31.



$$^A\underline{r}^m = (x+l\sin\theta)\underline{a}_1 + (-l\cos\theta)\underline{a}_2$$

$$^A\underline{v}^m = \frac{^A d}{dt}\,{}^A\underline{r}^m = (\dot{x}+\dot{\theta}l\cos\theta)\underline{a}_1 + \dot{\theta}l\sin\theta\,\underline{a}_2$$

$$T = \frac{1}{2}m\;{}^A\underline{v}^m \cdot {}^A\underline{v}^m$$

$$= \frac{1}{2}m\left((\dot{x}+\dot{\theta}l\cos\theta)^2 + (\dot{\theta}l\sin\theta)^2\right)$$

$$= \frac{1}{2}m\left(\dot{x}^2 + 2\dot{x}\dot{\theta}l\cos\theta + \dot{\theta}^2l^2\right)$$

$$V = -mgl\cos\theta + \frac{1}{2}kx^2$$

$$L = T - V = \frac{1}{2}m\left(\dot{x}^2 + 2\dot{x}\dot{\theta}l\cos\theta + \dot{\theta}^2l^2\right) + mgl\cos\theta - \frac{1}{2}kx^2$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = 0$$

$$\frac{d}{dt}\left(\frac{1}{2}m\left(2\dot{x} + 2\dot{\theta}l\cos\theta\right)\right) + kx = 0$$

$$m\ddot{x} + m\ddot{\theta}l\cos\theta - m\dot{\theta}^2l\sin\theta + kx = 0$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = 0$$

$$\frac{d}{dt}\left(\frac{1}{2}m\left(2\dot{x}l\cos\theta + 2\dot{\theta}l^2\right)\right) - \frac{1}{2}m\left(-2\dot{x}\dot{\theta}l\sin\theta\right) + mgl\sin\theta = 0$$

$$m\ddot{x}l\cos\theta + m\dot{x}\dot{\theta}l(-\sin\theta) + m\ddot{\theta}l^2 + m\dot{x}\dot{\theta}l\sin\theta + mgl\sin\theta = 0$$

$$\ddot{\theta}l^2 + \ddot{x}l\cos\theta + gl\sin\theta = 0$$

$$\ddot{\theta}l + \ddot{x}\cos\theta + g\sin\theta = 0$$

$$\Rightarrow \boxed{\begin{aligned} &\ddot{x} + \ddot{\theta}l\cos\theta - \dot{\theta}^2l\sin\theta + \frac{k}{m}x = 0 \\ &\ddot{\theta}l + \ddot{x}\cos\theta + g\sin\theta = 0 \end{aligned}}$$