Configuration Space and Degrees of Freedom

Degrees of freedom and constraints

Consider a system S with N particles, P_r (r=1,...,N), and their positions vector $\mathbf{x}_r$ in some reference frame A. The 3N components specify the configuration of the system, S.

The configuration space is defined as:

$$\aleph = \left\{ \mathbf{X} \;\middle|\; \mathbf{X} \in R^{3N}, \mathbf{X} = \left[\mathbf{x}_1.\mathbf{a}_1, \mathbf{x}_1.\mathbf{a}_2, \mathbf{x}_1.\mathbf{a}_3, \ldots \mathbf{x}_N.\mathbf{a}_1, \mathbf{x}_N.\mathbf{a}_2, \mathbf{x}_N.\mathbf{a}_3 \right]^T \right\}$$

The 3N scalar numbers are called configuration space variables or coordinates for the system.

The trajectories of the system in the configuration space are always continuous.

What are Constraints?

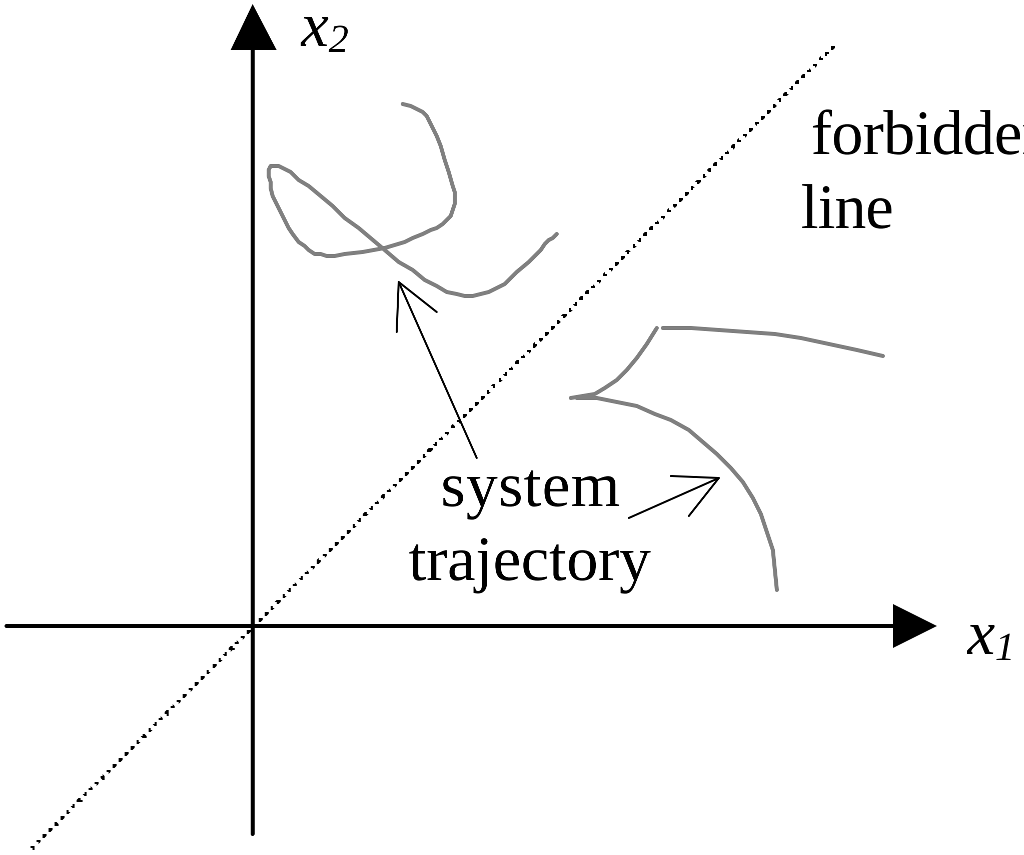
Claim

In a system of two or more particles, unconstrained motion is simply not possible.

Definition

If the motion of the system is affected by one of more constraints on the positions of the particles, the constraints are called *configuration constraints.*

A System of Two Particles on a Line



Examples of Constraints

1. A particle moving on a plane in three dimensional space. The configuration space is R^3. However, the particle is constrained to lie on a plane:

$$A x_1 + B x_2 + C x_3 + D = 0$$

2. A particle suspended from a string in three dimensional space. The configuration space is R^3. The particle is constrained to move so that its distance from a fixed point is always the same. This is called a spherical pendulum.

$$(x_1 - a)^2 + (x_2 - b)^2 + (x_3 - c)^2 - r^2 = 0$$

3. Two particles attached by a massless rod. The configuration space is R^6. The two particles are constrained so that the distance between them is a constant.

$$(x_1 - x_4)^2 + (x_2 - x_5)^2 + (x_3 - x_6)^2 - r^2 = 0$$

4. A particle constrained by the equation below is constrained to move on a circle in three-dimensional space whose radius changes with time t.

$$x_1\, dx_1 + x_2\, dx_2 + x_3\, dx_3 - c^2\, dt = 0$$

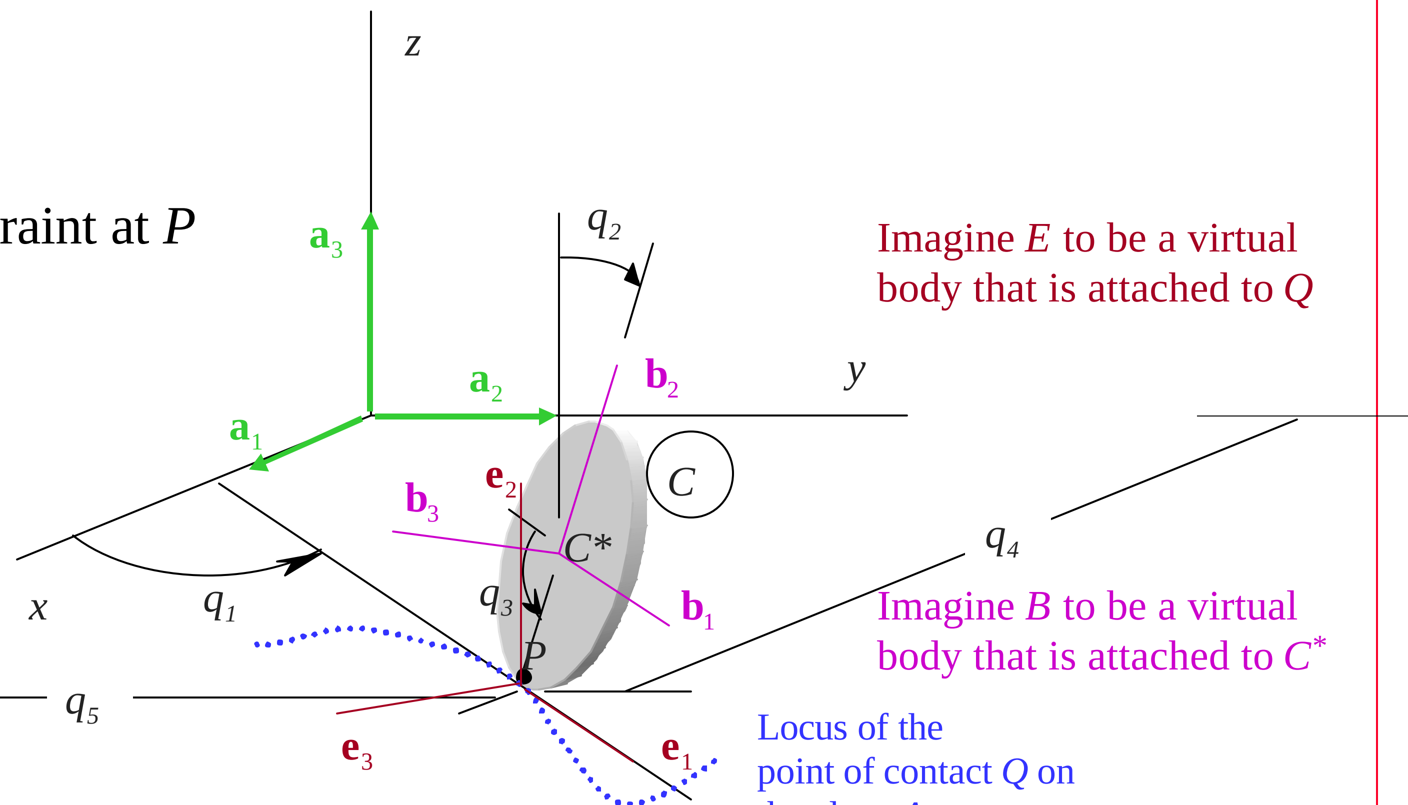
More Examples of Constraints

Problem 11(b)

A particle moving in a horizontal plane (call it the x-y plane) is steered in such a way that the slope of the trajectory is proportional to the time elapsed from t=0.

Problem 2

Rolling constraint at *P*



Degrees of Freedom

Consider a system of particles, *S*, and a convenient reference frame *A*. We have already seen that there is a 3*N*-dimensional configuration space associated with the system *S*. However, when there are one or more configuration constraints (as in the examples above) not all of the 3*N* variables describing the system configuration are independent. The minimum number of variables (also called coordinates) to completely specify the configuration (position of every particle) of a system is called the number of *degrees of freedom* for that system.

Definitions

- Degrees of freedom of a system

  The number of independent variables (or coordinates) required to completely specify the configuration of the system.

  - Point on a plane
  - Point in 3-D space
  - Line on a plane
  - 2 planar links connected by a pin joint
  - Human shoulder
  - Car

- Kinematic chain

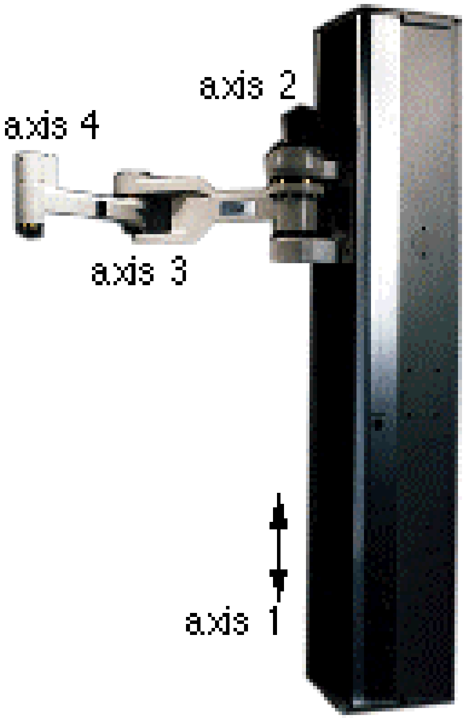
  A system of rigid bodies connected together by joints. A chain is called closed if it forms a closed loop. A chain that is not closed is called an open chain.

Rigidly Connected System of Particles

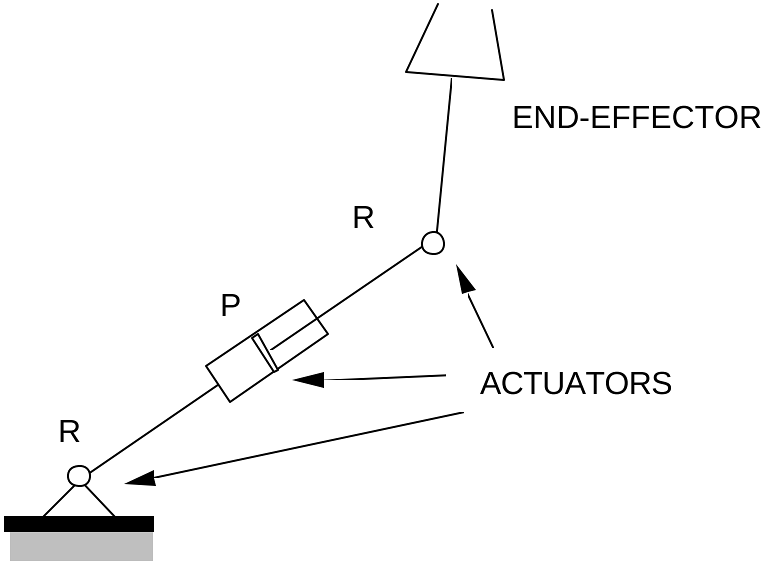
Number of particles rigidly connected, N	$3N - {}^{N}C_2$	Number of degrees of freedom, n
2	5	5
3	6	6
4	6	6
5	5	6
6	3	6

More Examples

The Adept 1850 Palletizer



Planar manipulator



The Planar 3-R manipulator

- Planar kinematic chain
- All joints are revolute with connectivity = 1
- What is the number of degrees of freedom?