Kinematics - II

Velocity and Acceleration Analysis

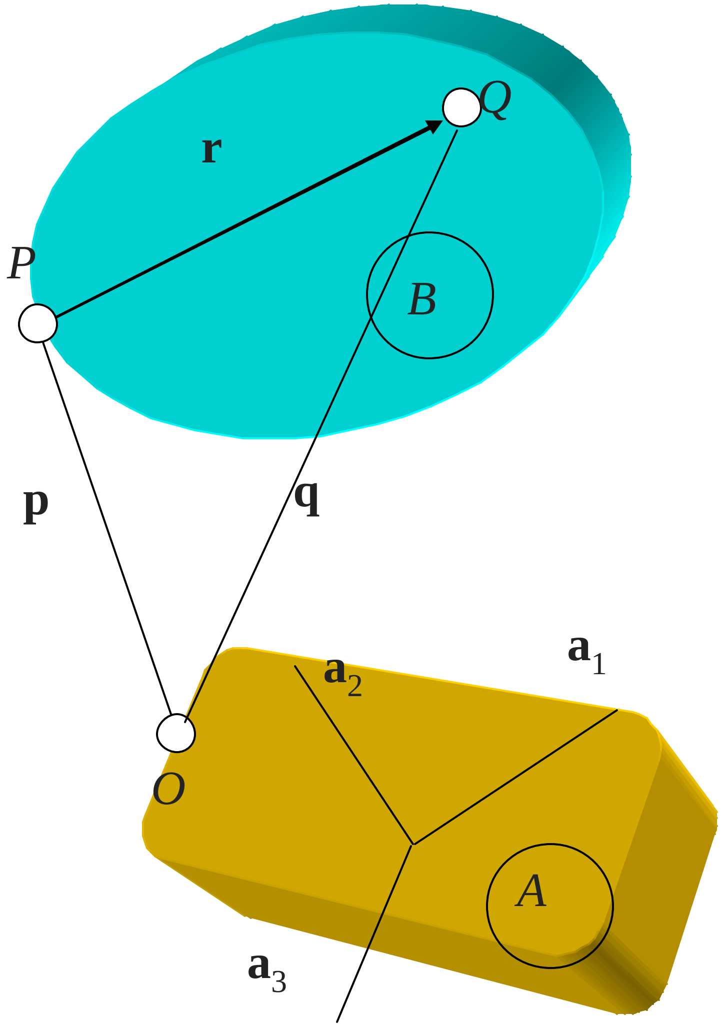
Relationship between Velocities and Accelerations in *A* of Two Points fixed to *B*

- Points *P* and *Q* fixed to (in) *B*
- *O* is a point fixed in *A*
- Position vectors for *P* and *Q* in *A* are denoted by **p** and **q**
- Velocities for *P* and *Q* in *A*

$$ {}^A\mathbf{v}^P = \frac{{}^A d\mathbf{p}}{dt}, \quad {}^A\mathbf{v}^Q = \frac{{}^A d\mathbf{q}}{dt} $$

- Accelerations for *P* and *Q* in *A*

$$ {}^A\mathbf{a}^P = \frac{{}^A d \ {}^A\mathbf{v}^P}{dt}, \quad {}^A\mathbf{a}^Q = \frac{{}^A d \ {}^A\mathbf{v}^Q}{dt} $$



Velocities of *P* and *Q*

- Triangle law of vector addition for points *P* and *Q*

$$\mathbf{q} = \mathbf{p} + \mathbf{r}$$

- Differentiate both sides

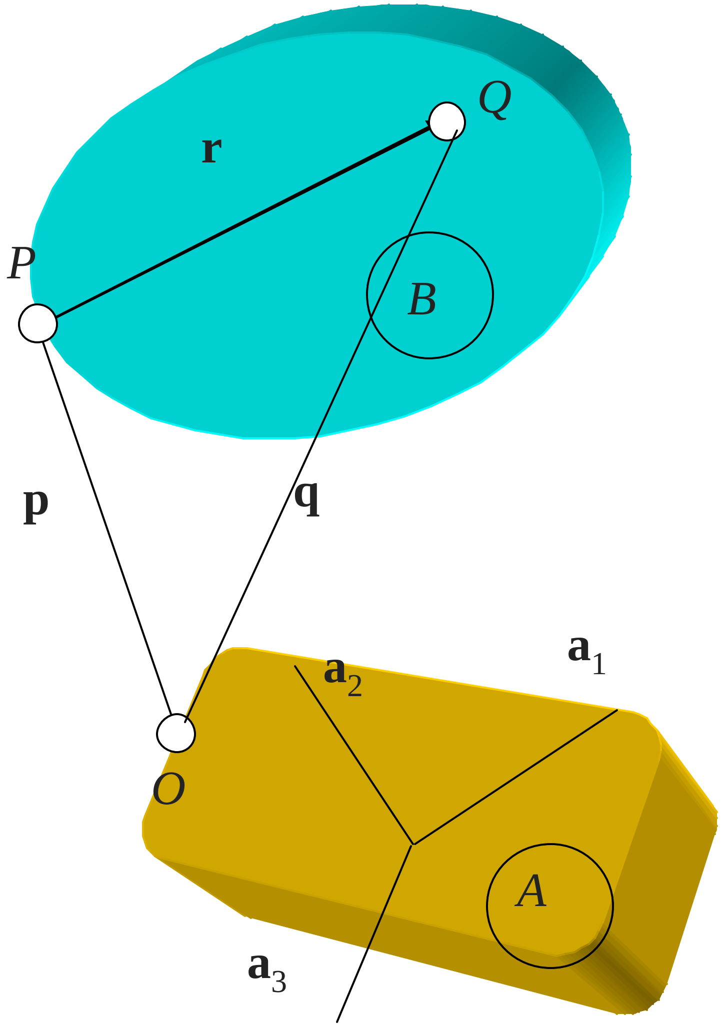
$$\frac{{}^{A}d\mathbf{q}}{dt} = \frac{{}^{A}d\mathbf{p}}{dt} + \frac{{}^{A}d\mathbf{r}}{dt}$$

- Substitute definitions of velocities

$${}^{A}\mathbf{v}^{Q} = {}^{A}\mathbf{v}^{P} + \left(\frac{{}^{B}d\mathbf{r}}{dt} + {}^{A}\omega^{B} \times \mathbf{r} \right)$$

- Velocities for *P* and *Q* in *A*

$${}^{A}\mathbf{v}^{Q} = {}^{A}\mathbf{v}^{P} + {}^{A}\omega^{B} \times \mathbf{r}$$



Accelerations of *P* and *Q*

- Velocities for *P* and *Q* in *A*

$$ {}^A\mathbf{v}^Q = {}^A\mathbf{v}^P + {}^A\omega^B \times \mathbf{r} $$

- Differentiate both sides

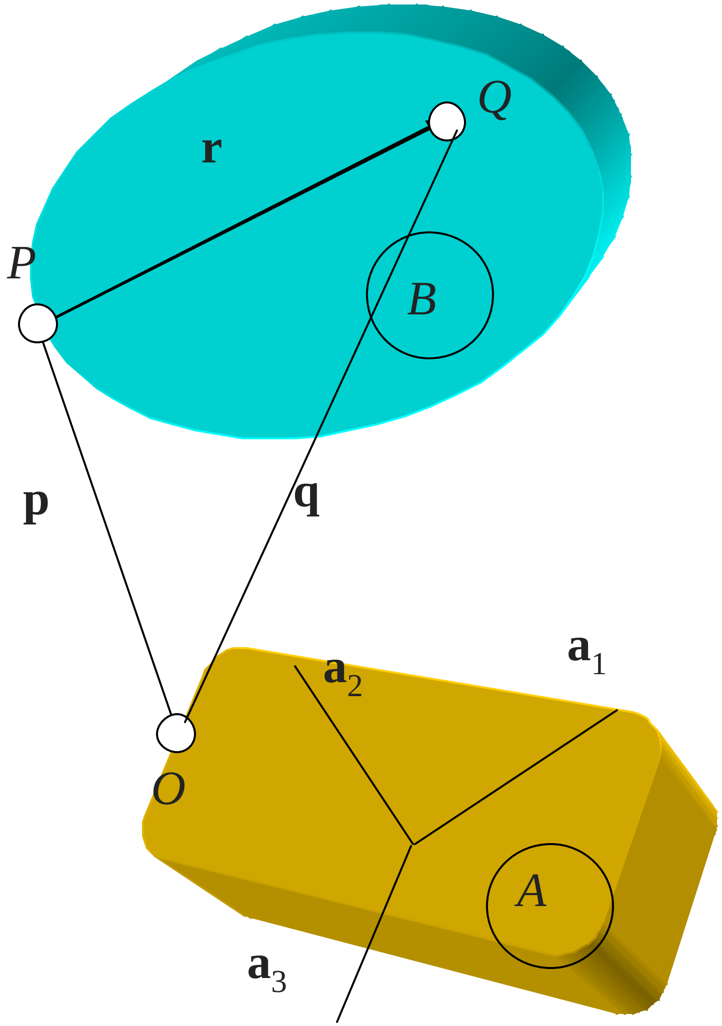
$$ \frac{{}^A d}{dt}\left({}^A\mathbf{v}^Q\right) = \frac{{}^A d}{dt}\left({}^A\mathbf{v}^P\right) + \frac{{}^A d}{dt}\left({}^A\omega^B \times \mathbf{r}\right) $$

$$ {}^A\mathbf{a}^Q = {}^A\mathbf{a}^P + \frac{{}^A d}{dt}\left({}^A\omega^B\right) \times \mathbf{r} + {}^A\omega^B \times \frac{{}^A d\mathbf{r}}{dt} $$

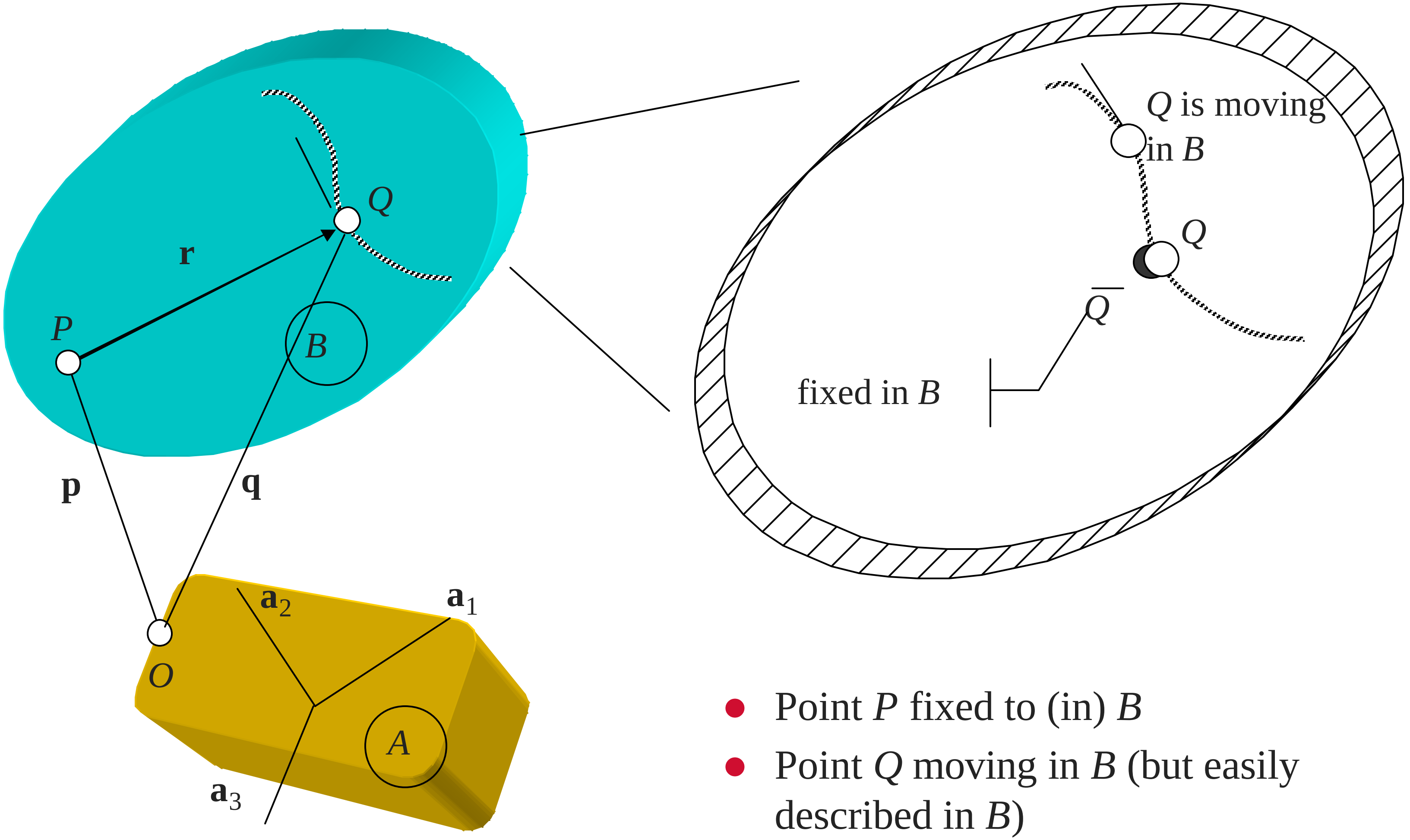
- Accelerations for *P* and *Q* in *A*

$$ {}^A\mathbf{a}^Q = {}^A\mathbf{a}^P + \boxed{{}^A\alpha^B \times \mathbf{r}} + \boxed{{}^A\omega^B \times \left({}^A\omega^B \times \mathbf{r}\right)} $$

tangential acceleration — centripetal (normal) acceleration



Relationship between Velocities and Accelerations in *A* of Points described in *B*



- Point *P* fixed to (in) *B*
- Point *Q* moving in *B* (but easily described in *B*)

Velocities of *P* and *Q*

- Triangle law of vector addition for points *P* and *Q*

$$\mathbf{q} = \mathbf{p} + \mathbf{r}$$

- Differentiate both sides

$$\frac{{}^{A}d\mathbf{q}}{dt} = \frac{{}^{A}d\mathbf{p}}{dt} + \frac{{}^{A}d\mathbf{r}}{dt}$$

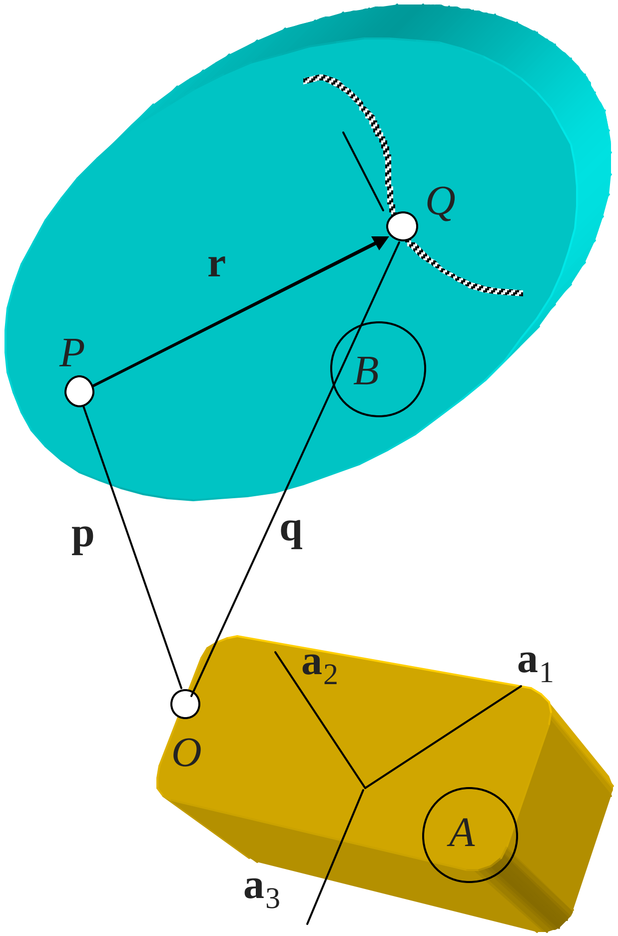
- Substitute definitions of velocities

$${}^{A}\mathbf{v}^{Q} = {}^{A}\mathbf{v}^{P} + \left(\frac{{}^{B}d\mathbf{r}}{dt} + {}^{A}\boldsymbol{\omega}^{B} \times \mathbf{r} \right)$$

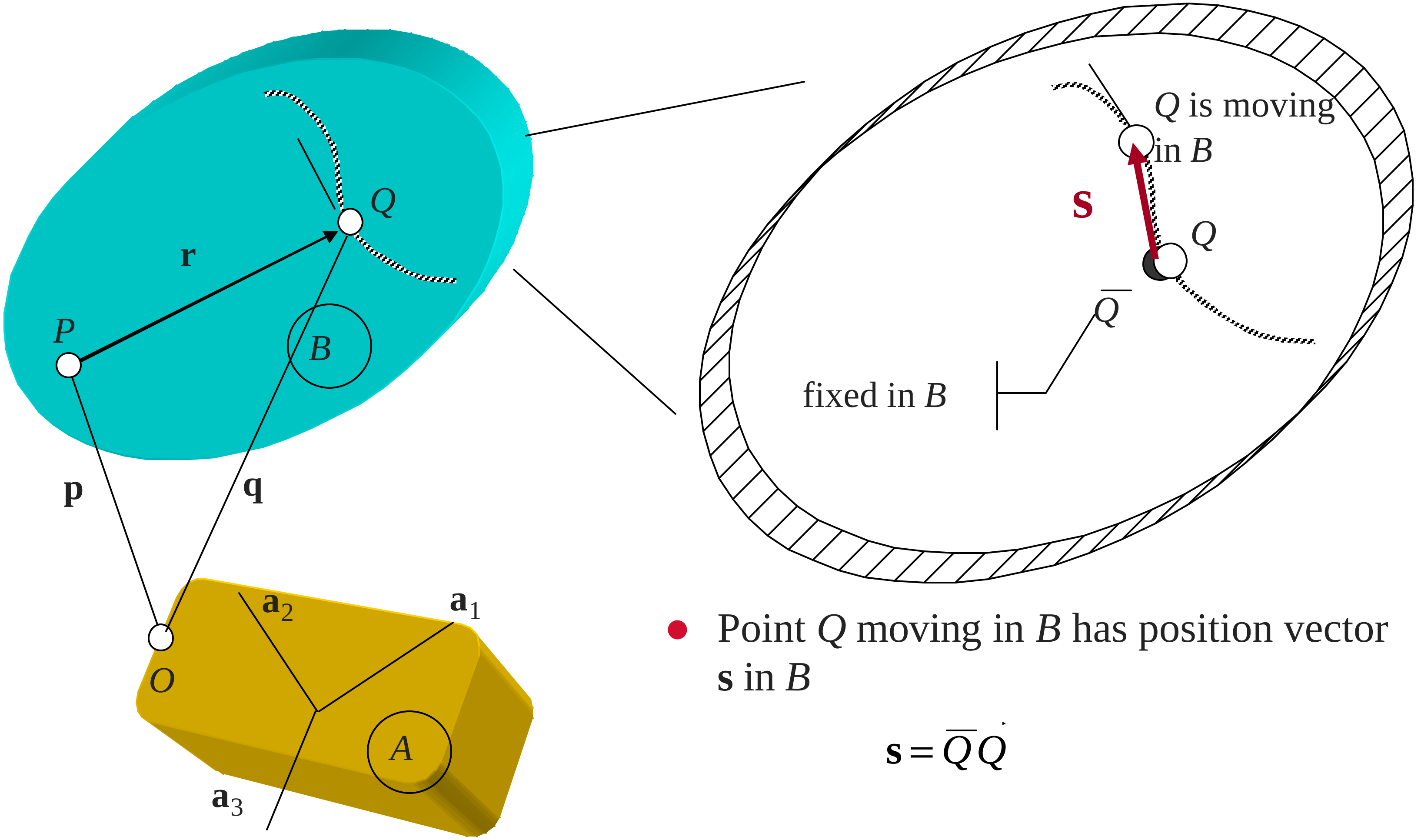
- Velocities for *P* and *Q* in *A*

$${}^{A}\mathbf{v}^{Q} = {}^{A}\mathbf{v}^{P} + \boxed{{}^{B}\mathbf{v}^{Q}} + {}^{A}\boldsymbol{\omega}^{B} \times \mathbf{r}$$

non zero



Velocity and Acceleration of *Q* in *B*



- Point *Q* moving in *B* has position vector **s** in *B*

$$\mathbf{s} = \overline{Q}Q$$

- Velocity, acceleration

$$\frac{{}^{B}d\mathbf{s}}{dt} = {}^{B}\mathbf{v}^{Q}, \; {}^{B}\mathbf{a}^{Q} = \frac{{}^{B}d}{dt}\left({}^{B}\mathbf{v}^{Q}\right)$$

Acceleration of *P* and *Q*

- Velocities for *P* and *Q* in *A*

$$ {}^{A}\mathbf{v}^{Q} = {}^{A}\mathbf{v}^{P} + {}^{B}\mathbf{v}^{Q} + {}^{A}\omega^{B} \times \mathbf{r} $$

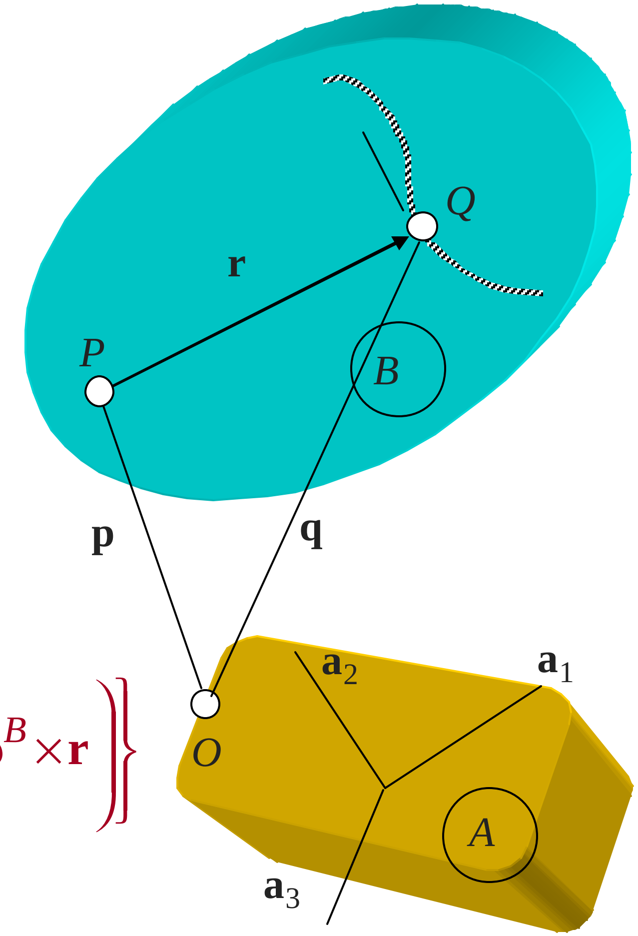
- Differentiate in *A*

$$ {}^{A}\mathbf{a}^{Q} = \frac{{}^{A}d}{dt}\left({}^{A}\mathbf{v}^{Q}\right) = \frac{{}^{A}d}{dt}\left({}^{A}\mathbf{v}^{P}\right) + \frac{{}^{A}d}{dt}\left({}^{B}\mathbf{v}^{Q}\right) + \frac{{}^{A}d}{dt}\left({}^{A}\omega^{B} \times \mathbf{r}\right) $$

$$ \left\{ {}^{A}\mathbf{a}^{P} \right\} $$

$$ \left\{ \frac{{}^{A}d}{dt}\left({}^{A}\omega^{B}\right) \times \mathbf{r} + {}^{A}\omega^{B} \times \left(\frac{{}^{B}d\mathbf{r}}{dt} + {}^{A}\omega^{B} \times \mathbf{r} \right) \right\} $$

$$ \left\{ \frac{{}^{B}d}{dt}\left({}^{B}\mathbf{v}^{Q}\right) + {}^{A}\omega^{B} \times {}^{B}\mathbf{v}^{Q} \right\} $$



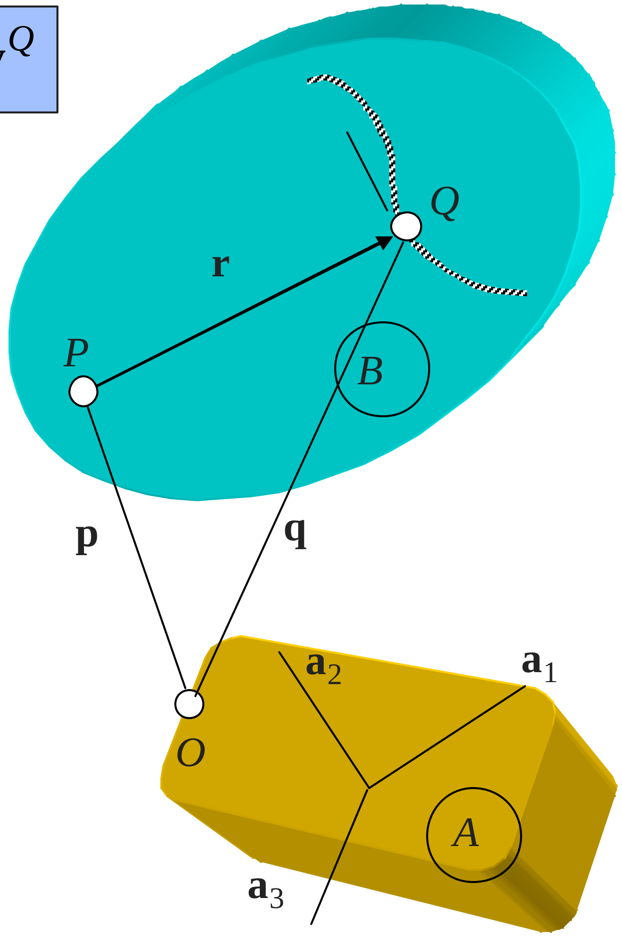
Acceleration of *P* and *Q*

$$ {}^{A}\mathbf{a}^{Q} = {}^{A}\mathbf{a}^{P} + {}^{B}\mathbf{a}^{Q} + {}^{A}\alpha^{B}\times\mathbf{r} + {}^{A}\omega^{B}\times\left({}^{A}\omega^{B}\times\mathbf{r}\right) + 2\,{}^{A}\omega^{B}\times{}^{B}\mathbf{v}^{Q} $$

tangential acceleration

centripetal (normal) acceleration

Coriolis acceleration



Special case: **r** = 0

$$ {}^{A}\mathbf{a}^{Q} = {}^{A}\mathbf{a}^{\overline{Q}} + {}^{B}\mathbf{a}^{Q} + 2\,{}^{A}\omega^{B}\times{}^{B}\mathbf{v}^{Q} $$