MEAM 535

Matlab Exercise II:
Simulation of a Rolling/Sliding Disk

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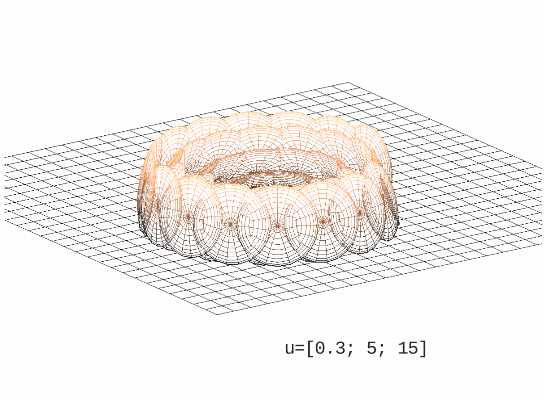
Simulation Result



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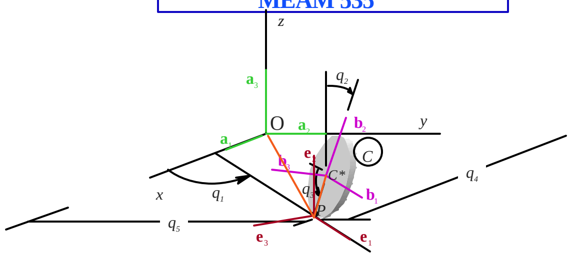

Find the ***position vector*** of the center (C^*) of the disk in frame A and express it in terms of triad **($\mathbf{a}_1$, $\mathbf{a}_2$, $\mathbf{a}_3$)**

$$\dot{OC^*} = \dot{OQ} + \dot{QC^*}$$
$$= (?)\,\mathbf{a}_1 + (?)\,\mathbf{a}_2 + (?)\,\mathbf{a}_3$$

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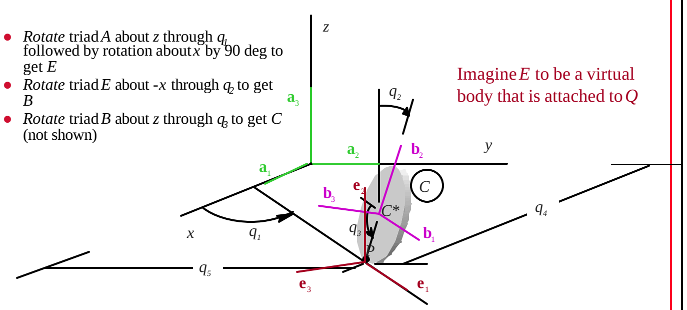
Reference Triads

- *Rotate* triad A about z through q_1 followed by rotation about x by 90 deg to get E
- *Rotate* triad E about $-x$ through q_2 to get B
- *Rotate* triad B about z through q_3 to get C (not shown)



Imagine E to be a virtual body that is attached to Q

$${}^A\mathbf{R}_C = {}^A\mathbf{R}_E \; {}^E\mathbf{R}_B \; {}^B\mathbf{R}_C$$

Imagine B to be a virtual body that is attached to C^*

Angular Velocity: Components

${}^{A}\omega^{C} = u_1\mathbf{b}_1 + u_2\mathbf{b}_2 + u_3\mathbf{b}_3$

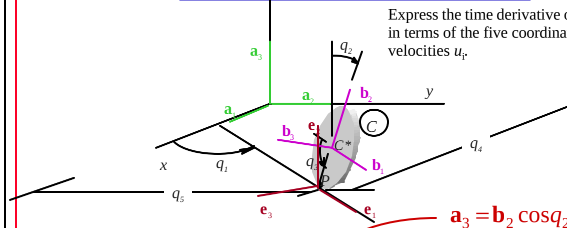
- u_i are the components of the angular velocity of the disk with respect to the reference triad B

${}^{A}\omega^{C} = u_x\mathbf{a}_x + u_y\mathbf{a}_y + u_z\mathbf{a}_z$

- u_a are the components of the angular velocity of the disk with respect to the reference triad A

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{bmatrix} = \begin{bmatrix} & & & & \\ & & \mathbf{X} & & \\ & & & & \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \\ \dot{q}_5 \end{bmatrix}$$

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \\ \dot{q}_5 \end{bmatrix} = \begin{bmatrix} & & & & \\ & & \mathbf{Y} & & \\ & & & & \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{bmatrix}$$



Express the time derivative of each coordinate in terms of the five coordinates and the velocities u_i.

$\mathbf{a}_3 = \mathbf{b}_2\cos q_2 + \mathbf{b}_3\sin q_2$

$${}^{A}\omega^{C} = {}^{A}\omega^{E} + {}^{E}\omega^{B} + {}^{B}\omega^{C} = \dot{q}_1\mathbf{a}_3 - \dot{q}_2\mathbf{b}_1 + \dot{q}_3\mathbf{b}_3$$

$${}^{A}\omega^{C} = u_1\mathbf{b}_1 + u_2\mathbf{b}_2 + u_3\mathbf{b}_3 = -\dot{q}_2\mathbf{b}_1 + \dot{q}_1\cos q_2\mathbf{b}_2 + (\dot{q}_1\sin q_2 + \dot{q}_3)\mathbf{b}_3$$

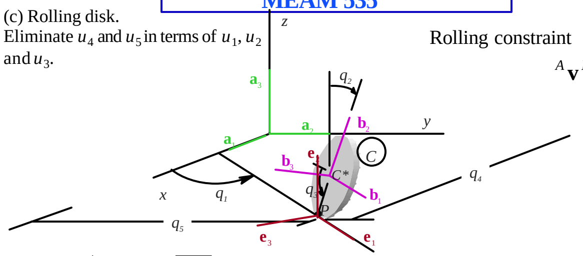
$u_1 = -\dot{q}_2 \quad u_2 = \dot{q}_1\cos q_2 \quad u_3 = \dot{q}_1\sin q_2 + \dot{q}_3 \quad u_4 = \dot{q}_4 \quad u_5 = \dot{q}_5$

$\dot{q}_1 = \frac{u_2}{\cos q_2} \quad \dot{q}_2 = -u_1 \quad \dot{q}_3 = u_3 - \frac{u_2\sin q_2}{\cos q_2}$

(c) Rolling disk.
Eliminate u_4 and u_5 in terms of u_1, u_2 and u_3.

Rolling constraint

${}^{A}\mathbf{v}^{P} = 0$



$${}^{A}\mathbf{v}^{P} = {}^{A}\mathbf{v}^{C^*} + {}^{A}\omega^{C} \times \overline{C^*P}$$

$$= {}^{A}\mathbf{v}^{C^*} + (u_1\mathbf{b}_1 + u_2\mathbf{b}_2 + u_3\mathbf{b}_3) \times (-R\mathbf{b}_2)$$

$$= 0$$

$$u_4\cos q_1 + u_5\sin q_1 - Ru_2\tan q_2 + Ru_3 = 0$$

$$-u_4\sin q_1\sin q_2 + u_5\cos q_1\sin q_2 = 0$$

Two constraints which reduce the disk's degrees of freedom from 5 to 3 … More about this later!

Program Structure