MEAM 550 **Fall 2001** **University of Pennsylvania**

Solution #5

Problem 1

$$z_{i+1} = \frac{\varepsilon A V^2}{2k(g_0 - z_i)^2}$$

Begin with $i = 0, z_0 = 0$ and proceed until difference between two successive z's is less than a tolerance value.

This is the simple principle of the "relaxation" or "fixed-point" method.

Problem 2

$$F = kx = \frac{F_{MM}^2}{2x_0}\left[\frac{\mu_0 A(g - g_\mu)}{g g_\mu + \left(\frac{\mu_0 L_m}{\mu}\right)(g + g_\mu) + \left(\frac{\mu_0 L_m}{\mu}\right)^2}\right]$$

Substitute $x = x_0$ in the above equation to get k.

Answer: k = 0.2462 N/m.

For dynamics, solve the equation:

$$m\ddot{x} + b\dot{x} + kx = \frac{F_{MM}^2}{2x_0}\left[\frac{\mu_0 A(g - g_\mu)}{g g_\mu + \left(\frac{\mu_0 L_m}{\mu}\right)(g + g_\mu) + \left(\frac{\mu_0 L_m}{\mu}\right)^2}\right]$$

Take $b = 0.1 \times 2\sqrt{mk}$ with m as the mass of the armature (assume iron's density to get mass).

We also have another equation for the electrical circuit:

$$RI + L\frac{dI}{dt} = V$$

where $L = \frac{n^2}{\Re} = \frac{n2}{x_0}\left(\frac{\mu_0 A}{g0 - z + \frac{\mu_0}{\mu}(L_m)}\right)$ from Eq. 5.57 of the textbook.

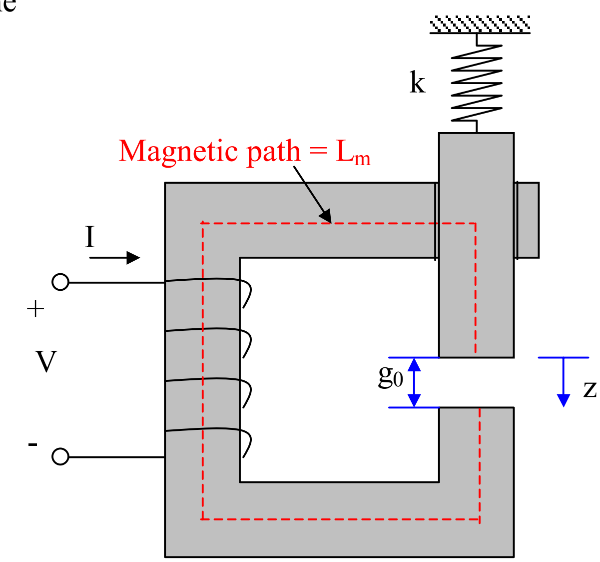
Write the three state variables $x_1 = x$, $x_2 = \dot{x}_1$ and $x_3 = I$ and write the state equations as

$$\begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{Bmatrix} = \begin{bmatrix} x_2 \\ \dfrac{F_{MM}^2}{2x_0}\left[\dfrac{\mu_0 A(g-g_\mu)}{gg_\mu + \left(\dfrac{\mu_0 L_m}{\mu}\right)(g+g_\mu) + \left(\dfrac{\mu_0 L_m}{\mu}\right)^2}\right] - bx_2 - kx_1 \\ \dfrac{1}{L}(V - Rx_3) \end{bmatrix}$$

and solve them.

Problem 3

By denoting length of the magnetic path at the beginning by L_{m0}, along the lines of Eq. 5.45 of the textbook, we can write for any *z*:



$$F_{MM} = H_\mu (L_{m0} + z) + H_g (g0 - z)$$

Since $H_\mu = \dfrac{\mu_0}{\mu} H_g$,

$$F_{MM} = \frac{\mu_0}{\mu} H_g (L_{m0} + z) + H_g (g0 - z)$$

and from this,

$$\phi = B_g A = \mu_0 H_g A = \frac{\mu_0 A}{\dfrac{\mu_0}{\mu}(L_{m0} + z) + (g0 - z)} F_{MM}$$

By writing the magnetic co-energy as

$$W^*(F_{MM}, g) = \frac{1}{2} \frac{\mu_0 A}{\dfrac{\mu_0}{\mu}(L_{m0} + z) + (g0 - z)} F_{MM}^2$$

whose derivative with respect to *z* gives the negative of the force on the vertically moving armature. As the form of this equation is similar to that of the electrostatic force equation, the behavior of this system is the same as before. Therefore, pull-in occurs here also.