MEAM 550 **Fall 2001** **University of Pennsylvania**

Solution #8

Problem 1

Analytical solution using eigen modes is given by

$$T(x) = \frac{4P_0L^2}{\kappa\pi^3}\sum_{n\ odd}\frac{\sin\left(\frac{n\pi x}{L}\right)}{n^3}$$

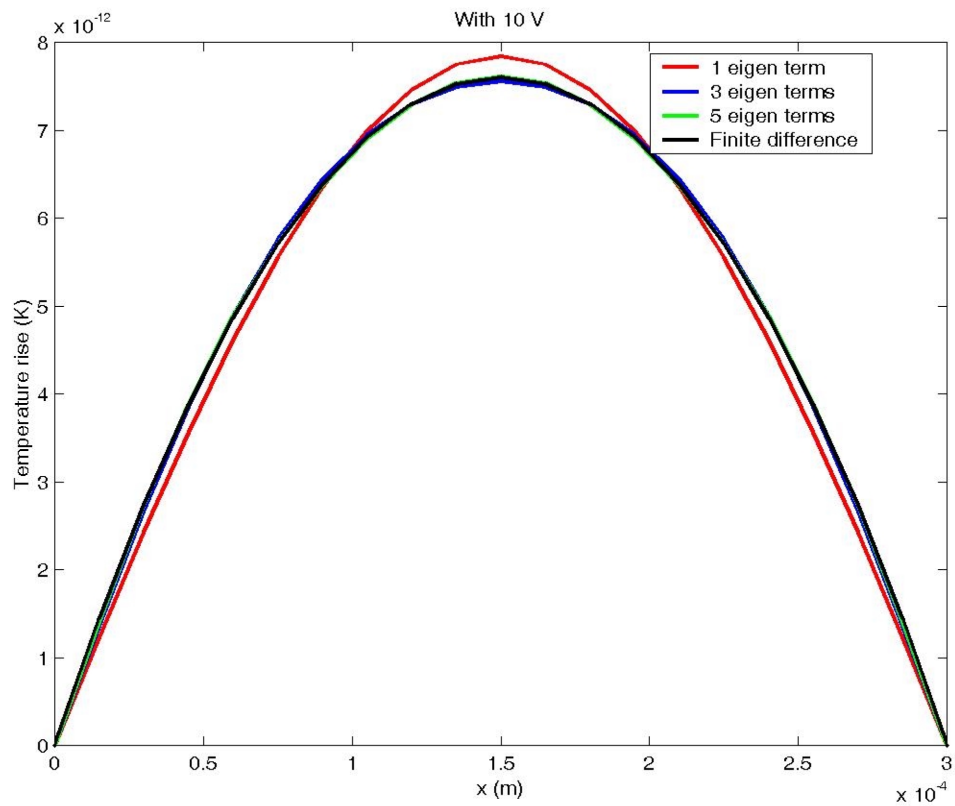
P_0 is computed using a voltage, *V*, applied across the resistor using

$$P_0 = \left(\frac{V^2}{R}\right)/\,volume = \frac{V^2}{\left(\frac{L}{A\sigma_e}\right)(AL)} = \frac{\sigma_e V^2}{L^2}$$

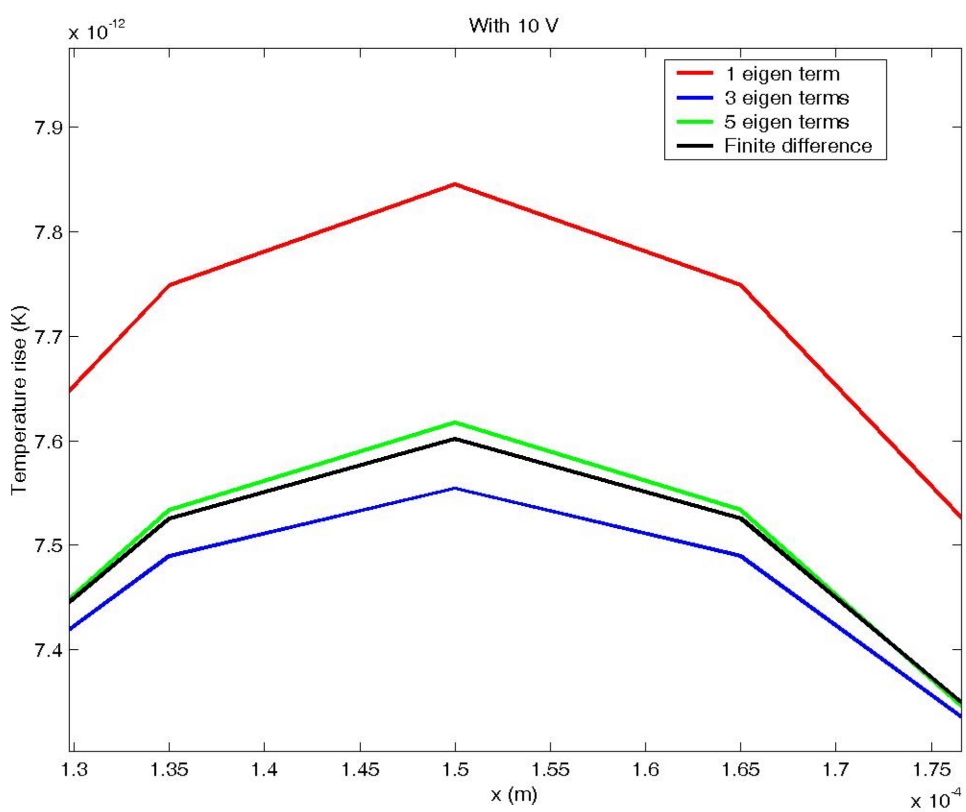
10 V was used in the following plots.

As can be seen, from the figures below and by running the simulation for other *h* size, the finite difference solution is closer to 3-eigenterm solution. No matter how small *h* is, the finite different solution tends to be away from the 1-eigenterm solution at the maximum value. Likewise, even if *h* is decreased to a value smaller than 7.5 μm, it does not approach the 3-eigenterm solution. So, a 5-eigenterm solution is plotted to show that finite difference solution with very small *h* does approach that.

Plots and Matlab script follow.



With 10 V
x 10^-12
Temperature rise (K)
x (m)
x 10^-4
1 eigen term
3 eigen terms
5 eigen terms
Finite difference



With 10 V
x 10^-12
Temperature rise (K)
x (m)
x 10^-4
1 eigen term
3 eigen terms
5 eigen terms
Finite difference

```
clear all
clc
clf
axis normal

L = 300E-6; % m
A = 4E-12; % m^2
k = 148; % W/(K-m)
sigmae = 1500; % Siemens/m
R = L/A/sigmae; % Ohm
V = 10; % V
current = V/R; % Amp
P0 = 0.1; %current^2*R/L/A; % W/m^3

% Analytical solution

x = 0:L/20:L;
T = zeros(size(x));
for n = 1:2:1,
   T = T + sin(n*pi*x/L) / n^3;
end
T = 4 * P0 * L^2 / k / pi^3 * T;

hndl = plot(x,T,'-r');
set(hndl, 'LineWidth', 2);

T = zeros(size(x));
for n = 1:2:3,
   T = T + sin(n*pi*x/L) / n^3;
end
T = 4 * P0 * L^2 / k / pi^3 * T;

hold on

hndl = plot(x,T,'-b');
set(hndl, 'LineWidth', 2);

T = zeros(size(x));
for n = 1:2:5,
   T = T + sin(n*pi*x/L) / n^3;
end
T = 4 * P0 * L^2 / k / pi^3 * T;

hold on

hndl = plot(x,T,'-g');
set(hndl, 'LineWidth', 2);

N = 21;
h = L/(N-1);
P = h*A*P0*ones(N,1);
P(1) = 0;
P(N) = 0;

KT = zeros(N, N);
KT(1,1) = 1;
G = k*A/h;
```

```matlab
for i = 2:1:N-1,
   KT(i,i-1) = -G;
   KT(i,i) = 2*G;
   KT(i,i+1) = -G;
end
KT(N,N) = 1;

fdT = KT \ P;
fdx = 0:h:L;
hndl = plot(fdx, fdT,'-k',fdx,fdT,'.k');
set(hndl, 'LineWidth', 2);

legend('1 eigen term', '3 eigen terms', '5 eigen terms', 'Finite
difference')
xlabel('x (m)');
ylabel('Temperature rise (K)');
title('With 10 V');
```