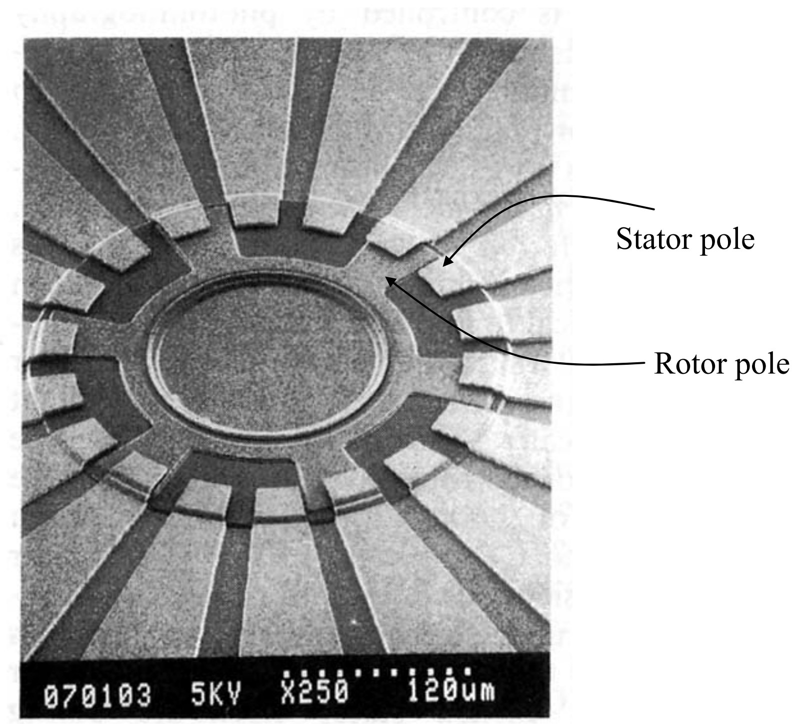
MEAM 550 **Modeling and Design of MEMS** **Spring 2004**

Solution to **Mid-term Examination**

Points: 25 Time: 90 minutes

Question 1 (8 points)

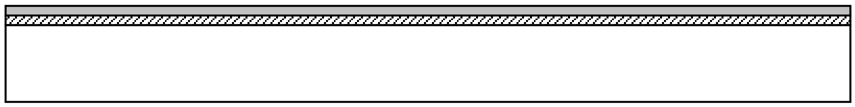
Propose a surface micromachining process consisting of polysilicon and sacrificial silicon dioxide layers on top of a silicon substrate coated with a silicon nitride layer of insulation, for realizing a variable capacitance micromotor whose scanning electron microscope picture is shown below. That is, write down the process steps (deposit this layer, etch that layer, sacrifice this layer, and so forth) and draw a 2-D cross-section along any diameter of the rotor after every critical process step. There is no need to draw the mask layouts.



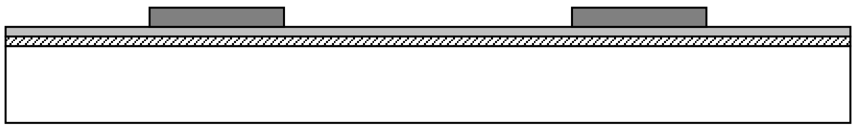
Three-phase variable capacitance micro motor (Mehregany et al.)

We should try to minimize the number of process steps as much as possible. The process shown below uses only two structural layers and two sacrificial layers. The drawings below are not to scale. The top views (the mask layouts) are not shown but can be imagined based on the photograph above.

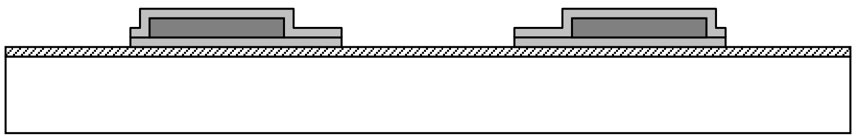
Deposit silicon nitride and then the first sacrificial oxide layer.



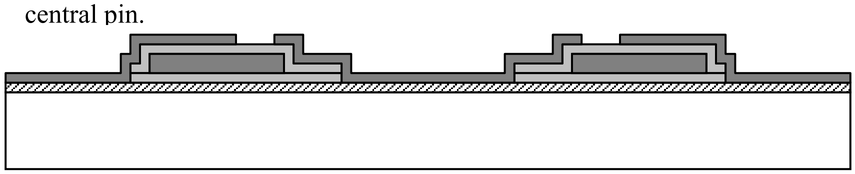
Deposit first structural polysilicon layer and etch it to define the rotor pattern.



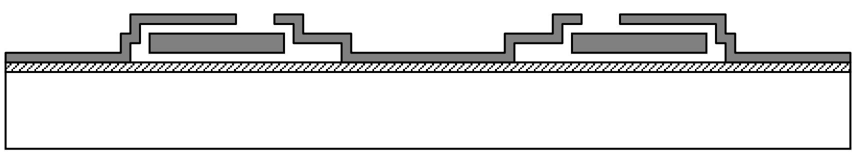
Deposit the second sacrificial oxide layer and etch this and the first oxide layer.



Deposit the second structural polysilicon layer and etch it to define stators and central pin.



Dissolve sacrificial layers; the rotor will fall down but not shown that way.



Question 2 (8 points)

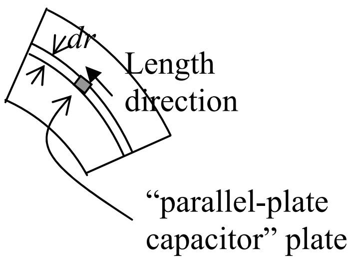
The above motor has three phases in the sense that there are 18 stator poles and six rotor poles. The stator poles are divided into three groups (phases) and each phase is activated sequentially. To start the motor, the stator pole set that is misaligned with the rotor poles but is close to them, is activated. As in a comb-drive, the “plates” try to align and the rotor moves. The “ parallel plates” are rotor pole and stator pole pair. Then, the next phase is activated after turning off the previous phase. The rotor continues to rotate and so on. Obtain an expression for the torque experienced by the rotor at any time when one stator phase is activated with a voltage V while the rotor is at the ground voltage of zero.

The vertical gap between the rotor and the stator electrodes is g. The inner and outer radii of the circles that bound the rotor poles are r_i and r_o, respectively.

Consider a differential ring of width dr at radius r from the center of the motor. This ring is to be "cut" to imagine six arches that make up the overlapping area between the rotor and stator pole faces. The six arches of width dr can be imagined to be parallel plate capacitors. The force that produces the torque is the electrostatic force in the tangential direction (length) direction of the parallel-plate capacitor. This force is given by

$$dF = \frac{\varepsilon_0 V^2}{2g} w = \frac{\varepsilon_0 V^2}{2g}\left(\frac{2\pi r}{np} dr\right)$$

where n is the number phases (here, it is 3) and p is the distance between the midpoints of two adjacent stator pole phases. We can assume that p is $1/(18\times 1.5)^{th}$ of the



perimeter. That is, the gap between two adjacent stator poles is half the circumferential width of the stator pole.

Now, the torque on the six arches is given by

$$dT = rdF = r\frac{\varepsilon_0 V^2}{2g}\left(\frac{2\pi r}{np} dr\right) = \frac{\pi\varepsilon_0 V^2}{gnp} r^2 dr$$

To get the total torque, we integrate from r_i to r_o:

$$T = \int_{r_i}^{r_o} \frac{\pi\varepsilon_0 V^2}{gnp} r^2 dr = \frac{\pi\varepsilon_0 V^2}{gnp}\left(r_o^3 - r_i^3\right) = \frac{\pi\varepsilon_0 V^2}{3gp}\left(r_o^3 - r_i^3\right)$$

Question 3 (9 points)
The figure in the next page shows four stages of a two-level, fixed-fixed beam structure used in a micromachined "polychromator" (a remote gas detector that uses correlation spectrography). As you can see, this design enables almost purely vertical motion of the upper beam without deformation. The lower level and upper level beams are both 20 μm wide and 1000 μm long. The lower beam is 2 μm thick and the upper one 3 μm thick. The gap between the lower beam and the electrode on the ground is 8 μm. The lower and upper beams are vertically separated by 4 μm.

(a) Using lumped modeling, compute the voltage (V_2) when the upper beam touches the side supports. (4 points)

(b) Compute the pull-in voltage of the combined s3tructure. (4 points)
(c) How would you modify this design to make the upper beam tilt rather than vertically move down? (1 point)

(a) Using the lumped model, the spring constant of the lower fixed-fixed beam is obtained as

$$k_l = \frac{384EI}{L^3} = \frac{32Ewt_l^3}{L^3}$$

The electrostatic force (neglecting fringing fields) is given by

$$F_e = \frac{\varepsilon_0 A V_2^2}{2g^2} = \frac{\varepsilon_0 wLV_2^2}{2g^2}$$

Although g should be taken as $(g-\delta)$ (which will then be solved as a cubic in δ), we will simply neglect that for the moment (solving a cubic in an exam setting is a waste of time!). Thus,

$$\frac{\varepsilon_0 wLV_2^2}{2g^2} = k_l\delta \Rightarrow V_2 = \sqrt{\frac{2k\delta g^2}{\varepsilon_0 wL}} = \sqrt{\frac{64Et_l^3\delta g^2}{\varepsilon_0 L^4}}$$

(b) Much before the pull-in voltage for the combined structure is reached, the upper beam would have touched the supports. Then, the upper beam also becomes a fixed-fixed beam. These two beam are mechanical springs in parallel. The combined spring constant is given by

$$k_{combined} = k_l + k_u$$

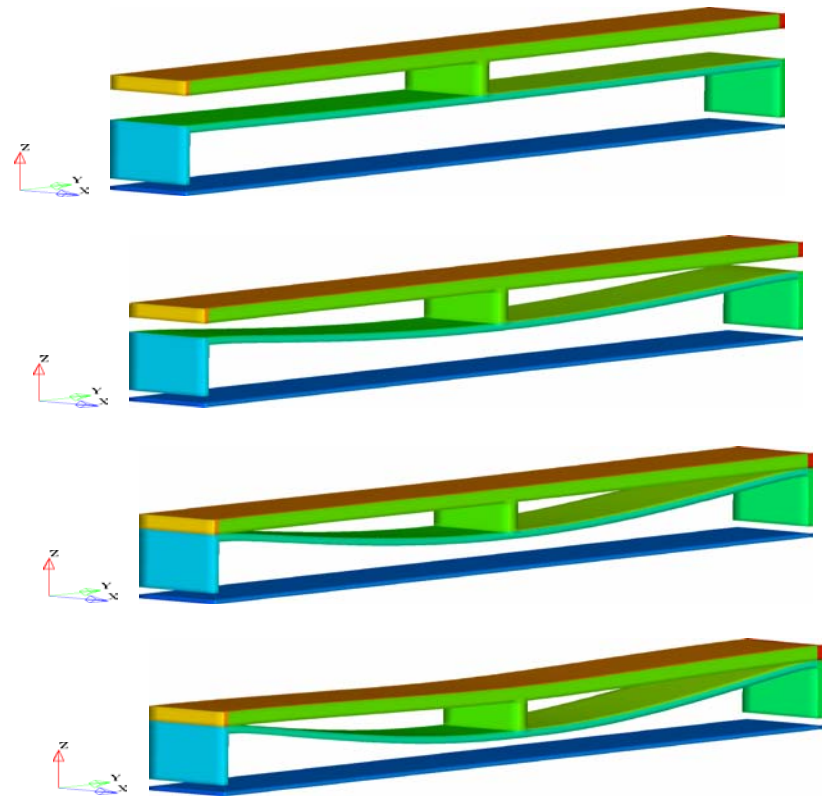
where $k_l = \frac{32Ewt_l^3}{L^3}$ and $k_u = \frac{32Ewt_u^3}{L^3}$.

We can neglect the electrostatic force acting on the upper beam as it is too far above the ground electrode. So, the pull-in voltage is simply a function of the combined spring constant and the gap between the lower beam and the ground electrode.

$$V_{pull-in}^{Combined} = \sqrt{\frac{8k_{combined}\, g^3}{27\varepsilon_0 A}} = \sqrt{\frac{8k_{combined}\, g^3}{27\varepsilon_0 wL}}$$

(c) It can be done in several ways.

- Shift the vertical connecting segment between the lower and upper beam away from the center.
- Have the ground electrode only left or right sides instead of across the entire length beneath the lower beam.
- Use a cantilever configuration instead of the fixed-fixed beam.



Unactuated

With a voltage V_1

With a voltage $V_2 > V_1$

With a voltage $V_3 > V_2$

(Figure courtesy of Polychromix, Inc.)