MEAM620
Homework No. 5
Instructor: Savvas Loizou
Due: 26 March 2007

March 12, 2007

Navigation Functions on sphere worlds

1. * Assume a 2-D spherical workspace. Given the workspace parameters as

$$W = \{q_d, \begin{bmatrix} q_0 \\ \rho_0 \end{bmatrix}, \ldots, \begin{bmatrix} q_M \\ \rho_M \end{bmatrix}\}$$

   construct a Matlab function *findk(W)* that will return the parameter k of the navigation function. q_d is the destination configuration while q_i and ρ_i are the center and radius of obstacle i.

2. Construct a Matlab function *NF(x, W, k)* that returns the value of a navigation function at position x in a spherical workspace. The structure W is defined as before. The parameter k should be calculated using function findk(W) constructed in the previous problem. Present the graphs of a 2-D navigation function for 3 and 4 obstacles, accompanied by a table containing the information of W and k.

3. Construct a Matlab function *gradphi(x, W, k)* that returns the gradient of navigation function at point x. The workspace and the structure $\mathcal{W}$ are as before. (Note: You can use either numerical or analytic approaches in calculating the gradient)

4. * Assume that we have the system

$$\dot{x} = -K\nabla\varphi$$

   where φ a navigation function, K a positive gain, and that we have the following discrete time implementation of that system:

$$x(t + \Delta t) = x(t) - K\nabla\varphi\Delta t$$

   where Δt is the sampling interval. Given the structure $\mathcal{W}$, the destination configuration and the sampling interval Δt, determine a gain K as a function of the given quantities that would guarantee the safety specifications of the navigation function implementation.

5. Using the discrete time implementation of the previous problem, simulate a system navigating among a. 3 and b. 4 obstacles for two different (non-trivial) initial conditions for each case. Present the graphs of the system's trajectories for each case.

* **Note**: Questions 1 and 4 are bonus questions. In case you find them difficult to solve, proceed with solving the rest of the problems using trial and error estimates of k and K. An extension can be provided only for those questions. The rest should be returned on time.