MEAM 620: HW 1

Sachin Chitta (sachinc@grasp.upenn.edu)

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1: MATLAB Programming assignment

Using MATLAB, write the following functions:

1. Given $R \in SO(3)$, return ω and θ such that $R = e^{\hat{\omega}\theta}$.
2. Given a twist ξ and $\theta \in \mathbb{R}$ return $A = e^{\hat{\xi}\theta}$.
3. Given $A \in SE(3)$, return twist ξ and θ such that $A = e^{\hat{\xi}\theta}$.
4. Given $R \in SO(3)$, return roll, pitch and yaw euler angles.
5. Given axis, magnitude and pitch for a screw motion, return corresponding homogeneous transformation matrix A.
6. Given $A \in SE(3)$, return axis, magnitude and pitch of corresponding screw.

2: Given

$$R = \begin{bmatrix} 0.4330 & -0.7500 & 0.5000 \\ 0.7891 & 0.0474 & -0.6124 \\ 0.4356 & 0.6597 & 0.6124 \end{bmatrix}$$

Find the exponential coordinates for this rotation.

3:

(1) Littledog (Figure 1) is a quadruped robot used in the **Learning Locomotion** program in the GRASP Lab. It has multiple sensors onboard including an accelerometer and a gyroscope. When the robot is not moving, the accelerometer reading corresponds to the direction of gravity with respect to a body fixed frame attached to the robot, *eg* when the robot is placed on a horizontal surface with zero pitch, zero roll and zero yaw, the accelerometer reads $(0, 0, 9.81)$. Table provides a set of accelerometer readings for different poses of Littledog. Determine the exponential coordinates and the roll, pitch and yaw euler angles for the robot corresponding to these readings. Comment on your answers.

(2) Download the dataset (littledog_gyro.txt) from the course website. The dataset provides 10 seconds worth of raw angular rate data for motion of Littledog measured using the onboard gyroscopes. The data is sampled at 100 Hz. Use the data to determine the euler angles corresponding

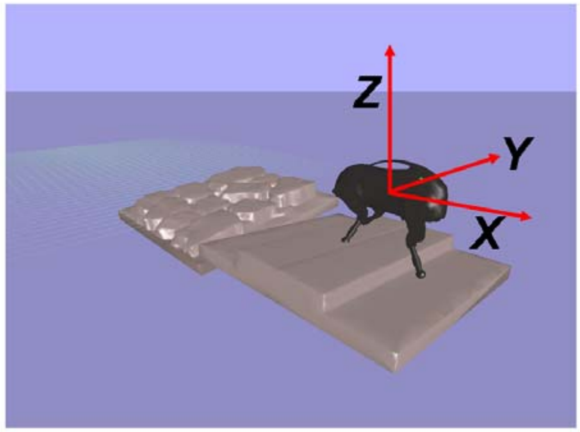

Figure 1: Littledog picture for Problem 3.

No.	a_x	a_y	a_z
1	-0.301060	0.014285	10.184623
2	-0.377591	-0.004374	9.985135
3	-0.984582	0.106256	9.878965
4	4.517402	-1.111705	9.849427
5	10.115886	0.355644	0.816780
6	4.031290	0.338663	-8.392149
7	7.989041	2.021141	5.229123
8	2.177099	9.017443	2.203660
9	7.244625	0.379338	0.394842
10	9.81	0	0

Table 1: Accelerometer data for Problem 3.

to the rotation of the robot at the end of every second with respect to a global reference frame. Assume that the robot was initially at the origin of the reference frame with zero roll, zero pitch and zero yaw. The matlab command *gyro_data = load('littledog_gyro.txt')* will load the data into a matlab workspace variable gyro_data. The data is in the format $(\omega_x, \omega_y, \omega_z)$ where the angular rates are in the body fixed reference frame of the robot shown in Figure 1.

4: The simplest gait for quadruped robots is a statically stable gait where the robot moves one leg at a time. Thus, three legs of the robot are always on the ground. Such a gait is often referred to as a *crawl* gait. The main requirement of a statically stable gait is that the robot be in static equilibrium at all points. This translates to making sure that the vertical projection of the center of gravity of the robot stays within the triangle of support formed by the three legs on the ground at all times. One method of ensuring this is to move the robot body so that the projection of the center of gravity of the robot coincides with the centroid of the triangle of support.

We will use the following notation for this problem. Let the feet of the robot be represented in the order (1) left front, (2) right front, (3) left back and (4) right back. Let the superscript G

denote quantities expressed in a global reference frame. Let the superscript B represent quantities expressed in a body fixed reference frame of the robot coincident with the geometric center (and center of gravity) of the robot.

In Table 2, you are given

1. The initial pose of the robot (translation and euler angles) in the global reference frame
2. The positions of the four feet of the robot $(x_i^G, y_i^G, z_i^G), i = 1, 2, 3, 4$ in the global reference frame
3. The index of the foot that will be picked up in the next step.

Let z_f^G denote the average of the z position of the front two feet while z_r^G denotes the average of the z positions of the back two feet. You need to move the body of the robot to facilitate the picking up of a foot. You choose to do this using the following strategy:

- Translate the robot so that its (x, y) position in the global reference frame coincides with the centroid of the triangle formed by the three feet that will stay on the ground
- Rotate the robot so that its pitch is given by $\gamma = -\text{atan}\frac{z_f^G - z_r^G}{L_{fr}}$ where $L_{fr} = 0.2$ m is a constant.
- Raise or lower the height of the robot so the z position of all of the feet in the body fixed reference frame is greater than -0.15 m, *ie* $z_i^B \geq -0.15, i = 1, \ldots, 4$.

(x, y, z)	$(0.548, 0.389, 0.164)$
(roll,pitch,yaw)	(-0.086,-0.2795,0.115)
(x_1^G, y_1^G, z_1^G)	(0.688119,0.522946,0.049961)
(x_2^G, y_2^G, z_2^G)	(0.701085,0.341127,0.065335)
(x_3^G, y_3^G, z_3^G)	(0.483927,0.502644,-0.010030)
(x_4^G, y_4^G, z_4^G)	(0.458653 0.317178 -0.005161)

Table 2: Table for Problem 4: All angles are in radians, all coordinates in m.

Derive:

1. The final pose of the robot body in the global reference frame.
2. A trajectory for the robot body from its initial pose to the final pose so that the final pose is reached in $T = 2$ seconds with zero velocity boundary conditions, *ie* the robot body starts from rest and comes to rest at the end of the motion.

5: The pose of Littledog is measured using a high speed motion capture (MOCAP) system. The system tracks a set of markers on the robot. Littledog undergoes a transformation from an initial position to a final position for which the MOCAP measures the initial and final positions of three markers A, B, C on the body. The positions are given by:

- Point A: initial position $\begin{bmatrix}1 & 0 & 0\end{bmatrix}$, final position $\begin{bmatrix}0 & -1 & 3\end{bmatrix}$.
- Point B: initial position $\begin{bmatrix}2 & 1 & -1\end{bmatrix}$, final position $\begin{bmatrix}-1 & 0 & 4\end{bmatrix}$.
- Point C: initial position $\begin{bmatrix}1 & 2 & 0\end{bmatrix}$, final position $\begin{bmatrix}-2 & -1 & 3\end{bmatrix}$.

Find:

1. The homogeneous transformation matrix corresponding to this displacement.
2. The exponential coordinates corresponding to this displacement.
3. The screw axis, angle of rotation and pitch of screw corresponding to this displacement.

An external observer placed on this rigid body reports that his initial configuration on the rigid body with respect to the global coordinate frame can be determined by the following sequence of transformations:

1. Translation along the **global** X axis by -10 m,
2. Rotation about the **global** Z axis by $pi/4$ radians,
3. Translation along the **global** Z axis by 5 m,
4. Rotation along the **global** X axis by $-pi/6$ radians.

Describe the displacement of the rigid body in the coordinate frame of the observer.

6: Show that $SO(3)$, the set of all 3×3 matrices with $\det(R) = +1$ and $RR^T = I$, is a lie group. Find its dimension. Find the dimension of $SO(n)$, the set of $n \times n$ matrices with $\det(R) = +1$ and $RR^T = I$.

7: Prove that $e^{\hat{\omega}\theta} \in SO(3)$, where $\hat{\omega}$ is the skew-symmetric matrix corresponding to $\omega = \begin{bmatrix} \omega_1 & \omega_2 & \omega_3 \end{bmatrix}$ and $\theta \in \mathbb{R}$.

8: Properties of skew-symmetric matrices: MLS Chapter 2, Problem 4 Show that the following properties of skew-symmetric matrices are true:

1. If $R \in SO(3)$ and $\omega \in \mathbb{R}^3$, then $R\hat{\omega}R^T = (\hat{R\omega})$.
2. If $R \in SO(3)$ and $v, w \in \mathbb{R}^3$, then $R(v \times w) = (Rv) \times (Rw)$.
3. Determine the dimension and give a basis for $so(3)$.

9: Planar rotational motion: MLS Chapter 2, Problem 8

Let $SO(2)$ be the set of all 2×2 orthogonal matrices with determinant equal to $+1$.

1. Let $\omega \in \mathbb{R}$ be a real number and define $\hat{\omega} \in so(2)$ as the skew-symmetric matrix

$$\hat{\omega} = \begin{bmatrix} 0 & -\omega \\ \omega & 0 \end{bmatrix}$$

Show that:

$$e^{\hat{\omega}\theta} = \begin{bmatrix} \cos\omega\theta & -\sin\omega\theta \\ \sin\omega\theta & \cos\omega\theta \end{bmatrix}.$$

Is the exponential map exp: $so(2) \to SO(2)$ surjective? Injective?

2. Show that for $R \in SO(2)$ and $\hat{\omega} \in so(2)$, $R\hat{\omega}R^T = \hat{\omega}$.

10: MLS Chapter 2: Problem 13

Let $\xi_a = (-\omega_a \times q_a + h\omega_a, \omega_a)$ be the twist associated with a screw having pitch h and axis $l = (q_a + \lambda\omega_a : \lambda \in \mathbb{R})$, where all quantities are specified relative to a coordinate frame A.

1. Let B be a second coordinate frame with configuration $g_{ab} \in SE(3)$. Show that the representation of the twist relative to B is given by $\xi_b = Ad_{g_{ab}}^{-1}\xi_a = Ad_{g_{ba}}\xi_a$.

2. Suppose instead that we move the screw via a rigid body transformation $g \in SE(3)$. Show that the transformed screw can be represented by the twist $\xi_a^{'} = Ad_g\xi_a$.