MEAM 620: HW 2

Assigned: February 5, 2007 Due: February 12, 2007

1: MLS Chapter 2, Problem 16

Figure 1, shows a two degree of freedom manipulator. Let $l_0 = 1$, $l_1 = 1$, $l_2 = 1$ be the link length parameters and θ_1, θ_2 the joint angle variables of link 1 and link 2, respectively.

- 1. Express the position and orientation of frame C_3 relative to frame C_0 in terms of the joint angle variables and the link parameters. Now find the position and orientation for $\theta_1 = \pi/6, \theta_2 = -\pi/3$.
- 2. Compute the spatial velocity of C_3 relative to C_0 as function of the joint angles and the joint rates. Using this expression, compute the spatial velocity for the particular values $\theta_1 = \pi/6, \theta_2 = -\pi/3, \dot{\theta}_1 = 0.5, \dot{\theta}_2 = 0.3$.
- 3. Compute the body velocities of C_3 relative to C_0 as functions of the joint angles and the joint rates. Using this expression, compute the body velocity for the particular values $\theta_1 = \pi/6, \theta_2 = -\pi/3, \dot{\theta}_1 = 0.5, \dot{\theta}_2 = 0.3$.
- 4. Find the spatial velocity of the origin of C_3 and use this to check your answer for parts (2) and (3) above.
- 5. Find the spatial velocity of C_3 relative to a reference frame C_4 as shown in Figure 1 with $l_3 = 1, l_4 = 1$.

2: Direct kinematics

Using the product of exponentials formulation, derive the direct kinematics for the Stanford arm (you can assume $a_5 = 0$).

- 1. Setup a *home position* for the manipulator.
- 2. Write the joint twists for the individual joints.
- 3. Apply the product of exponentials formulation.
- 4. Find the configuration of the end-effector for the values $a_1 = 0.1$, $a_2 = 0.1$, $a_3 = 0.1$ and $\theta_1 = 0.5$, $\theta_2 = 0.5$, $d_3 = 0.5$, $\theta_4 = 0.5$, $\theta_5 = 0.5$, $\theta_6 = 0.5$.



Figure 1: Problem 2.

Solve the inverse kinematics for the Stanford arm using the Paden-Kahan subproblems. Choose a set of reference values for the joint angles and joint extensions, solve the direct kinematics to get a reference configuration for the end-effector and solve the inverse kinematics for this configuration. How many solutions do you get?

Comment on the singularities in the solution of the inverse kinematics for the Stanford arm.



Figure 2: Stanford arm.