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## Measuring capacity flexibility of a transportation system

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### Abstract

This paper describes concepts and techniques to measure the flexibility of a transportation system to accommodate changing demands and traffic patterns. Flexibility is increasingly desired as a characteristic of transport systems, particularly in light of changes in supply chains and traffic patterns, and the concern for the vulnerability of the system to both natural disasters and terrorist actions. Two approaches are described. One is based on current methods for estimating system capacity, and the related concept of reserve capacity. This results in a conservative estimate of flexibility. The second approach permits variations in the traffic pattern, in order to more fully capture demand variations that can be accommodated. The described measures are implemented on a containerized freight rail network, as a means of testing their feasibility and potential value as descriptors of system characteristics. The uses of the measures in planning, investment, and policy-making are discussed.

*Keywords:* Flexibility; Agility; Planning; Capacity; System adaptation; Reserve capacity.

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## 1. Objective and definition of flexibility

Flexibility is an important but little studied characteristic of transportation systems. The rapidly increasing and shifting demands for transportation services, the long lead times required to make any major changes to infrastructure, and changing external conditions such as energy supplies, all make flexibility increasingly desirable. This paper presents results from an ongoing project intended to define flexibility of a transportation system and to develop methods for measuring flexibility. *Flexibility* is defined as

the ability of a system to adapt to external changes, while maintaining satisfactory system performance. System performance is characterized by parameters such as capacity, level of service, maintainability, and profitability. External changes are uncontrolled conditions that affect the system, including changes in level of demand or use, shifts in spatial traffic patterns, infrastructure loss and degradation, and changes in the price and availability of important resources such as fuel, etc (Morlok 2003).

The focus of this paper is on flexibility with respect to changes in demand or traffic using a transportation system. Such changes in demand include variations in

- the overall quantity of traffic (e.g., ton-miles),
- the mix of commodities carried
- the spatial pattern of flows (e.g., shifting from predominately east-west to north-south) the mix of specific transport services used (e.g., shifting shipments from slow service to faster service -- ground parcel service to next day air parcel service, or rail carload service to intermodal rail-truck service).

System capacity flexibility is defined as follows:

*System capacity flexibility* is the ability of a transport system to accommodate variations or changes in traffic demand while maintaining a satisfactory level of performance.

Before presenting methods for the measurement of flexibility, it is useful to discuss the importance of the topic and prior research. These sections are followed by presentation of two approaches to measuring flexibility. These are applied to an actual system in order to assess the validity and usefulness of the measures. The paper concludes with a discussion of how the measures could be used in various contexts.

## 2. Importance of capacity flexibility

Capacity flexibility is an important concept, for at least two reasons. One is the continuing increase in US transport traffic while the US transport infrastructure remains relatively stagnant (Committee for the Study of Freight Capacity for the Next Century 2003). A traffic volume that taxes the capacity of transport system links and nodes leaves little opportunity for the system to accommodate changes. In the past three decades domestic freight ton-miles

have increased by 64% and international freight increased by 93%. But the infrastructure has lagged in expansion. For example, the miles of roadway operated by Class I railroads declined by 43%. The U. S. Department of Transportation projects freight traffic growth in tons by nearly 70% over current levels of 15 billion tons by 2020 (U.S. Department of Transportation, 2002). Many industry commentators predict this increase in traffic will continue to result in increased congestion and greater inefficiencies throughout the nation's transportation system (Schulz 2002). This is not lost on industry leaders. For example, a panel of railroad experts recently convened by *Railway Age* concluded that more flexibility is needed to recover after an unusual event and that flexibility is more important than speed in achieving and maintaining reliable service (Judge 2002).

The second reason is the extent of changes in (a) trade patterns, (b) the types of cargo carried and (c) the nature of transport service needed. For example, the trade between Mexico and the United States had an annual mean growth rate of about 9% in each direction from 1991-1997 (Mendoza et al.1999). This change occurred because of external forces, such as the new trade policy (e.g., GATT/NAFTA) and recent production/factory relocations from the US to Mexico. Also, cargo has been undergoing “de-massification”—meaning products are becoming less heavy or dense. They are also increasingly more customized, even in mass-produced quantities. This mass customization leads to a shift toward rapid delivery and small shipments, particularly for final goods from a few warehouses. The rapid growth of parcel traffic, which now exceeds in revenue all modes but trucking (truck-load and less-than-truck-load) in the U.S., provides a striking example (Morlok et al. 2000). It is important to note, however, that this typically high-speed and high reliability traffic moves via road, rail, and air, and in limited cases via water. The trend is affecting all modes. Again, it places greater importance on flexibility, i.e., on the ability to adapt to changes in the quantity and mix of commodities and in the types of services required.

### 3. Previous research

Flexibility is an increasingly important characteristic of many systems, and there is considerable interest in measuring flexibility and in increasing the flexibility of systems. It has been a particularly important topic in manufacturing. This includes relating flexibility to strategy, organizational structure, environmental uncertainty, and technology factors (Vokura and O’Leary-Kelly 1995) and quantitative analyses of how to achieve flexibility in a cost-effective manner (e.g., Jordan and Graves 1995).

One of the few flexibility studies that included reference to transport was a qualitative study that looked at network flexibility in the node, arc and temporal sense (Feitelson and Salomon 2000). They ranked different network systems with respect to their inherent flexibility. Interestingly, they concluded that international air transportation is the least flexible and the telecommunications is the most flexible.

There have been a few studies specifically in telecommunications that address questions falling within the scope of flexibility as defined in this paper. These focus on network design, particularly topology (the link connections of the network between nodes). Particular emphasis has been given to network design (in the sense of links to be included in a network, or added to it) to improve survivability under conditions of link or node failure. Survivability is defined with respect to the fraction of the demand that can still be satisfied after failure of one or more nodes or links. Thus it addresses one type of flexibility described above (infrastructure loss

or degradation). In one of the earliest studies of this sort, Frisch presented methods (algorithms) for determining how many nodes or links must fail in order to break the network into disconnected parts (Frisch 1967), as an aid to network design. This has been followed by many efforts to design networks that are survivable, and to develop useful measures of survivability. These usually assume a given deterministic set of origin-destination (O-D) demands. For example, Rios et al. discussed the construction of a minimum cost capacitated network that accommodates traffic demands between given node (O-D) pairs under specific survivability requirements (Rios et al. 2000). Interestingly, no studies of flexibility with respect to demand variations were found in the telecommunications literature.

Thus there has been considerable work on some questions somewhat related to capacity flexibility. However, none of these prior works provide quantitative measures of flexibility with respect to the topic of interest here—system capacity flexibility.

#### 4. Basic approaches to measuring flexibility

A fundamental concept in the measurement of flexibility is found in traditional engineering economic literature—the idea of sensitivity analysis, especially the particular form used in breakeven analysis. The essential idea is to determine whether or not a productive unit (factory, etc.) will be profitable, when facing an uncertain sales volume of its product. That volume can vary from zero up to the capacity of the production unit. In effect this measures flexibility, in that it provides the range of demand (sales) over which the system (factory) will have satisfactory performance (a positive profit). This concept is illustrated in Figure 1. The difficulty in using this in the transportation context is of course that the demand, or output, of a transportation system is multi-dimensional, reflecting many O-D pairs, commodities, quality of service, etc. Performance may also be multi-dimensional. However, the fundamental idea of identifying the range of demand (or sales volume) over which the system will perform satisfactorily is relevant.

>Fig. 1 about here<

In the context of a multi-dimensional output (traffic), there are essentially two approaches.

1. The first approach follows that taken in most network capacity modeling. It assumes that the spatial pattern of cargo is fixed. This can be generalized to account for mixes of commodities, types of service required, etc., by similarly fixing the percentage mix of traffic of each type. This approach provides much useful information about the range of changes in demand that can be accommodated. However, it yields a very conservative estimate of flexibility.
2. The second approach permits the mix of cargo (spatial pattern, commodity, etc.) to vary. In order to keep the variations in reasonable, a means to measure and limit deviations from a baseline pattern are necessary (as will be described later). This baseline may be the current pattern or a projected pattern--the latter presumably based on possible or likely changes in production and consumption of goods by region and sector.

The first approach uses the models and measures of reserve capacity, already developed and coming into use for assessing the capacity of a transport system. The second builds upon these to develop more robust approaches. Our purpose here is to present the essential ideas and

their usefulness, illustrating them with some of the results of applications to a portion of the U.S. double stack container rail network that has been used for other capacity modeling.

## 5. Fixed traffic pattern approach to measuring flexibility (MAXCAP)

The MAXCAP model estimates the *base traffic pattern capacity* of a system by estimating the maximum traffic that can be accommodated by the system subject to an underlying (base) traffic pattern (i.e., the fixed unit OD matrix) and the system's resources (fleet, terminal and link capacities, etc.). The fixed unit OD matrix requires that the fraction of total traffic (e.g., tons originated, or passenger trips originating, in the entire system) that moves between each OD pair is constant. The concept can also be extended to include the distinction between commodities, etc. The difference between the current traffic and the maximum traffic is referred to as the *reserve capacity* of the system (e.g., Yang and Wang 2002). When it follows the base traffic pattern it is termed the base traffic pattern reserve capacity. For simplicity, the concepts of capacity and reserve capacity will be referred to the *base pattern capacity* and *base pattern reserve capacity*, respectively.

From the perspective of flexibility, the MAXCAP model yields the maximum traffic that can be carried in each and every traffic lane. Thus, the increase in each and every lane, over any base traffic, is the same percentage increase from that base level. This gives a conservative estimate of the maximum volume of traffic that can be carried on any given lane. Hence, the *base pattern capacity* is a conservative estimator of the flexibility of the system.

The MAXCAP model (in the form for freight systems) is presented in equations (1) through (12). The objective function maximizes the quantity of cargo transported from origins to destinations (equation 1). The constraints reflect the traffic pattern (2), the mix of cargo by lane (O-D pair, commodity, etc.) and the limitations of link (arc) capacity (or time versus volume relationships) (3 to 5), terminal capacity (6 and 7), the available fleet (8 and 9), conservation of flow (10 and 11) and the routing options available in the system (determined by the set of available paths for each O-D pair,  $P_r$ ). Also included (as optional constraints) are limitations on the O-D time or other level of service limitations. Limitations on consumable resources can also be included, such as fuel available, labor available, or even pollution emissions, though none of these are used in the work reported here. (For the interested reader, more details of the model are presented in Morlok and Riddle (2000).)

The MAXCAP model equations are given below.

Maximize total system cargo traffic:

$$\max z_M = \sum_{r=1}^R x_r \quad (1)$$

Subject to:

Traffic pattern:

$$x_r = \mathbf{a}_r \cdot \left[ \sum_{r=1}^R x_r \right], \forall r \quad (2)$$

Arc vehicle and cargo flows (arcs being track segments):

$$\sum_{p \in P_r} \mathbf{f}_p \cdot f_p = x_r, \forall r \quad (3)$$

$$\sum_{p \in B_a} (f_p + e_p) \leq \mathbf{g} \cdot w_a, \forall a \quad (4)$$

$$w_a \leq \mathbf{b} \cdot v_a, \forall a \quad (5)$$

Arc and terminal capacity (or volume-delay relationship):

$$v_a \leq K_a, \forall a \quad (6)$$

$$\sum_{p \in D_n} [f_p + e_p] \leq L_n \quad (7)$$

Fleet size:

$$\sum_{a=1}^A [T_a \cdot w_a] \leq H_{car} \quad (8)$$

$$\sum_{p=1}^P [S_p \cdot (f_p + e_p)] \leq H_{con} \quad (9)$$

Conservation of flow:

$$\sum_{a \in E_n} w_a = \sum_{a \in V_n} w_a, \forall n \quad (10)$$

$$\sum_{p \in F_n} (f_p + e_p) = \sum_{p \in M_n} (f_p + e_p), \forall n \quad (11)$$

Non-negativity:

$$v_a, w_a, x_r, f_p, e_p \geq 0, \forall r, p, a \quad (12)$$

The variables are (with typical units in brackets):

$v_a$	train volume on each arc $a$
$w_a$	car volume on each arc $a$
$x_r$	loaded container volume on traffic lane $r$ [containers/month],
$f_p$	loaded container volume on path $p$ [containers/month]
$e_p$	empty container volume on path $p$ [containers/month])
$z_M$	system capacity [containers/month]

The parameters are (also with typical units in brackets):

$a$	rail line arc
$A$	set of all arcs
$B_a$	set of paths sharing arc $a$
$D_n$	set of paths sharing terminal $n$
$E_n, V_n$	set of arcs entering, leaving terminal $n$
$F_n, M_n$	set of paths entering, leaving terminal $n$
$H_{car}, H_{con}$	car, container fleet hours available [car-hour/year], [container-hour/year]
$K_a$	capacity of arc $a$ [trains/month]
$L_n$	lift capacity at terminal $n$ [containers/month]
$p$	path of arc(s) and node(s) over which train(s) travel (from O to D)

$P$	set of all paths
$P_r$	set of paths that service the same OD pair
$r$	lane , referring to origin and destination node (OD) pair, service lane, etc.
$R$	set of all origin and destination pairs (OD pairs)
$S_p$	container travel time on path $p$ [hours]
$T_a$	car travel time on arc $a$ [hours]
$\mathbf{a}_p$	traffic pattern factors for each path $p$
$\mathbf{b}$	train capacity [cars/train]
$\mathbf{g}$	container capacity of car [containers/car]
$\mathbf{f}_p$	average container-load factor on each path $p$ [tons/container]

It should be noted that MAXCAP (like the ADDVOL model to be presented later) provides the estimated maximum system capacity as the optimal solution to its objective function. This value is measured in units such as tons originated, loaded containers (e.g., FEUs-- forty foot equivalent units) originated, etc., all per unit time (typically per week, month, etc.). However, underlying this single value of capacity are multiple possibilities (solutions) for components of that total flow. These include variations in empty container movements, for example.

Therefore, a second optimization of the flows is performed based on optimal capacity solutions to minimize cost of operations (flows) to achieve the optimal result, subject to the overall required capacity level of cargo flows (from the basic model). This is done by replacing (1) with the cost function to be minimized, setting the values of  $x_r$  to those from the MAXCAP solution, and solving the model. This then yields the desired maximal solution of system capacity under an optimal operation costs. All results presented below use this the bi-level optimization.

## 6. Application

These concepts are illustrated by an application to the double-stack rail freight transportation (DDS) network presented in Morlok and Riddle (2000), based on the FRA/MARAD study by Manalytics, Inc. (Smith 1990). However, for this study, the network was expanded to provide additional routing options. These additional options are used to illustrate the measures and approaches developed. The network is shown in Figure 2.

>Fig. 2 about here<

### 6.1. Base Pattern Estimates of Flexibility

The overall results for the MAXCAP model are presented in Table 1. The first row is the base capacity for the system when only the shortest path is used between each OD pair. This corresponds to the situation modeled in the original DSS network analysis, and indicates the maximum traffic (demand) the system can accommodate. With the shortest path routing rule, the estimated maximum capacity is  $1.48 \times 10^5$  containers/month (where containers are in FEUs). This

compares to the 1987 base traffic volume of  $1.15 \times 10^5$  in the original network. The base pattern reserve capacity is thus  $0.33 \times 10^5$  containers/month, or 28.6% of the base traffic volume. It indicates that each and every lane (OD pair) could increase its traffic by 28.6%. Thus the network is sufficiently flexible to accommodate up to 28.6% more traffic on any combination of traffic lanes (OD pairs, and commodities if these represent different commodities). This 28.6% represents a significant degree of flexibility.

>Table 1 about here<

## 6.2. Routing Options

If options for routing traffic over the network are provided, the system would be even more flexible. The routing options considered are

1. the shortest + one node disjoint paths routing option (*SP+1*) and
2. the level-of-service paths routing option (*LOS*).

The *shortest + one node disjoint paths routing option* is the shortest path plus an alternative second shortest path that does not share any of the arcs or nodes (except for the origin and destination nodes) traveled by the loaded container on the shortest path. The *level-of-service paths routing option* is defined as the set of all paths between an O and a D that meet a given level of service (e.g., travel time). These paths can have common elements. In this application, the level-of-service paths routing option consist of all paths that have a travel time less than or equal to the travel time of the alternate second shortest path (node disjoint path). These two routing options introduce flexibility. It should be noted that the solutions under both the SP+1 and the LOS paths routing options are still subject to the total flow on each link not exceeding its capacity. The same applies to terminals.

Referring back to Table 1, the shortest + one node disjoint paths routing option yields a base pattern reserve capacity of 178.8%, in contrast to the 28.6% of the shortest path routing option. Clearly there is a very large increase in flexibility by permitting just one additional (independent) path between each OD pair. The effect of this added flexibility for each of the traffic lanes (OD pairs) is presented in Table 2. As can be seen, the base pattern reserve capacity varies greatly among the OD pairs, ranging from 2468 to 45440 FEUs/month.

>Table 2 about here<

The level-of-service paths routing option makes even more capacity available, and hence increases demand flexibility. Returning to Table 1, the LOS option is seen to yield a base traffic pattern reserve capacity of 192.1%. But this is not that much larger than the overall increase from the SP+1 paths routing option. Providing routing options does increase flexibility, sometimes substantially. But there are diminishing gains from increasing routing options, as other limitations (terminals, fleet, etc.) come into play.

This result reveals an important connection to the concepts of *limited flexibility* and *total flexibility* in manufacturing systems. Specifically, total flexibility in manufacturing refers to a situation in which all of many different products (e.g., auto models) can be made in all of the plants owned. This permits the maximum range of adjustment to variations in demand for each model. Jordan and Graves (1995) found that almost as much flexibility was provided by having each plant able to produce only two different models—a situation called limited flexibility. Having only one more plant able to produce a product is much easier than trying to achieve

complete flexibility, i.e., allowing all products to be produced in all plants. This is analogous to the situation we have found. Allowing only one additional entirely distinct path provides almost as much flexibility as allowing use of all paths that will meet a given level of service requirement.

Returning to reserve capacity (or the MAXCAP model) as a measure, it is a conservative estimator of capacity and hence of flexibility. It does not include the additional traffic volume that could be accommodated by allowing deviations from the base traffic pattern. Another estimator is needed to capture this possibility.

## 7. Adjusted traffic pattern approach to measuring flexibility (ADDVOL)

The ADDVOL model is a second estimator of the maximum demand that can be accommodated by the system. The ADDVOL model estimates how much more traffic volume could added to the MAXCAP estimate for the system, when deviations from the base traffic pattern are permitted. This means that the fraction of total traffic that moves between each OD pair is now a variable. Thus, there are many more opportunities to add traffic to the system than the MAXCAP approach permitted. This includes converting empty container moves to loaded moves and adding traffic to arcs that are not constrained. ADDVOL is more flexible in achieving a maximum cargo flow of traffic on every traffic lane.

The ADDVOL model equations are similar to those of the MAXCAP model, but differ primarily in the objective function. It is described by adding equations (13) and (14) below to equations (3) through (12) from the MAXCAP model.

The ADDVOL model seeks to add as much cargo traffic as possible over and above that of the base capacity (obtained from the MAXCAP model):

$$\max z_A = \sum_{r=1}^R (X_r + \bar{x}_r) \quad (13)$$

Subject to:

Arc vehicle and cargo flows:

$$\sum_{p \in P_r} f_p \cdot f_p = X_r + \bar{x}_r, \forall r \quad (14)$$

where two new variables and one new parameter are added:

$\bar{x}_r$	additional cargo volume on OD pair $r$ (tons/month)
$z_A$	estimated maximum system capacity for ADDVOL model
$X_r$	base traffic pattern system capacity for each OD pair $r$ (MAXCAP results treated as a constant in this model)

Thus, it too, maximizes overall cargo volume, but no longer subject to the base traffic pattern. Following the format used earlier, the resulting capacity is called the *adjusted traffic pattern capacity*. The reserve capacity is called the *adjusted traffic pattern reserve capacity*. As

will be shown below, the adjusted pattern capacity is a function of the magnitude of the deviation from the base traffic pattern.

The ADDVOL model was also applied to the DSS network described in the Application Section. Similar to the MAXCAP Model, a second optimization was used to optimize empty vehicle flows.

### 7.1. Adjusted Pattern Capacity Estimates of Flexibility

The results for the ADDVOL model, using the shortest + one node disjoint paths routing option, is presented in Table 3. As can be seen from columns C through E, traffic can be added to most lanes, compared to the base pattern capacity traffic ( $x_r$  from MAXCAP), as intended. But a few lanes are already constrained by resources, reflecting the many basic constraints of the model. For example, in Lane 4, 35.7% more traffic could be added (as shown in column E), but in Lane 3 no more traffic could be added. The possible increases range up to almost 400% in Lane 12, though this lane has the smallest base traffic volume. The increase over the base traffic varies considerably (column F) also, and the range of the increase varies from 178.8 to 1283.8% (Column G).

>Table 3 about here<

In all of these analyses, different path routing options were applied, and the results were similar. The addition of more path routing options increases the total volume that can be accommodated, and hence increases flexibility.

There is one problem with the ADDVOL model. It generally (virtually always) will give a degenerate solution, i.e., there are many possible combinations of traffic in individual lanes that will yield the same total volume for the system. To counter this problem, we developed various methods to achieve a unique solution. These all employ minimizing the deviation between the additional traffic volume for each lane and the volume that would result were the flows to adhere to the original base unit OD matrix. This is a reasonable objective. Traffic patterns reflect the spatial pattern of production and consumption, and would not change arbitrarily. Some limitation of possible patterns is reasonable and appropriate.

### 7.2. PENALTY ADDVOL Model

The issue of uniqueness of the available capacity (ADDVOL traffic volumes) is solved by applying a penalty to the model when it deviates from the underlying traffic pattern. As the deviation increases, the penalty increases. In this manner, the traffic pattern can be constrained to be as close to the base or underlying one as desired. There are many possible measures of deviation. Numerous ones were tried, generally based on standard deviation concepts, but also including other measures such as absolute value of deviation.

The measure of deviation that emerged as preferred is the following:

$$FSD = \sum_{r=1}^R \left[ \frac{\left[ \left( X_r + \bar{x}_r \right) - \left( \frac{X_r}{Z_m} \cdot \sum_{r=1}^R [ X_r + \bar{x}_r ] \right) \right]}{Z_m} \right]^2 \quad (15)$$

We chose this penalty function because it captures the deviation of the resulting or new traffic pattern from the base traffic pattern but normalized by the base traffic pattern capacity (MAXCAP solution). The numerator of the penalty function is the difference between the new traffic on each OD pair and the traffic that would be on that pair if there were compliance with the base traffic pattern. The denominator is the estimated maximum system capacity from the MAXCAP Model. Thus this is the fractional deviation of the new pattern from the base traffic pattern. It is squared to avoid the problem of positive and negative deviations canceling one another. The sum of the squared fractional deviations gives the Fractional Squared Deviation (*FSD*).

The penalty function chosen has other advantages. The deviation measure is fairly easy to understand, and it has definite upper and lower limits. The lower limit is zero, when the new traffic conforms to the base traffic pattern. The upper limit is the square of the additional traffic as a fraction of the MAXCAP solution.

This deviation measure is added to the ADDVOL objective function and weighted by beta to make it a penalty function (16). The constraints in the original ADDVOL model remain the same.

$$\max z_{PO} = z_P - \mathbf{b} \cdot \sum_{r=1}^R \left[ \frac{\left[ (X_r + \bar{x}_r) - \left( \frac{X_r}{Z_m} \cdot \sum_{r=1}^R [X_r + \bar{x}_r] \right) \right]}{Z_m} \right]^2 \quad (16)$$

where

$$z_P = \sum_{r=1}^R (X_r + \bar{x}_r)$$

$z_{PO}$  estimated maximum system capacity for the modified ADDVOL model with fractional squared difference penalty function added

$\mathbf{b}$  traffic pattern deviation penalty factor

Figure 3 presents the result of using this on the example system (with the shortest path + one node disjoint paths option, as used earlier). It plots system capacity (in units of one-hundred thousand containers/month) versus the traffic pattern deviation measured by FSD. As the deviation increases, the capacity of the system increases. At zero deviation, the capacity is that given by base pattern capacity  $z_M$ . But as deviations from the base traffic pattern appear, the capacity increases. The increase is steady but at a declining rate, as might be expected. Once the deviation is sufficiently large, the capacity is the maximum value of the adjusted pattern capacity,  $z_A$ . Above this no further increase in capacity is possible, regardless of the magnitude of the deviation; that is, even complete freedom as to traffic pattern will not enable the system to accommodate more traffic.

>Fig. 3 about here<

The curve of Figure 3 is generated by varying beta. When beta is small, the estimated capacity and the deviation (FSD) are large. When beta is large, the estimated capacity and the deviation is small. For large beta, the penalty is so large that, in certain instances, it is better not to add traffic between certain OD pairs and incur the “cost” of increasing the penalty (in the objective function). So, as beta is increased to successively larger values, the optimization algorithm ultimately chooses to not ship additional cargo and hence the traffic pattern conforms to the base pattern. Similarly, smaller values of beta have almost no effect on the estimated system capacity, as the penalty for deviations from the base pattern are small compared to the gain in objective function from more traffic carried. The values for beta are determined through experimentation, in this case from  $10^{-6}$  to  $10^{-3}$ .

## 8. Discussion of measures and uses in planning, operations and policy-making

The fixed traffic pattern and adjusted traffic pattern approaches (MAXCAP and ADDVOL models, respectively) provide very different measures of capacity or demand flexibility. Both models estimate the capacity of the transportation system and may be used to measure system flexibility, when flexible routing options are introduced. The fixed pattern approach is a conservative estimator that gives a lower bound estimate of the system flexibility, while the adjusted pattern approach gives an upper bound on system flexibility by permitting deviations from the base traffic pattern.

These approaches and models were applied to a specific portion of the freight transportation system in order to obtain illustrative quantitative results. The results are quite plausible in that variations in the traffic patterns and in routing options yield results that are constant with *a priori* expectations. Also, the magnitudes seem reasonable, particularly in the case of the actual routing options (primarily a single path for each OD, presumably a minimum path) used in the study from which the data were drawn. Also important is the fact that the methods presented here require only data on components of the system that were readily available. Thus these measures should be feasible given data that are typically available or could be developed in the course of specific application studies.

While quantitative results were obtained, caution is necessary in inferring that the degree of flexibility found is present in the current system. Mergers and other changes since 1987, the year of most data in that study, have altered the infrastructure. New train control technologies have increased capacity. Also, current non-container traffic is considerably greater. The intent was to illustrate the approach and models, not estimate precisely the flexibility of this specific system.

Measures of flexibility should be useful in a variety of ways. One is to measure the extent to which the system can function satisfactorily in the face of major disasters, such as earthquakes or a terrorist attack. Another is to determine the extent to which it has the ability to accommodate a substantial increase in traffic, as might occur in the event of war. A third use is in connection with investments in the network. While traditionally investments are evaluated with respect to a specific increase in traffic volume and pattern, changes in the world economy are causing major shifts in traffic patterns. Thus being able to assess the flexibility of the system to accommodate a range of traffic is increasingly important, for both private and public

investments. Developing ways of measuring flexibility that will prove useful in such a planning or policy evaluation context has been the motivation of this work.

## 9. Conclusions and possible future research

This paper has defined, characterized and formulated concepts of flexibility for a transportation system. Two approaches were developed to measure capacity (or demand) flexibility. The fixed traffic pattern approach (MAXCAP) is a conservative estimator that estimates the maximum demand that the system could accommodate subject to a fixed OD traffic pattern. The adjusted traffic pattern approach (ADDVOL) allows for deviations from the base traffic pattern, starting from the MAXCAP solution, and thus provides greater latitude for traffic or demand changes, as from shifts in the location of production or consumption patterns. The measures were applied to a freight transportation system and results show that these measures show promise as means to quantify flexibility. The research also illustrated that providing path options significantly increases system capacity flexibility.

There are many avenues for future research. There are also a number of technical issues related to the measures and the models that should be addressed, such as the multiple optimal solutions (degeneracy) issue discussed in the ADDVOL Model sections, among others. Other directions are more conceptual. One is to consider other external influences on the system, not just demand. An important exogenous factor is change in the relative cost of resources, or in their availability. Another example is the effect of losses of elements of the system, such as links lost due to natural or manmade disasters (e.g., vulnerability to terrorism). Another avenue of research would be to consider a broader range of performance measures, not simply the ability of the system to carry traffic at a satisfactory level of service, was done here. Other measures would include rate of return, profitability, and system cost—a direction we are currently following. Another direction for future work is to quantify the more likely uncontrolled changes that may occur, including variations in demand and traffic patterns. Thus many issues remain.

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Table 1  
Base traffic pattern estimates of flexibility with three routing options (using MAXCAP model)

Routing options	Capacity, (FEUs/month)	% Increase over base traffic <sup>a</sup>	% Increase over SP capacity
Shortest Path (SP)	147887	28.6%	--
Shortest + One Node Disjoint Paths (SP+1)	320610	178.8%	116.8%
Level-of-service Paths (LOS)	335865	192.1%	127.1%

<sup>a</sup>Base traffic pattern reserve capacity, measured in % over the base traffic of 115001 containers (FEUs)/month.

Table 2

Base traffic pattern estimate of flexibility by lane with the shortest path + one node disjoint path set routing option (MAXCAP model results in FEUs/month.)

Traffic lane	Base traffic	Base traffic pattern capacity <sup>a</sup>	Base traffic pattern reserve capacity
1	15410	42962	27552
2	14031	39115	25084
3	21734	60595	38861
4	8050	22443	14393
5	25415	70855	45440
6	4025	11221	7196
7	4715	13145	8430
8	4025	11221	7196
9	6210	17313	11103
10	2761	7695	4934
11	7244	20198	12954
12	1379	3847	2468
Total	115001	320611	205610

<sup>a</sup>This is the maximum traffic that can be accommodated on each lane while adhering to the base traffic pattern.

Table 3  
 Maximum traffic increases by lane allowing deviations from the base traffic pattern and with the shortest path + one node disjoint path set routing option (ADDVOL model results)

(A)	(B)	(C)	(D)	(E)	(F)	(G)
Traffic lane	Base traffic (FEUs/month)	Base pattern capacity ( $x_r$ ) (FEUs/month)	Adjusted pattern capacity - ( $X_r + x_r$ ) (FEUs/month)	Adjusted pattern capacity as % increase over base capacity (cols. (D-C)/C)	Adjusted pattern reserve capacity, (cols. D-B) (FEUs/month)	Adjusted pattern capacity, as % increase over base traffic (cols. (D-B)/B)
1	15410	42962	45006	4.8%	29596	192.1%
2	14031	39115	45006	15.1%	30976	220.8%
3	21734	60595	60595	0.0%	38861	178.8%
4	8050	22443	30458	35.7%	22408	278.4%
5	25415	70855	70855	0.0%	45440	178.8%
6	4025	11221	36350	223.9%	32325	803.1%
7	4715	13145	13145	0.0%	8430	178.8%
8	4025	11221	36350	223.9%	32325	803.1%
9	6210	17313	23405	35.2%	17195	276.9%
10	2761	7695	8831	14.8%	6071	219.9%
11	7244	20198	20198	0.0%	12954	178.8%
12	1379	3847	19091	396.2%	17711	1283.8%
Total	115001	320611	409291	27.7%	294291	255.9%

Note: Totals are subject to round-off error.

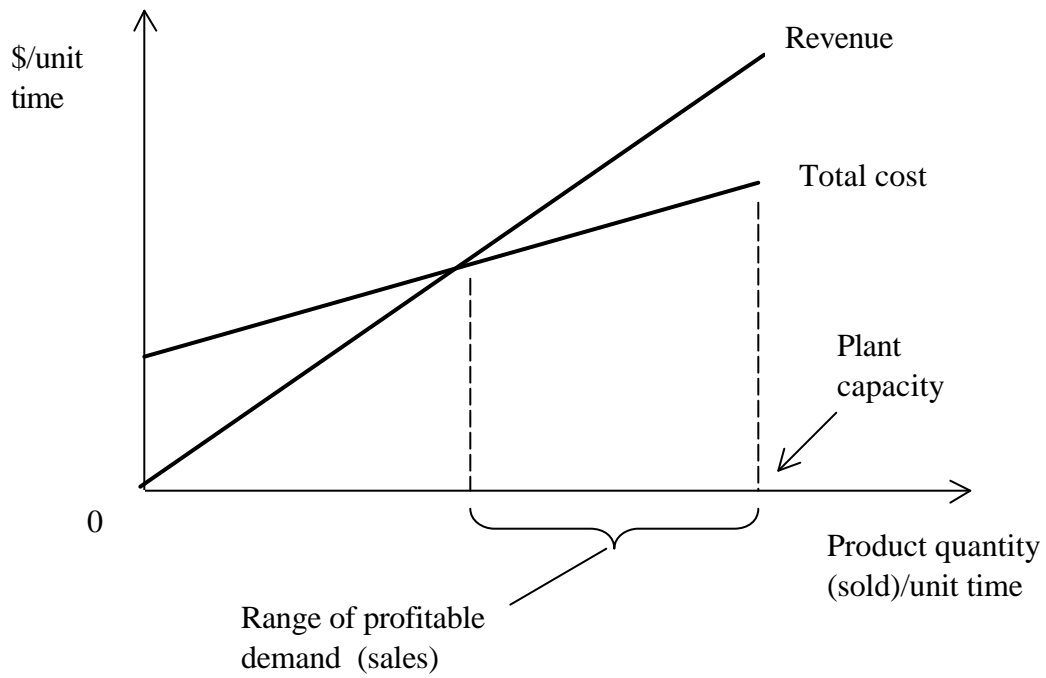


Fig. 1. Breakeven sensitivity analysis as a basis for flexibility assessment.

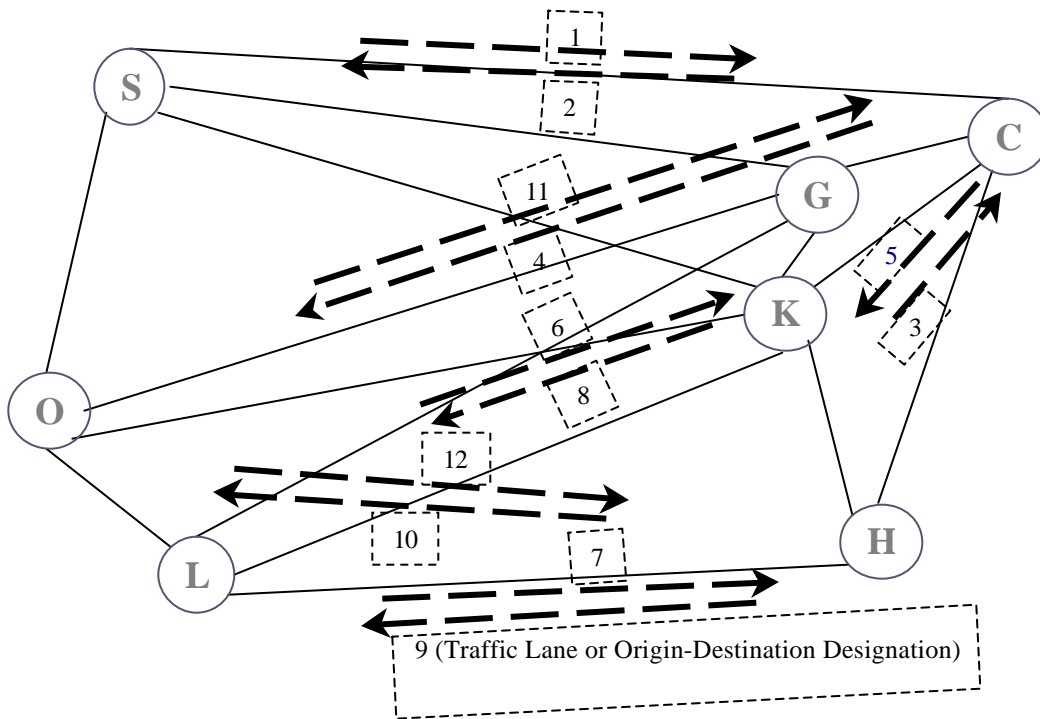


Fig. 2. The double-stack container network used in examples.

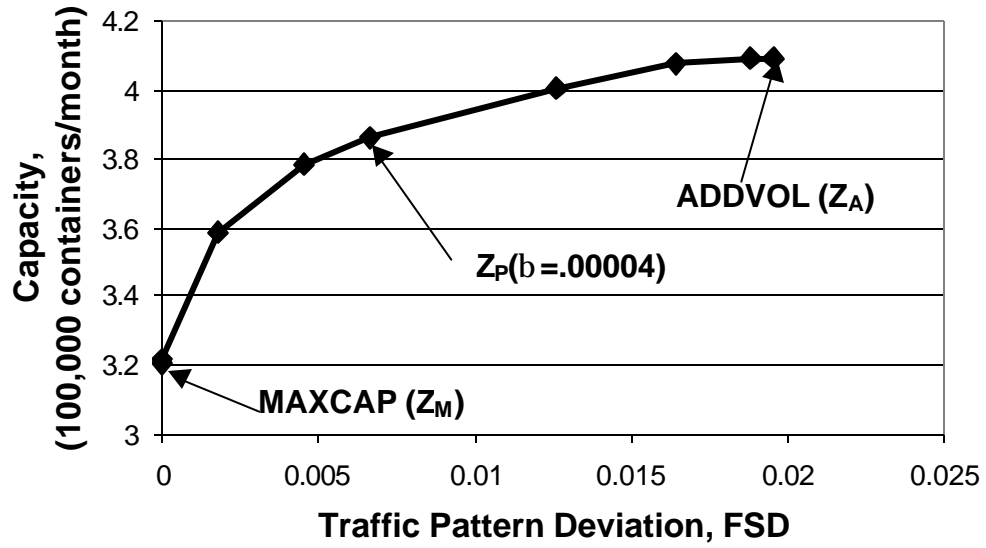


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## Table 3

Maximum traffic increases by lane allowing deviations from the base traffic pattern and with the shortest path + one node disjoint path set routing option (ADDVOL model results)

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