Structured Hedging for Resource Allocations with Leverage

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Problem Statement

Problem: How do we distribute a resource across a variety of assets in order to maximize return and minimize risk?

Novel Aspects: (1) Borrow additional resources to use as leverage to increase returns.
(2) Utilize different allocation positions through structured hedging to reduce risk.

Solution: Design an online learning algorithm which learns asset correlations and uses leverage to hedge its allocation.

Leverage and Structured Hedging

Leverage is a way to increase returns such as borrow additional resources to increase allocation power. **Hedging** is a method to offset risk by taking opposing positions.

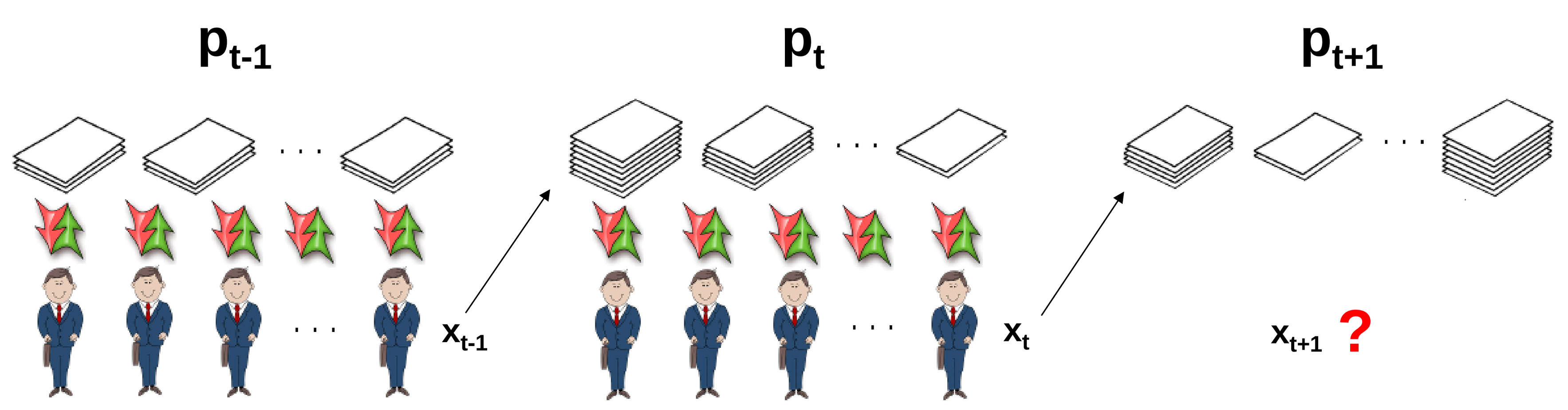
For **structured hedging**, we use a correlation graph over the n assets with 2 nodes per asset, one for each position.

Let *W* be a correlation matrix and $q_\ell(i)$ and $q_s(j)$ be long/short positions for assets *i* and *j*, then a structured hedging penalty function is

$$\Omega_h = \sum_{i=1}^{n} \sum_{\substack{j=1+n \\ j\neq i+n}}^{2n} W_{ij}\left(q_\ell(i) + q_s(j)\right)^2 = \mathbf{p}^\top \mathbf{L}\mathbf{p}$$

Online Portfolio Selection

Consider a stock market with *n* stocks over a span of *T* days. Each day *t* we must choose a portfolio $\mathbf{p}_t$ over the stocks to maximize the log gain in wealth $LS_T(\mathbf{p}_{1:T}, \mathbf{x}_{1:T})$ where $\mathbf{x}_t$ is the vector of price relatives (closing price / opening price).



Long and Short Portfolios

Long position: Investing in an asset using cash on hand. One profits if the value of the asset increases.

Short position: Investing borrowed cash/shares in an asset. One profits if the value of the asset decreases.

Portfolio: $\mathbf{p} \in \mathcal{P} \subset \mathbb{R}^{2n}$ and $\ell = \{1,\ldots,n\}$, $s = \{n+1,\ldots,2n\}$

$$D_\ell(i,i) = 1 \text{ for } i \in \ell \text{ and } D_s(i,i) = 1 \text{ for } i \in s$$

Long-only: $\mathbf{q}_\ell = \mathbf{D}_\ell \mathbf{p} \geq 0$ Short-only: $\mathbf{q}_s = \mathbf{D}_s \mathbf{p} \leq 0$

Long and short portfolio multiplicative gain in wealth:

$$\underbrace{\mathbf{q}_\ell^\top \mathbf{x}_t}_{\text{market change in wealth}} + \underbrace{(1-\mathbf{q}_\ell^\top \mathbf{1})(1+r)}_{\text{cash borrowed or not invested}} + \underbrace{\mathbf{q}_s^\top(\mathbf{x}_t - 1)}_{\text{market change in wealth}} + \underbrace{\mathbf{q}_s^\top \mathbf{1} r}_{\text{interest owed on borrowed shares}}$$

$$= \mathbf{q}_\ell^\top \mathbf{x}_t + \mathbf{q}_s^\top(\mathbf{x}_t - 1 + r) + (1 - \mathbf{q}_\ell^\top \mathbf{1})(1+r)$$

SHERAL Algorithm

Structured hedging for resource allocations with leverage problem:

$$\min_{\substack{\mathbf{q}_\ell \geq 0 \\ \mathbf{q}_s \leq 0 \\ \mathbf{a}^\top \mathbf{p} \leq \frac{1+r}{B_\ell + r}}} \eta\langle \mathbf{p}, \nabla \ell_t(\mathbf{p}_t)\rangle + \lambda \mathbf{p}^\top \mathbf{L}\mathbf{p} + \frac{1}{2}\|\mathbf{p} - \mathbf{p}_t\|_2^2$$

Instantiated for the online portfolio selection problem using loss function:

$$\ell_t(\mathbf{p}_t) = -\log\left(\alpha_1 \mathbf{q}_\ell^\top \mathbf{x}_t + \alpha_2 \mathbf{q}_s^\top(\mathbf{x}_t - 1 + r) + (1 - \mathbf{q}_\ell^\top \mathbf{1})(1+r)\right)$$

Online projected gradient descent based solution:

$$\mathbf{p}_{t+1} = \prod_{\mathcal{P}}\left(\eta \nabla \ell_t(\mathbf{p}_t) + \mathbf{p}_t\right)\left(\lambda(\mathbf{L} + \mathbf{L}^\top) + \mathbf{I}\right)^{-1}$$

where

$$\nabla \ell_t(\mathbf{p}_t) = \frac{\alpha_1 \mathbf{D}_\ell^\top \mathbf{x}_t + \alpha_2 \mathbf{D}_s^\top(\mathbf{x}_t - 1 + r) - \mathbf{D}_\ell^\top \mathbf{1}(1+r)}{\alpha_1 \mathbf{p}_t^\top \mathbf{D}_\ell^\top \mathbf{x}_t + \alpha_2 \mathbf{p}_t^\top \mathbf{D}_s^\top(\mathbf{x}_t - 1 + r) + (1 - \mathbf{p}_t^\top \mathbf{D}_\ell^\top \mathbf{1})(1+r)}$$

Datasets

Description:

- DJIA: 30 stocks with 507 trading days between 2001-2003.
- NYSE: 36 stocks with 5651 trading days between 1962-1984.
- SP500[a]: 25 stocks with 1276 trading days between 1998-2003.
- SP500[b]: 263 stocks with 505 trading days between 2007-2009.
- TSX: 88 stocks with 1259 trading days between, 1994-1998.

Percentage of stocks that lost value over the time period:

- DJIA: 83%.
- NYSE: 0%, all stocks increased in value.
- SP500[a]: 28%.
- SP500[b]: 96%
- TSX: 36%.

Experiment 1: EG with Leverage vs SHERAL

	DJIA	NYSE	$SP500^a$	$SP500^b$	TSX
EG	0.81	26.70	1.64	0.68	1.59
EG* (LO)	1.55	6.9×10^{14}	**20.90**	2.21	$\mathbf{1.0\times10^{3}}$
EG* (SO)	0.63	0.04	0.34	1.10	1.07
EG* (LS)	2.00	6.6×10^{14}	20.65	2.26	1.62
SHERAL ($\lambda>0$)	**2.47**	$\mathbf{1.8\times10^{15}}$	19.89	**7.84**	8.74

Table 2: Cumulative wealth for EG, leveraged long-only (LO), short-only (SO), and long/short (LS) variants of EG*, and SHERAL with $\lambda > 0$.

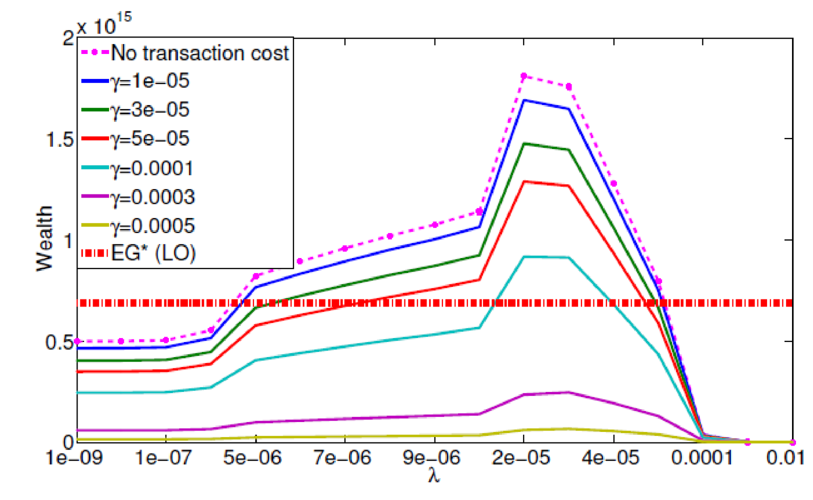


Figure 1: Transaction cost-adjusted wealth for the NYSE dataset with varying values of λ. SHERAL returns more wealth than the best competing algorithm even with transaction costs.

Experiment 2: SHERAL vs Baselines

	DJIA	NYSE	$SP500^a$	$SP500^b$	TSX
U-BAH	0.76	14.49	1.34	0.63	1.61
U-CRP	0.81	26.78	1.64	0.69	1.59
UP	0.80	26.99	1.62	0.69	1.59
OLU	0.84	50.80	2.45	3.02	2.24
Best Stock (LO)	1.18	54.14	3.77	1.74	6.27
U-BAH* (LO)	0.55	38.44	0.44	0.38	1.71
U-BAH* (SO)	0.43	3.68×10^{-6}	0.01	1.21	0.54
U-BAH* (LS)	0.97	0.43	0.78	1.03	1.12
U-CRP* (LO)	0.61	695.59	1.04	0.43	1.68
U-CRP* (SO)	0.28	1.04×10^{-6}	1.00×10^{-2}	0.97	0.54
U-CRP* (LS)	0.83	0.05	0.45	0.93	0.92
Best Stock* (LO)	1.09 (P&G)	65.71 (PM)	0.56 (WMT)	1.15 (SWN)	7.84 (GTA)
Best Stock* (SO)	1.13 (MCD)	1.45×10^{-5} (DD)	0.02 (KO)	4.71 (GCI)	2.11 (IFP)
SHERAL ($\lambda>0$)	**2.47**	$\mathbf{1.81\times10^{15}}$	**19.89**	**7.84**	**8.74**

Table 3: Cumulative wealth (without transaction costs) of SHERAL, benchmark algorithms, and several variants for each of the five datasets.

Key References
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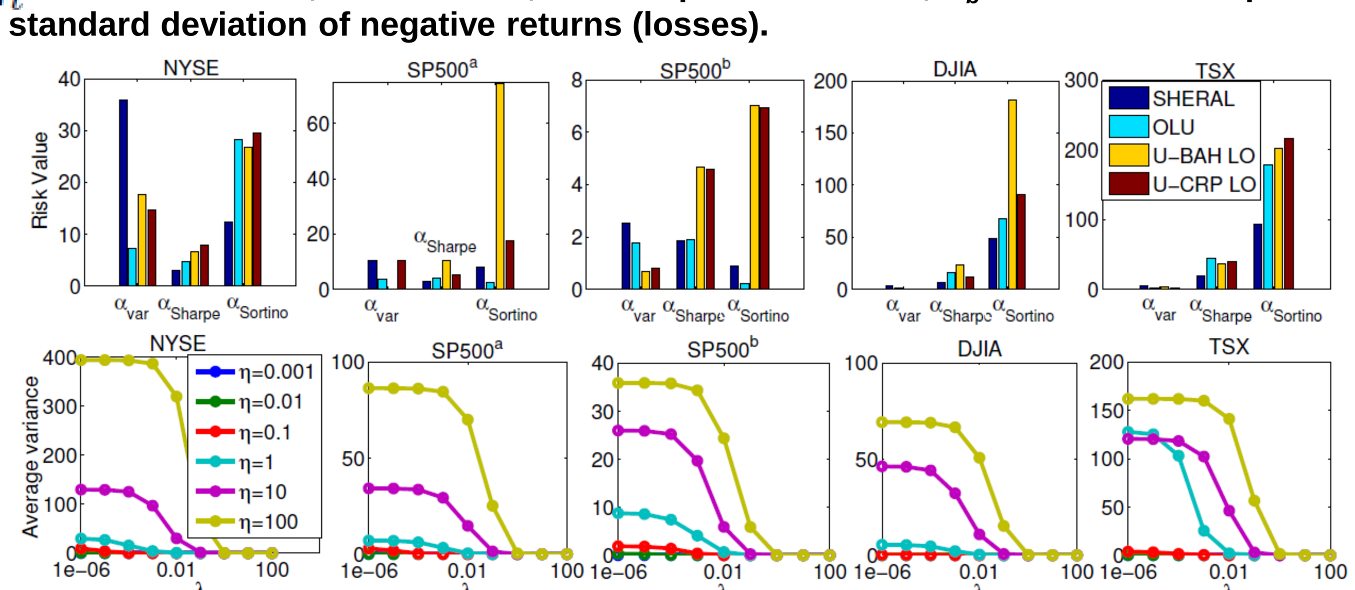
Experiment 3: Risk

Measure of risk

$$\alpha_{var} = \frac{\mathbf{p}_t^\top \Sigma_t \mathbf{p}_t}{\mathbf{u}^\top \Sigma_t \mathbf{u}} \qquad \alpha_{Sharpe} = \frac{(R - R_b)}{\sqrt{var(R - R_b)}} \qquad \alpha_{Sortino} = \frac{(R - R_b)}{DR}$$

where Σ_t is the covariance, u is a U-CRP, *R* is the portfolio return, R_b is a benchmark portfolio return, and *DR* is the standard deviation of negative returns (losses).



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