Loopy: Programmable and Formally Verified Loop Transformations

Kedar Namjoshi
Bell Labs, Nokia

Nimit Singhania
University of Pennsylvania

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Problem

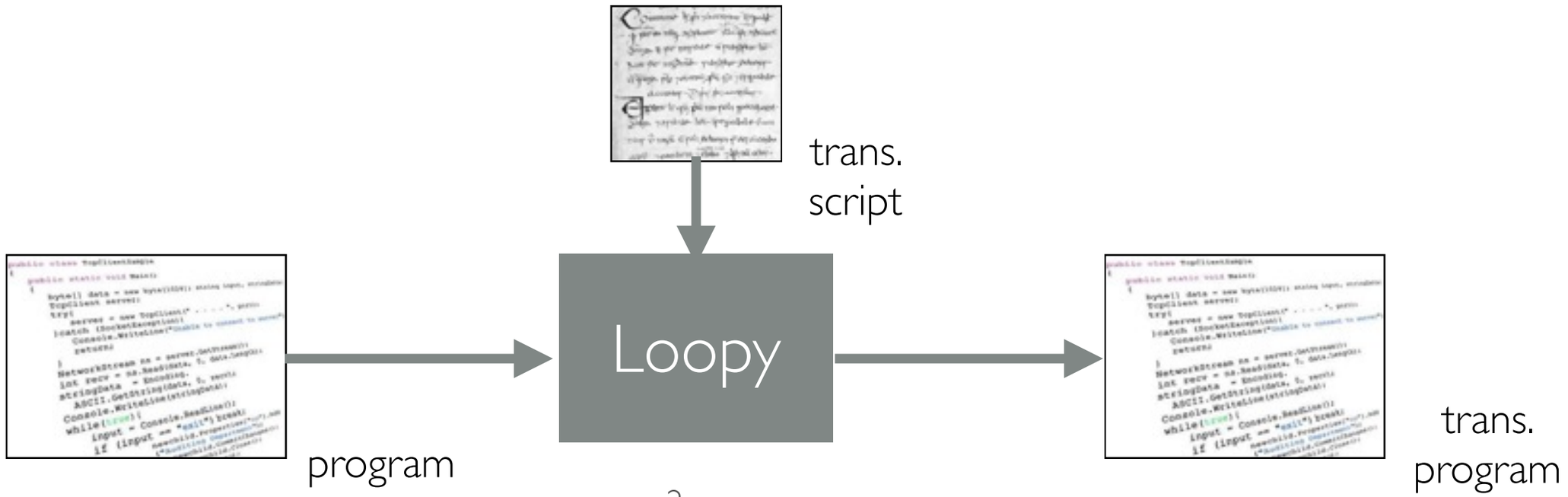
How to optimize loops in a program?

Optimizing Compilers?

Variable performance gains

Manual Rewriting?

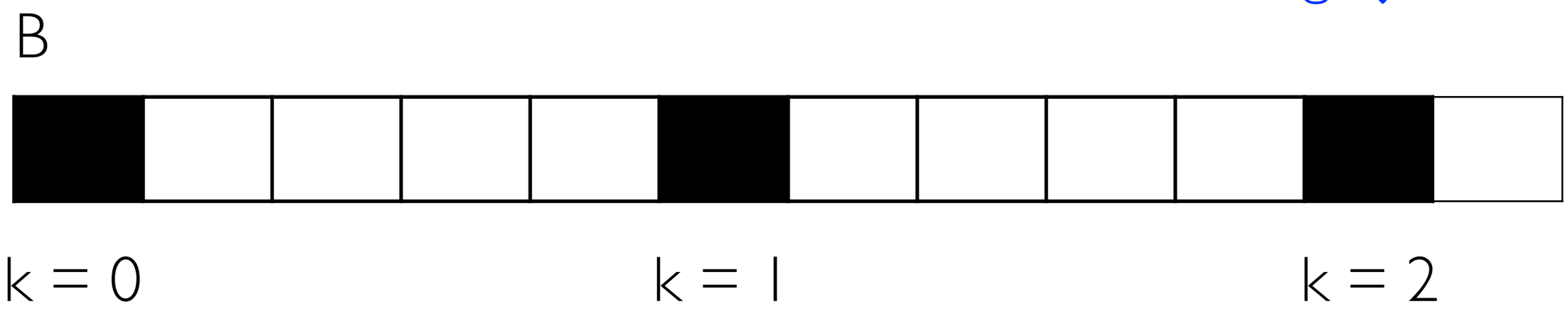
Tedious and Error Prone



Example

```
for (i = 0; i < N; i++) {
  for (j = 0; j < N; j++) {
    {
      Init: C[i][j] = 0;
    }
    for (k = 0; k < N; k++) {
      Mult:
        C[i][j] += A[i][k] * B[k][j];
    }
  }
}
```

Interchange **j** and **k**



Operation 1: Loop Splitting

```
for (i = 0; i < N; i++) {
  for (j = 0; j < N; j++) {
      Init: C[i][j] = 0;
  }
}
for (i = 0; i < N; i++) {
  for (j = 0; j < N; j++) {
    for (k = 0; k < N; k++) {
      Mult:
        C[i][j] += A[i][k] * B[k][j];
    }
  }
}
```

realign(Init, Mult, 0)

Operation 2: Loop Interchange

```
for (i = 0; i < N; i++) {
  for (j = 0; j < N; j++) {
      Init: C[i][j] = 0;
  }
}
for (i = 0; i < N; i++) {
  for (k = 0; k < N; k++) {
    for (j = 0; j < N; j++) {
      Mult:
        C[i][j] += A[i][k] * B[k][j];
    }
  }
}
```

5x speedup!

```
affine(Mult, {[i, j, k] -> [i, k, j]})
```

Operation 3: Loop Tiling

```
for (i = 0; i < N; i++) {
  for (j = 0; j < N; j++) {
    Init: C[i][j] = 0;
  }
}
for (i1 = 0; i1 < N/32; i1++) {
  for (k1 = 0; k1 < N/32; k1++) {
    for (j1 = 0; j1 < N/32; j1++) {
  for (i2 = 0; i2 < min(32, N - i1*32); i2++) {
    for (k2 = 0; k2 < min(32, N - k1*32); k2++) {
      for (j2 = 0; j2 < min(32, N - j1*32); j2++) {
        Mult:
          C[i1*32+i2][j1*32+j2] += A[i1*32+i2][k1*32+k2]
                                 * B[k1*32+k2][j1*32+j2];
    }}
  }}
}}
```

2x speedup!

```
affine(Mult, {[i, j, k] -> [i1, j1, k1, i2, j2, k2]:
                           i1 = [i/32] and i2 = i%32 and
                           j1 = [j/32] and j2 = j%32 and
                           k1 = [k/32] and k2 = k%32})
```

Transformation Script

```
realign(Init, Mult, 0)

★ affine(Mult, {[i, j, k] -> [i, k, j]})

affine(Mult, {[i, j, k] ->
  [i1, j1, k1, i2, j2, k2]:
    i1 = [i/32] and i2 = i%32 and
    j1 = [j/32] and j2 = j%32 and
    k1 = [k/32] and k2 = k%32})
```

★ Not implemented by Polly

Loopy

```
realign(Init, Mult, 0)

affine(Mult, {[i, j, k] -> [i, k, j]})

affine(Mult, {[i, j, k] ->
          [i1, j1, k1, i2, j2, k2]:
           i1 = [i/32] and i2 = i%32 and
           j1 = [j/32] and j2 = j%32 and
           k1 = [k/32] and k2 = k%32})
```

```
for (i = 0; i < N; i++) {
  for (j = 0; j < N; j++) {
    {
      Init: C[i][j] = 0;
    }
    for (k = 0; k < N; k++) {
      Mult:
        C[i][j] += A[i][k] * B[k][j];
    }
  }
}
```

Loopy

```
for (i = 0; i < N; i++) {
  for (j = 0; j < N; j++) {
    Init: C[i][j] = 0;
  }
}

for (i1 = 0; i1 < N/32; i1++) {
  for (k1 = 0; k1 < N/32; k1++) {
    for (j1 = 0; j1 < N/32; j1++) {
    for (i2 = 0; i2 < min(32, N - i1*32); i2++) {
      for (k2 = 0; k2 < min(32, N - k1*32); k2++) {
        for (j2 = 0; j2 < min(32, N - j1*32); j2++) {
          Mult:
            C[i1*32+i2][j1*32+j2] += A[i1*32+i2][k1*32+k2]
                                   * B[k1*32+k2][j1*32+j2];
      }}
  }}
}}
```

Verify
Equivalence

Other Scripting Approaches

- CHiLL, POET, Orio, X Language, …
  - No correctness guarantees
- URUK
  - Transformations specified on internal program representation
  - Difficult for a programmer to describe
- Loopy
  - **Source-level** transformations with **correctness** guarantees

Loop Components

```
for (i = 0; i < N; i++) {
  for (j = 0; j < N; j++) {
    {
      L1: C[i][j] = 0;
    }
    for (k = 0; k < N; k++) {
      L2:
        C[i][j] += A[i][k] * B[k][j];
    }
  }
}
```

Program

```
op1(L1, L2, 0)
L3 = op2(L1, f1)
(L4, L5) = op4(L3, p)
op3(L4, f2)
op3(L5, k)
```

Script

Supported Operations

realign(L1, L2, n): Splits or merges adjacent loops to have **n** common loops

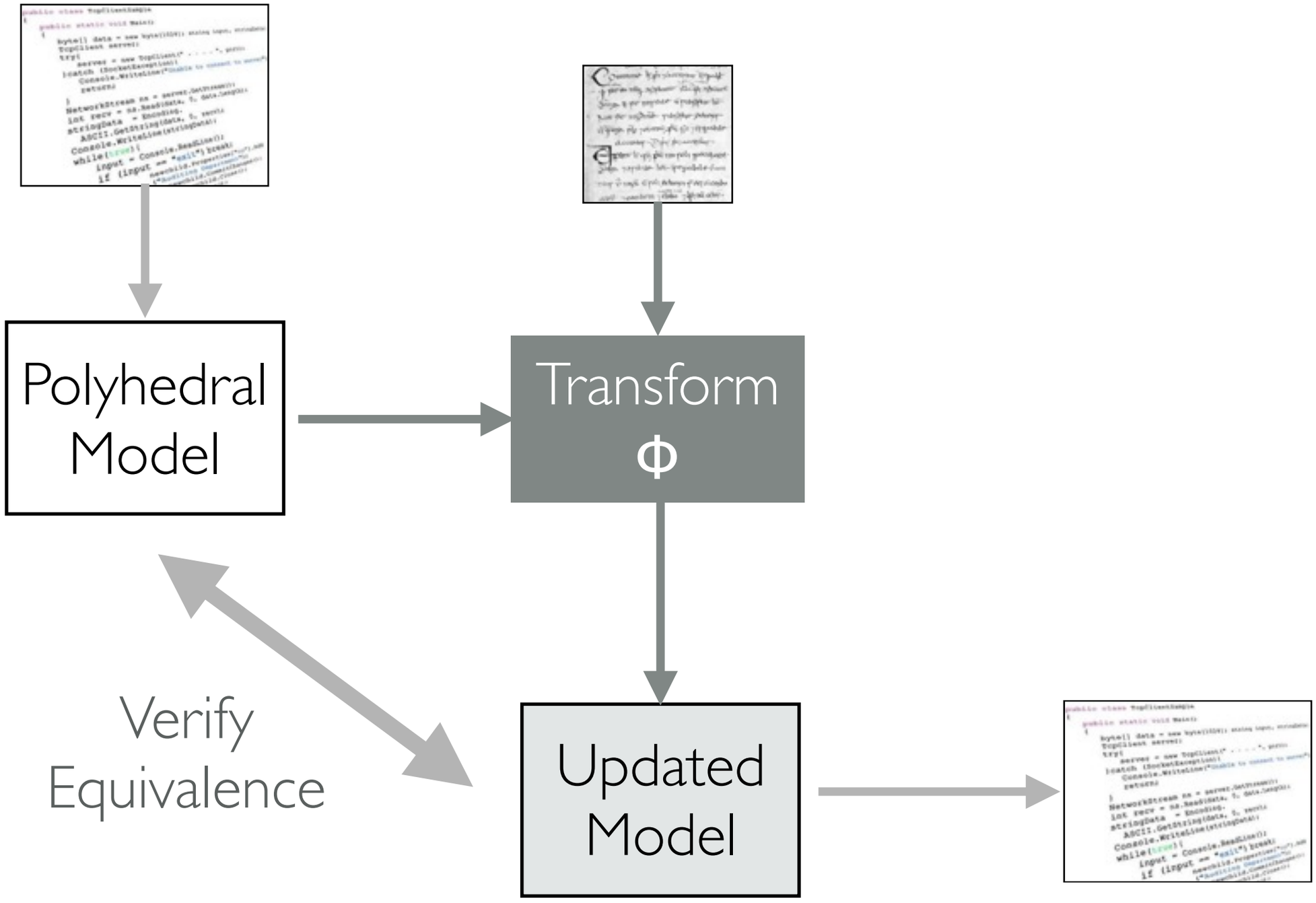
affine(L, f): Applies affine transformation **f** on L

(L1, L2) = **isplit**(L, p, n): Splits L into L1 and L2 with index sets satisfying **p** and **~p**

L1 = **lift**(L, n): Returns handle L1 to **n**th loop of L

Piecewise Affine Reordering

Internal Workflow

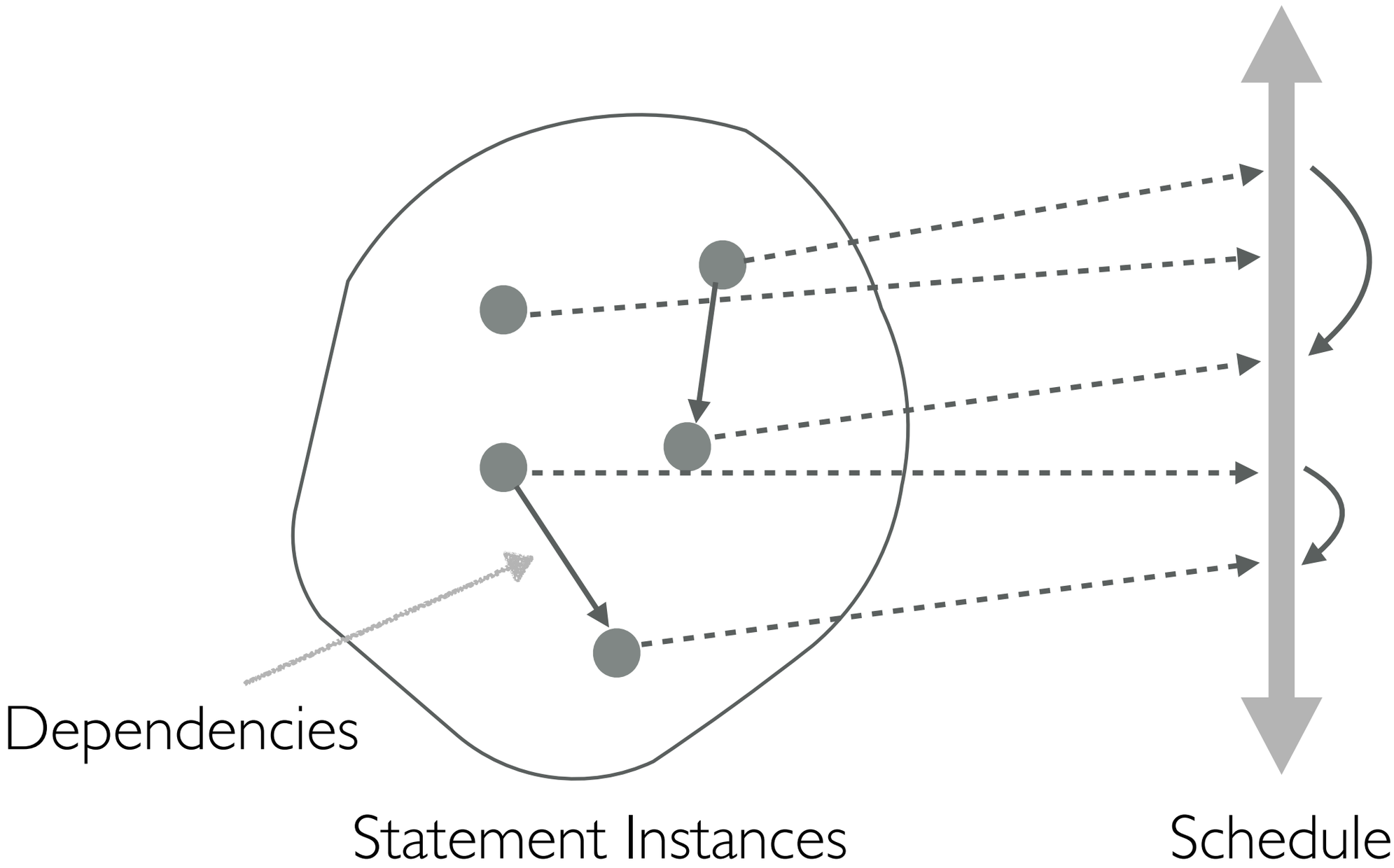


Polyhedral Model

```
for (i = 0; i < N; i++) {
  for (j = 0; j < N; j++) {
    {
      Init: C[i][j] = 0;          [0, 0, 0, 0]
    }
    for (k = 0; k < N; k++) {
      Mult:
        C[i][j] += A[i][k] * B[k][j];
                                  [0, 0, 1, 0]
    }
  }
}
```

- Statement Instances:
  {Init[i, j], Mult[i, j, k]}
- Iteration Domain:
  $0 \leq i < N \wedge 0 \leq j < N \wedge 0 \leq k < N$
- Dependencies:
  $Init[i, j] \longrightarrow Mult[i, j, k]$
  $Mult[i, j, k] \longrightarrow Mult[i, j, k+1]$
- Schedule:
  Init[i, j] : (**0**, i, **0**, j, **0**, 0, **0**)
  Mult[i, j, k] : (**0**, i, **0**, j, **1**, k, **0**)

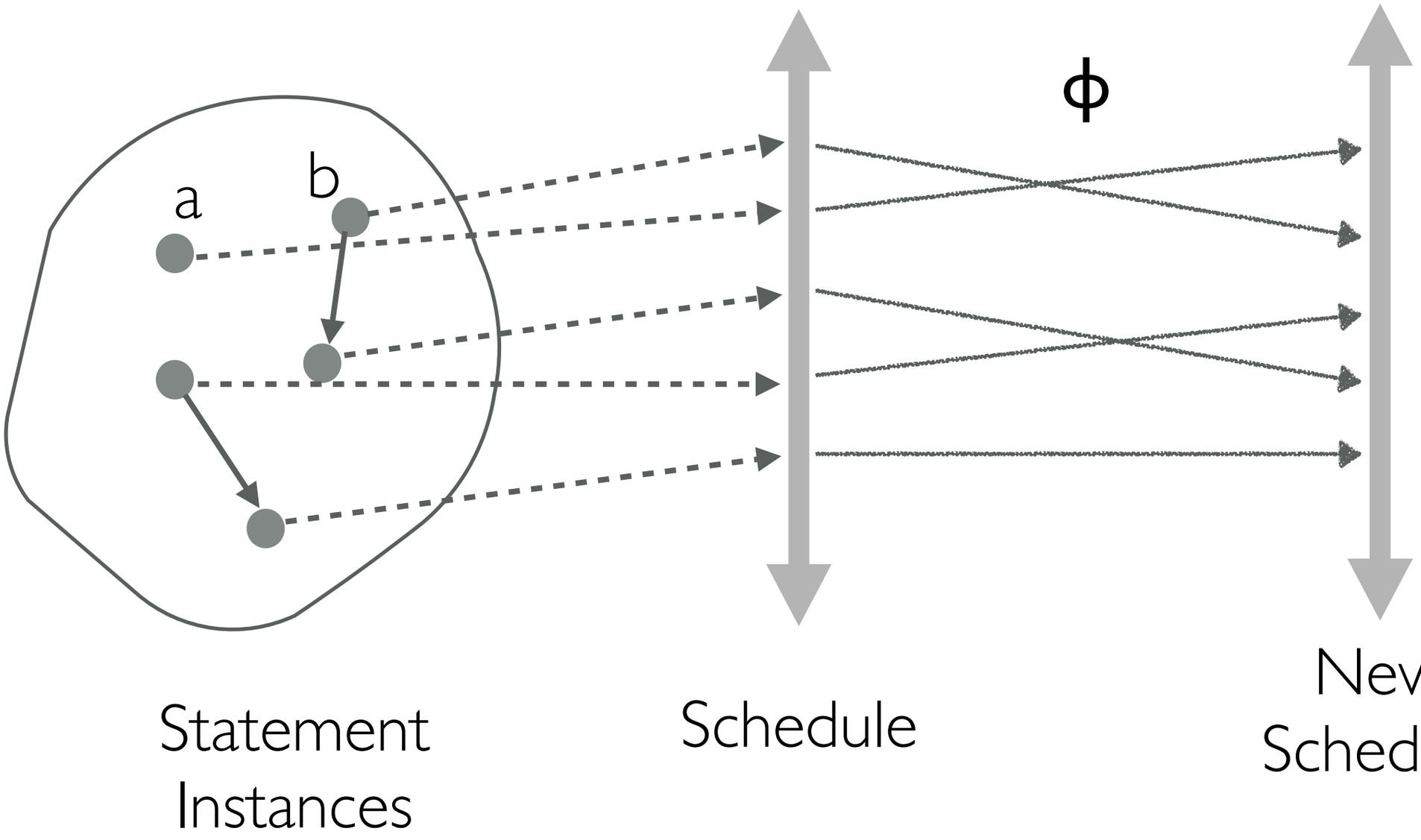
Polyhedral Model



Transform Function

- Transformation of the schedule
  - Re-ordering execution of statements
- Two types of transformations
  - Positional changes - e.g. loop splitting
  - Iterator changes - e.g. loop reversal
- $\phi = \phi_m \circ \phi_{m-1} \circ \ldots \circ \phi_1$

Transform Function



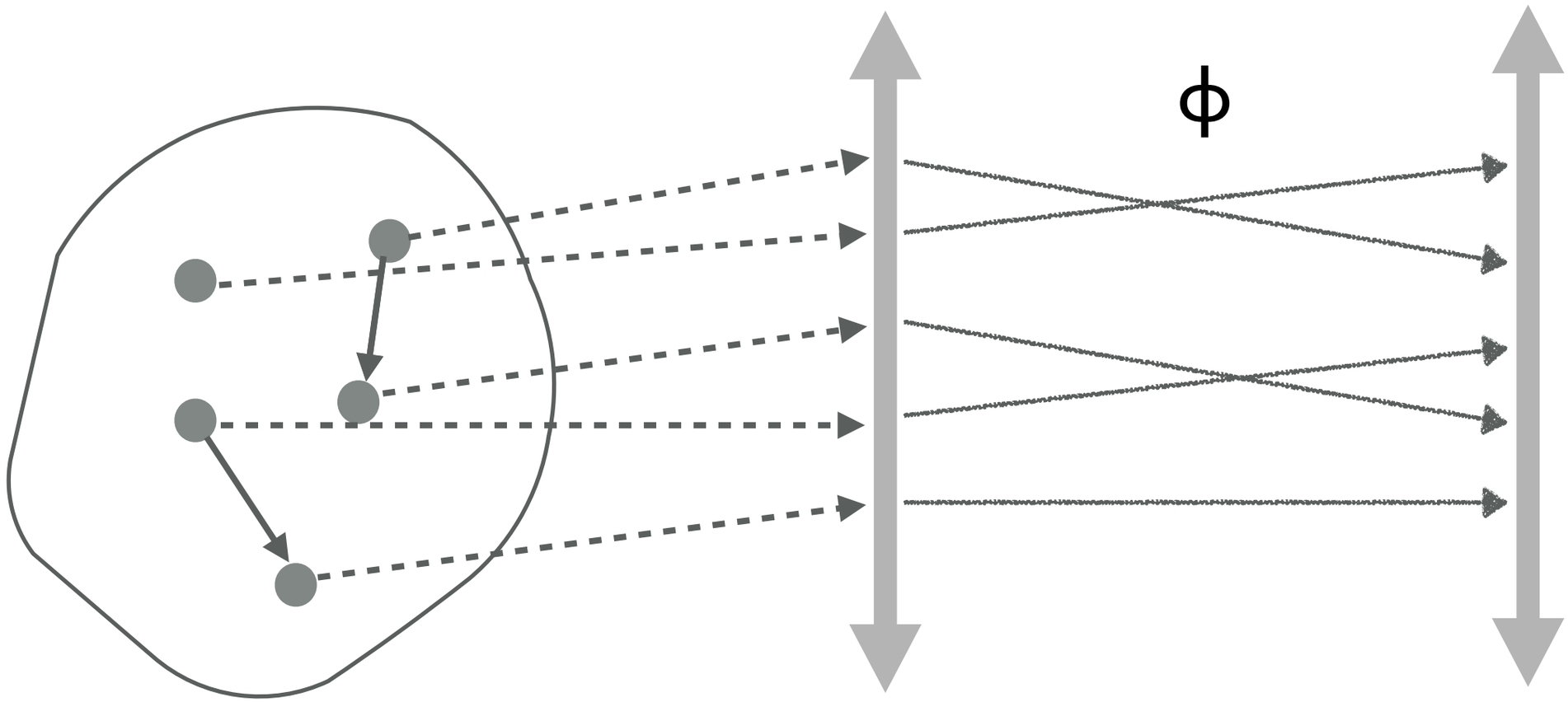
Verification

- Composed transformation ϕ must be one-one
- For each dependency ($s \longrightarrow t$), execution order must be preserved:

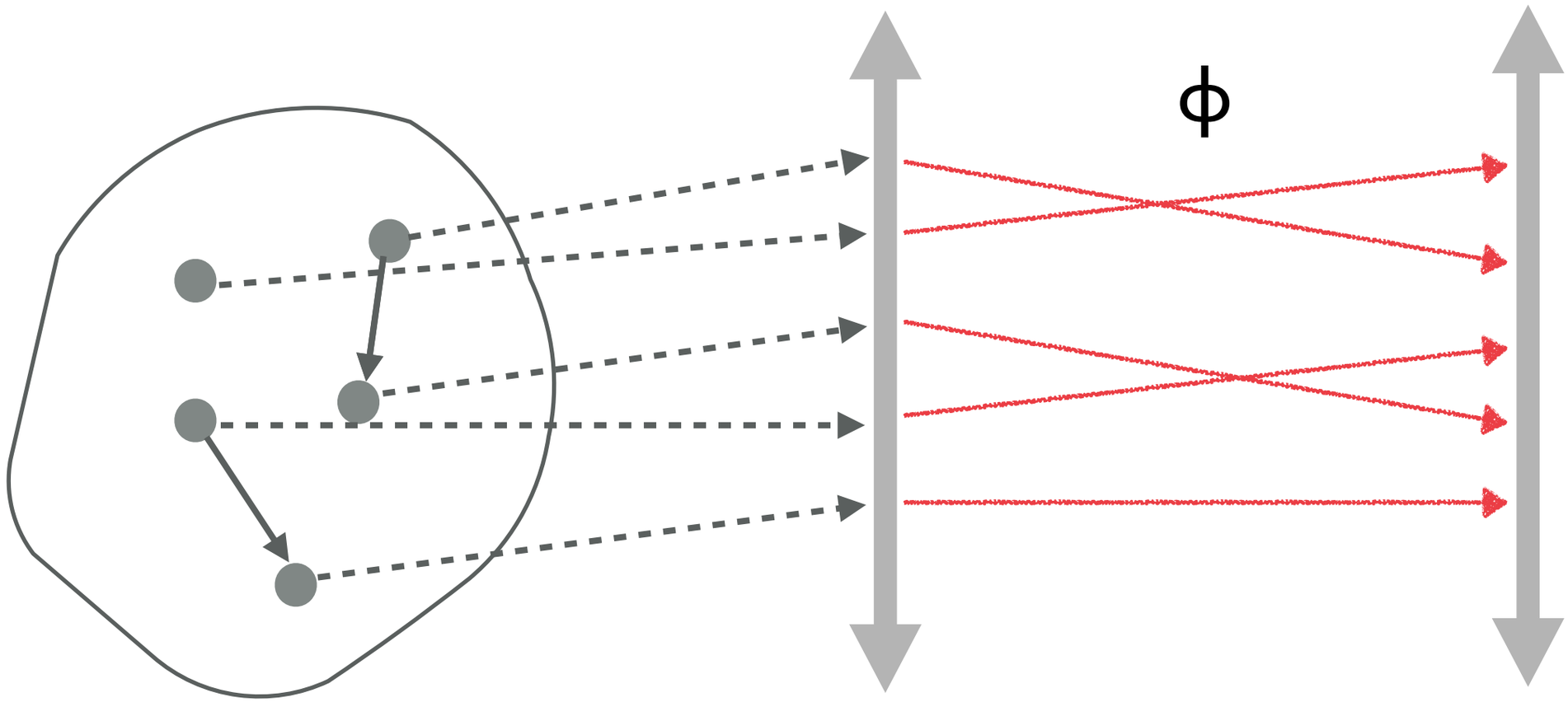
$$\phi(\text{schedule}(s)) < \phi(\text{schedule}(t))$$

- Checked using an Integer Constraint Solver
  - Counterexample generated if incorrect
- Verification for the composed transformation ϕ only
  - Dependencies might be broken in between

Verification

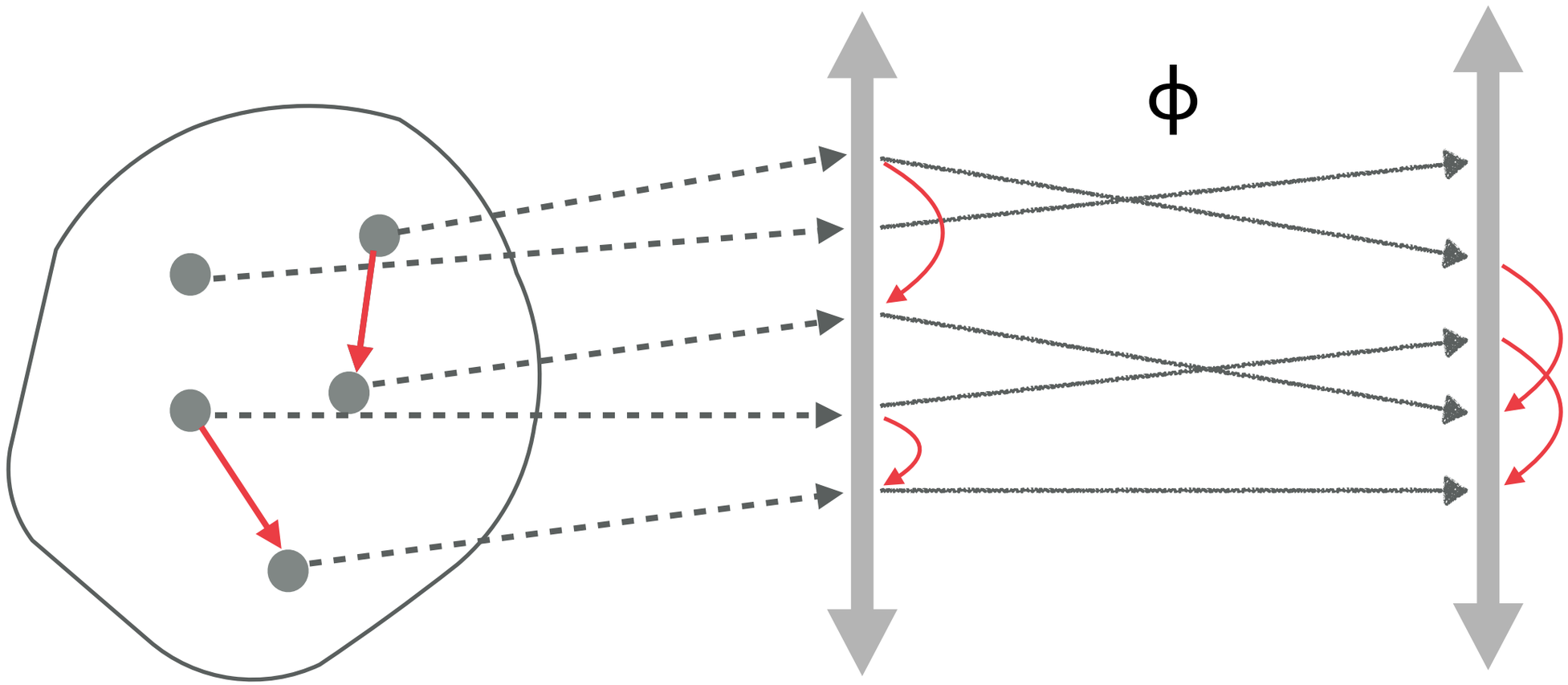


Verification



One-to-one

Verification

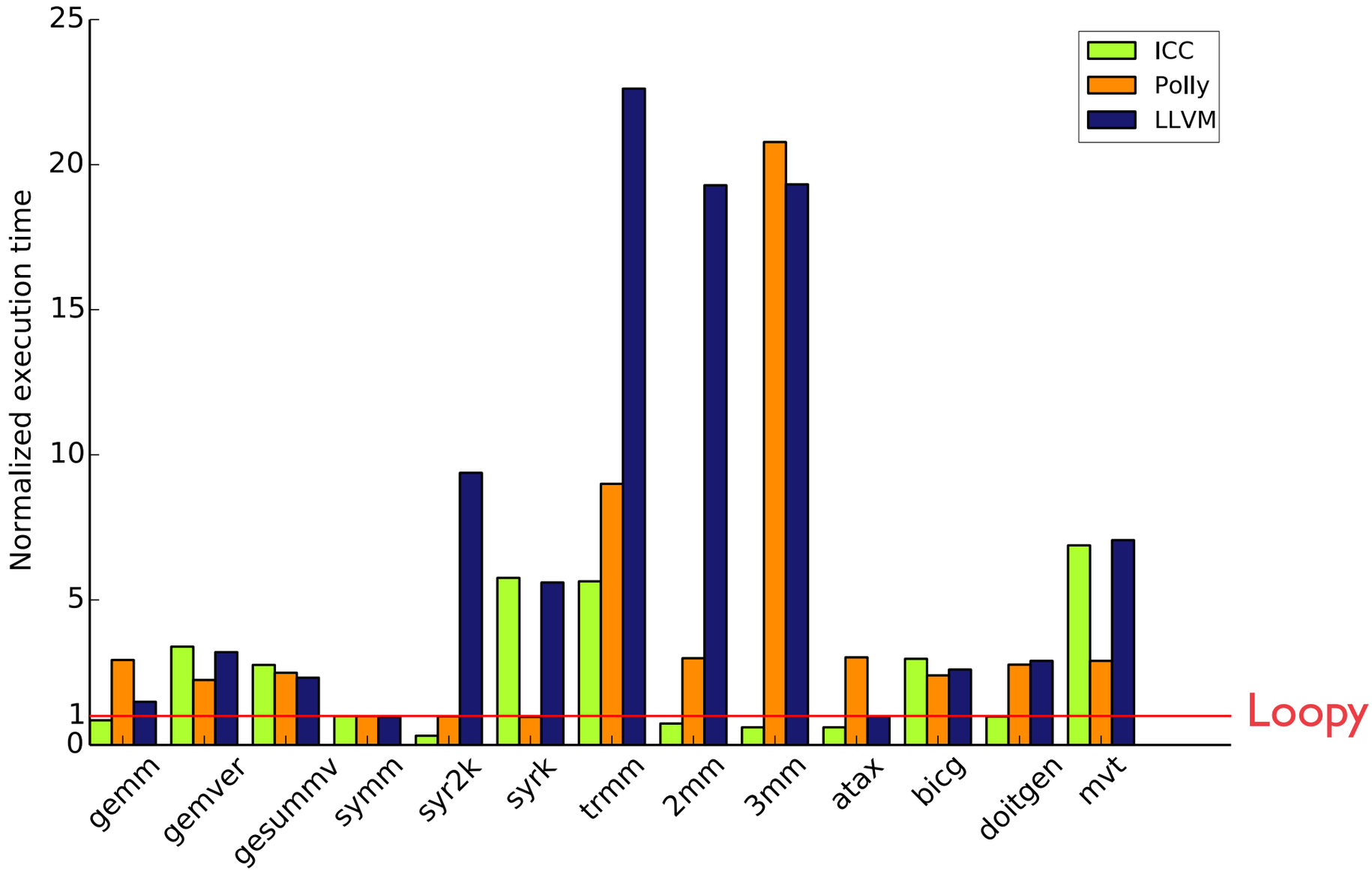


Dependencies must be preserved

Evaluation

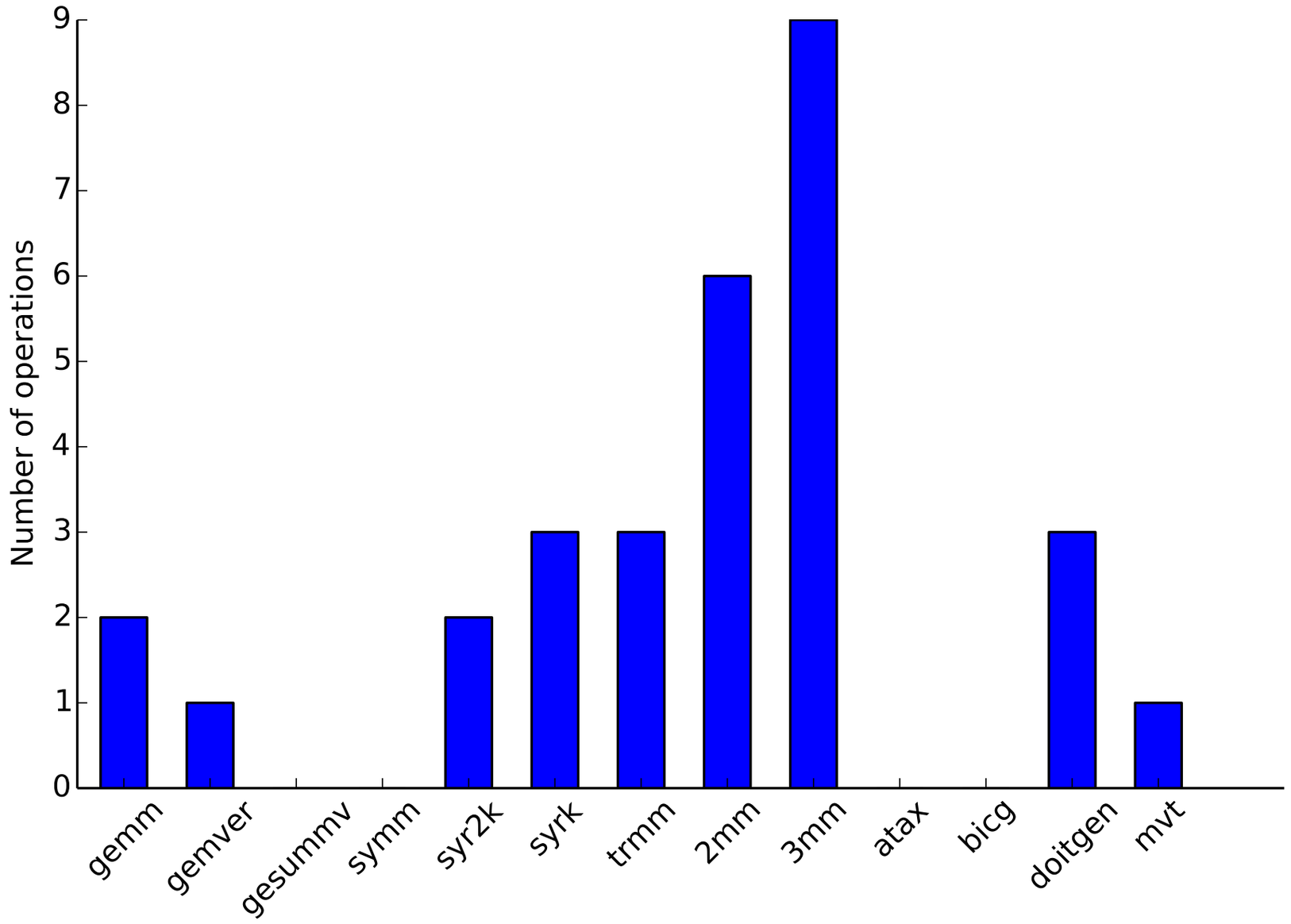
- Implemented in Polly, a polyhedral library for LLVM
- Evaluated on Polybench benchmark suite
  - 30 programs from diverse domains
- Comparison against ICC, PLUTO(Polly) and LLVM on *single-threaded* performance

Benchmark Performance



Linear Algebra Kernels

Script size



Linear Algebra Kernels

Conclusion

- Fun with Loop Transformations!
  - Scripting language lets one *play* with transformations
  - Correctness checking *guards* against mistakes
- Future directions
  - New transformations for Multicore and GPUs
  - Synthesizing scripts

Thank you!

(https://github.com/nimit-singhania/loopy.git)

Acknowledgements

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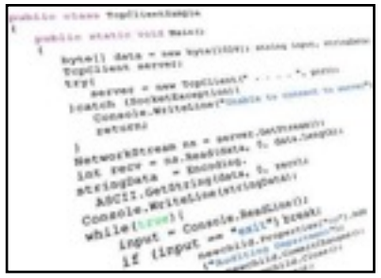
Polyhedral Model Verification



Polyhedral Model

Loop Invariant

Ranking Function

Iteration Domain

Schedule

realign(Init, Mult, 0)

```
[0]   for (i = 0; i < N; i++) {
        for (j = 0; j < N; j++) {
          Init: C[i][j] = 0;
        }
      }
[1]   for (i = 0; i < N; i++) {
        for (j = 0; j < N; j++) {
          for (k = 0; k < N; k++) {
            Mult:
              C[i][j] += A[i][k] * B[k][j];
          }
        }
      }
```

[0, 0, 0, 0]

[1, 0, 0, 0]

$\phi_{realign}([0, 0, 0, 0], \mathbf{j}) = ([0, 0, 0, 0], \mathbf{j})$

$\phi_{realign}([0, 0, 1, 0], \mathbf{j}) = ([1, 0, 0, 0], \mathbf{j})$

realign(Init, Mult, 0)

```
[0]    for (i = 0; i < N; i++) {
         for (j = 0; j < N; j++) {
           Init: C[i][j] = 0;
         }
       }
[1]    for (i = 0; i < N; i++) {
         for (j = 0; j < N; j++) {
           for (k = 0; k < N; k++) {
             Mult:
               C[i][j] += A[i][k] * B[k][j];
           }
         }
       }
```

[0, 0, 0]

[1, 0, 0, 0]

$$\phi_{realign}(p, j) = \begin{cases} (p + p_{init} - p_{mult} + 1(0), j) & \text{if } p \geq p_{mult} \\ (p, j) & \text{otherwise} \end{cases}$$