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Adaptive Communication-Constrained Deployment of Mobile Robotic Networks

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***Abstract*— Cooperation between multiple autonomous vehicles requires inter-vehicle communication, which in many scenarios must be established over an ad-hoc wireless network. This paper proposes an optimization-based approach to the deployment of such mobile robotic networks. A primal-dual gradient descent algorithm jointly optimizes the steady-state positions of the robots based on the specification of a high-level task in the form of a potential field, and routes packets through the network to support the communication rates desired for the application. The motion planning and communication objectives are tightly coupled since the link capacities depend heavily on the relative distances between vehicles. The algorithm decomposes naturally into two components, one for position optimization and one for communication optimization, coupled via a set of Lagrange multipliers. Crucially and in contrast to previous work, our method can rely on on-line evaluation of the channel capacities during deployment instead of a prespecified model. A randomized sampling scheme along the trajectories allows the robots to implement the algorithm with minimal coordination overhead.**

I. INTRODUCTION

Unmanned Vehicle Systems (UVS) have become critical assets for intelligence, surveillance and reconnaissance missions and could be used in the near future for environmental monitoring, search and rescue missions, intelligent distribution and transportation, or the exploration of dangerous indoor environments. UVS consist of a number of autonomous vehicles or mobile robots that can communicate with each other to enable cooperative behaviors, and with base stations for complex data analysis and higher level control purposes. However, in many situations, e.g., for disaster relief operations, a wireless communication infrastructure is initially absent and the robots need to form an ad-hoc network [1].

Much work in mobile robotics has focused on trajectory planning and deployment under communication constraints, but generally by assuming very simplified connectivity constraints and communication models. In particular, the disc model enables the use of graph theoretical methods to account for connectivity constraints [2], [3]. However, such models are inadequate to optimize global performance metrics for the network, such as bandwidth specifications between distant terminals, or to exploit the spatial diversity in the inter-robot channels to adaptively route the traffic. Interferences are also not taken into account.

Recently, more realistic wireless channel models have been used in robotics [4]–[6]. Firouzabadi and Martins [7] consider a network optimization problem including node placement and optimal power allocation, and solve it using geometric programming. Zavlanos et al. [8], [9] consider a problem similar to the one presented in this paper, including robot motion planning and packet routing. Other related communication-constrained deployment problems are treated in e.g. [10], [11]. However, virtually all proposed approaches, including our previous work [12], rely on known, and often deterministic models of the channel gains. In this case, the resulting trajectory planning algorithms can operate "open-loop", without ever relying on wireless channel measurements. Unfortunately wireless channel modeling is notably difficult, especially for indoor environments [1], and the robustness of the proposed methods to modeling errors has been discussed only in a few cases [4], [6].

As described in Section II, we consider communication-constrained *deployment problems*, where the goal is to move a mobile robotic network from an initial configuration to a desired final configuration appropriate for a given task, without directly attempting to optimize its transient trajectory. We develop in Section III a primal-dual optimization algorithm that progressively drives the robots toward a satisfying spatial configuration for the task, while ensuring that the resulting network can support the desired communication flows between vehicles and with the base stations. Section IV details how the algorithm can plan the trajectories using channel measurements rather than prespecified channel models. The measurement-based algorithm relies in particular on a random sampling direction approach proposed in the stochastic optimization literature [13], [14], which requires little coordination between the robots. Finally, simulation results for an illustrative scenario are discussed in Section V. Formal convergence results for the algorithms can be found in the full version of the paper [15].

II. JOINT DEPLOYMENT AND COMMUNICATION OPTIMIZATION

A. Task Potential

A mobile robotic network consists of n robots evolving in a workspace $\mathsf{W} \subset \mathbb{R}^d$, with positions denoted $\mathbf{x} = [\mathbf{x}_1^T, \ldots, \mathbf{x}_n^T]^T \in \mathsf{W}^n$. We use boldface letters for vector quantities, and denote $[n] := \{1, \ldots, n\}$. For simplicity, we assume throughout the paper that W is compact and convex. Some of the robot positions may be fixed, so that these "robots" can also represent a fixed communication infrastructure composed of base stations, with which the mobile elements must maintain communication. We consider high-level motion planning problems where we neglect the

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dynamics of the robots, whose positions evolve in discrete time as

$$\mathbf{x}_i[k+1] = \mathcal{P}_{\mathsf{W}}(\mathbf{x}_i[k] + \mathbf{u}_i[k]), \;\; \mathbf{x}_i[0] \in \mathsf{W}, \;\; \forall i \in [n]. \quad (1)$$

Here $\mathcal{P}_{\mathsf{W}}$ denotes the projection on W used to keep the iterates within the workspace, and $\mathbf{u}_i[k]$ is the control input for robot $i \in [n]$ at period k, satisfying the velocity constraint $\|\mathbf{u}_i[k]\| \leq v_i$.

The quality of the deployment of the mobile robotic network is captured by a potential field $G(\mathbf{x})$ [16]–[18], whose minimum corresponds to a desired steady-state configuration $\mathbf{x}^*$ for the group. We call this function the *task potential*. The deployment problem for this task potential consists then in designing feedback control laws allowing the system to reach $\mathbf{x}^*$ starting from an initial configuration $\mathbf{x}[0]$. For example, a simple quadratic potential

$$G(\mathbf{x}) = \|\mathbf{x}_1 - \mathbf{q}^*\|^2 \quad (2)$$

can be used to bring robot 1 to a known target location $\mathbf{q}^*$. Diverging barrier potentials can be used for inter-robot and obstacle avoidance in non-convex workspaces [16], [17], and other potential functions can force groups of robots to maintain certain formations [18].

For a deployment problem with task potential G, typical feedback controllers take a truncated gradient form [16]

$$\mathbf{u}(\mathbf{x}[k]) = \mathrm{sat}_{\mathbf{v}}(-\alpha_k \nabla G(\mathbf{x}[k])) \quad (3)$$

where α_k are prespecified stepsizes, $\mathbf{v} = [v_1, \ldots, v_n]$, and for $i \in [n]$

$$(\mathrm{sat}_{\mathbf{v}}(\mathbf{u}))_i = \begin{cases} \mathbf{u}_i, & \text{if } \|\mathbf{u}_i\| \leq v_i \\ v_i \frac{\mathbf{u}_i}{\|\mathbf{u}_i\|}, & \text{otherwise.} \end{cases}$$

The resulting gradient descent algorithm (1) only leads to the set of critical points of G in general. We follow this approach nonetheless, since global minimization of interesting task potentials for most multi-robot deployment problems is computationally intractable.

B. Wireless Networking

Because the deployed robotic network must also satisfy certain communication constraints, the controller (3) must be modified. Indeed, path loss and interferences make the achievable wireless communication rates highly dependent on inter-robot distances. Here we review some terminology and basic principles of wireless networking, in order to express these communication constraints quantitatively.

Each robot is equipped with a wireless terminal, and wishes to deliver communication packets to other robots and base stations for different application level flows, where a flow ϕ is associated to a given final destination $\mathrm{dest}(\phi)$. Note that several flows can have the same destination. The set of flows is denoted Φ. The amount of information for flow $\phi \in \Phi$ accepted at robot $i \neq \mathrm{dest}(\phi)$ at time k is denoted $a_i^\phi[k]$, with $a_i^\phi[k] \geq 0$. The amount of information for flow ϕ routed between robots i and j at time k is denoted $r_{ij}^\phi[k]$, with $r_{ij}^\phi[k] \geq 0$. Finally, the capacity of the link (i,j) at time k if the robotic network is in configuration $\mathbf{x}$ is denoted $c_{ij}[k;\mathbf{x}]$. This capacity is nonnegative and random, determined by the fading state and the chosen communication scheme, through coding and the allocation of transmission time-slots, frequencies and powers [1].

Let us illustrate in more details how the robot positions influence the channel capacities. Assume that a set $\mathcal{F}$ of frequency tones is available to communicate. For every pair $(i,j) \in [n]$, let $h_{ij}^f[k;\mathbf{x}_i,\mathbf{x}_j]$ denote the channel power gain at period k on frequency f, from terminal i at position $\mathbf{x}_i$ to terminal j at position $\mathbf{x}_j$. Following standard practice in wireless communications, we assume that $\{h_{ij}^f[k;\mathbf{x}_i,\mathbf{x}_j]\}_{f,ij}$ is the realization of a random vector $\mathbf{H}[k;\mathbf{x}]$ [1]. Channel gain models often take the form

$$\begin{aligned} h_{ij}^f[k;\mathbf{x}_i,\mathbf{x}_j]|_{dB} &:= 10\log_{10} h_{ij}^f[k;\mathbf{x}_i,\mathbf{x}_j] \\ &= l^f(\mathbf{x}_i,\mathbf{x}_j) + Y_{ij}^f[k], \end{aligned} \quad (4)$$

where $Y_{ij}^f[k]$ is a zero-mean random variable modeling fading and shadowing effects [1]. The function $l^f(\mathbf{x}_i,\mathbf{x}_j)$ models the deterministic path loss between positions $\mathbf{x}_i$ and $\mathbf{x}_j$. Let p_{ij}^f denote the power used by terminal i to communicate with terminal j over the frequency tone f. Many communication schemes result in link capacities that are functions of the signal to interference plus noise ratios (SINR)

$$\mathrm{SINR}_{ij}^f[k;\mathbf{x}] = \frac{h_{ij}^f[k;\mathbf{x}_i,\mathbf{x}_j]p_{ij}^f}{\sigma_j + \frac{1}{S}\sum\limits_{(l,m)\neq(i,j)} h_{lm}^f[k;\mathbf{x}_l,\mathbf{x}_m]p_{lm}^f}, \quad (5)$$

where σ_j denotes the noise power at receiver j and $1/S$ is the interference reduction due to signal processing, e.g. S is approximately equal to the processing gain in a CDMA system [1, chapter 14]. With capacity-achieving channel codes, we could have for example $c_{ij}[k;\mathbf{x}] = \sum_{f\in\mathcal{F}} \log\left(1 + \mathrm{SINR}_{ij}^f[k;\mathbf{x}]\right)$.

Remark 1: Frequency and power allocation optimization can also be considered in our framework, following [19]. For clarity of exposition, these variables are not included below, our focus being on the role of controlled mobility on communication performance.

To ensure boundedness of the queues at all terminals, it is sufficient to ensure that the long-term average amount of information accepted at each terminal is less than the long-term average amount of information forwarded to other terminals [20]. We denote the long-term averages in the following by $a_i^\phi := \lim_{T\to\infty}\frac{1}{T}\sum_{k=0}^{T-1} a_i^\phi[k]$ and $r_{ij}^\phi := \lim_{T\to\infty}\frac{1}{T}\sum_{k=0}^{T-1} r_{ij}^\phi[k]$ for $i \neq j$. Then we have the constraint $a_i^\phi + \sum_{j\neq i} r_{ji}^\phi \leq \sum_{j\neq i} r_{ij}^\phi$, for all ϕ and all $i \neq \mathrm{dest}(\phi)$. Moreover, the long-term average amount of information circulating on link (i,j) cannot exceed the long-term average capacity. We assume for simplicity that for each fixed configuration $\mathbf{x}$, the capacity values $c_{ij}[k;\mathbf{x}]$ form a temporal sequence of identically and independently distributed (iid) random variables, with mean $c_{ij}(\mathbf{x})$. By the strong law of large number, we have $c_{ij}(\mathbf{x}) = \mathbb{E}[c_{ij}[0;\mathbf{x}]] =$

$\lim_{T\to\infty} \frac{1}{T}\sum_{k=0}^{T-1} c_{ij}[k;\mathbf{x}]$. In fact, the algorithm below works with general ergodic Markov chains instead of iid sequences, with more technical assumptions for convergence. For a robotic network in a fixed configuration $\mathbf{x}$, we then have the constraint $\sum_{\phi\in\Phi} r_{ij}^{\phi} \le c_{ij}(\mathbf{x}), \forall i \neq j$.

We adopt the convention $r^{\phi}_{\text{dest}(\phi)j} = 0$ for all j, a natural requirement. Finally, we can also have rate constraints of the form $a^{\phi}_{i,\min} \le a^{\phi}_i \le a^{\phi}_{i,\max}$ for $i \neq \text{dest}(\phi)$ and $r^{\phi}_{ij,\min} \le r^{\phi}_{ij} \le r^{\phi}_{ij,\max}$ for $i \neq j$, with $a^{\phi}_{i,\min}, r^{\phi}_{ij,\min} = 0$ and $a^{\phi}_{i,\max}, r^{\phi}_{ij,\max} = +\infty$ possible values. Any set of average rates $\mathbf{a} := \{a^{\phi}_i\}_{i,\phi}, \mathbf{r} := \{r^{\phi}_{ij}\}_{i\neq j,\phi}$ and final configuration $\mathbf{x} \in \mathsf{W}^n$ such that the constraints above are satisfied is said to be feasible. We would like to select, among these feasible points, one that is (at least locally) optimal according to a given criterion, including the task potential for the configuration component.

C. Joint Optimization Problem

We introduce concave utilities $U^{\phi}_i(a^{\phi}_i)$ to value the average admission rates a^{ϕ}_i, and convex costs $V^{\phi}_{ij}(r^{\phi}_{ij})$ for establishing communication links. The optimal configuration and wireless network parameters are then defined as the solution of the following optimization problem

$$\min_{\mathbf{x},\mathbf{a},\mathbf{r}} \quad G(\mathbf{x}) - \sum_{\phi\in\Phi}\sum_{i\neq\text{dest}(\phi)} U^{\phi}_i(a^{\phi}_i) + \sum_{\phi\in\Phi}\sum_{i\neq j} V^{\phi}_{ij}(r^{\phi}_{ij}) \quad (6)$$

$$\text{s.t.} \quad a^{\phi}_i \le \sum_{j\neq i} r^{\phi}_{ij} - r^{\phi}_{ji}, \quad \forall\phi, \forall i \neq \text{dest}(\phi) \quad (7)$$

$$\sum_{\phi\in\Phi} r^{\phi}_{ij} \le c_{ij}(\mathbf{x}), \quad \forall i \neq j, \quad (8)$$

$$a^{\phi}_{i,\min} \le a^{\phi}_i \le a^{\phi}_{i,\max}, \quad \forall i \neq \text{dest}(\phi), \quad (9)$$

$$r^{\phi}_{ij,\min} \le r^{\phi}_{ij} \le r^{\phi}_{ij,\max}, \quad \forall i \neq j. \quad (10)$$

For a fixed configuration $\mathbf{x}$, the problem (6)-(10) is convex in the communication rates $\mathbf{a}, \mathbf{r}$. However, the presence of the configuration vector $\mathbf{x}$ makes the problem non-convex in general, because most useful multi-robot task potentials G are not convex and the mean capacities $c_{ij}(\mathbf{x})$ in (8) are not concave functions of $\mathbf{x}$. Still, following standard practice in robotics, one can at least look for a locally optimal solution to the constrained problem (6)-(10). Moreover, by using a gradient-based algorithm similar in spirit to (3), we can obtain a feedback controller driving the robotic network progressively toward this locally optimal configuration. The next section introduces a primal-dual optimization algorithm for this purpose.

III. Primal-Dual Algorithm

We assume in this section that the average channel capacities $c_{ij}(\mathbf{x})$ in (8) are known functions. Problem (6)-(10) is then a deterministic optimization problem, for which many nonlinear programming techniques are available [21]. Here we concentrate on a primal-dual augmented Lagrangian algorithm [21]. Our report [15] motivates this choice.

Define the dual variables $\lambda^{\phi}_i \ge 0, i \neq \text{dest } \phi$, and $\mu_{ij} \ge 0, i \neq j$, associated with (7) and (8) respectively. We gather the primal and dual variables in the vectors $\mathbf{y} = (\mathbf{x}, \mathbf{a}, \mathbf{r})$ and $\boldsymbol{\xi} = (\boldsymbol{\lambda}, \boldsymbol{\mu})$ respectively. The region defined by W and the box constraints (9), (10) for the primal variables is denoted Y. Next, let $F(\mathbf{y})$ denote the objective function (6), and denote the functions appearing in the constraints (7), (8) as $g^{\phi}_i(a^{\phi}_i, \mathbf{r}^{\phi}_{i\cdot}, \mathbf{r}^{\phi}_{\cdot i}) = a^{\phi}_i - \sum_{j\neq i}\left(r^{\phi}_{ij} - r^{\phi}_{ji}\right)$, $h_{ij}(\mathbf{x}, \mathbf{r}_{ij}) = \sum_{\phi\in\Phi} r^{\phi}_{ij} - c_{ij}(\mathbf{x})$, with $\mathbf{r}_{ij} = \{r^{\phi}_{ij}\}_{\phi\in\Phi}$, $\mathbf{r}^{\phi}_{\cdot i} = \{r^{\phi}_{ji}\}_{j\in[n]}$, and $\mathbf{r}^{\phi}_{i\cdot} = \{r^{\phi}_{ij}\}_{j\in[n]}$. We can always rewrite an inequality constraint $K(\mathbf{y}) \le 0$ as an equality constraint $K(\mathbf{y}) + z^2 = 0$, with z a slack variable. Doing this for (7), (8) leads to the definition of the augmented Lagrangian [21, Section 4.2] for the equivalent equality-constrained problem

$$\hat{\mathcal{L}}_{\rho}(\mathbf{y}, \mathbf{z}, \boldsymbol{\xi}) = F(\mathbf{y}) + \sum_{\substack{\phi\in\Phi \\ i\neq\text{dest}(\phi)}} \Big\{\lambda^{\phi}_i\left(g^{\phi}_i(a^{\phi}_i, \mathbf{r}^{\phi}_{i\cdot}, \mathbf{r}^{\phi}_{\cdot i}) + (z^{\phi}_i)^2\right)$$
$$+ \frac{\rho}{2}|g^{\phi}_i(a^{\phi}_i, \mathbf{r}^{\phi}_{i\cdot}, \mathbf{r}^{\phi}_{\cdot i}) + (z^{\phi}_i)^2|^2\Big\}$$
$$+ \sum_{i=1}^{n}\sum_{j\neq i}\left\{\mu_{ij}\left(h_{ij}(\mathbf{x}, \mathbf{r}_{ij}) + z^2_{ij}\right) + \frac{\rho}{2}|h_{ij}(\mathbf{x}, \mathbf{r}_{ij}) + z^2_{ij}|^2\right\},$$

where $\rho > 0$ is a penalization parameter, and the variables z^{ϕ}_i and z_{ij} are slack variables. Partial minimization with respect to the slack variables, as detailed in [21, p. 406], allows us to work with the following simpler version of the augmented Lagrangian function, which depends only on the original primal and dual variables

$$\mathcal{L}_{\rho}(\mathbf{y}, \boldsymbol{\xi}) = F(\mathbf{y}) + \quad (11)$$
$$\frac{1}{2\rho}\sum_{\phi\in\Phi}\sum_{i\neq\text{dest}(\phi)}\left\{(\max\{0, \lambda^{\phi}_i + \rho g^{\phi}_i(a^{\phi}_i, \mathbf{r}^{\phi}_{i\cdot}, \mathbf{r}^{\phi}_{\cdot i})\})^2 - (\lambda^{\phi}_i)^2\right\}$$
$$+ \frac{1}{2\rho}\sum_{i=1}^{n}\sum_{j\neq i}\left\{(\max\{0, \mu_{ij} + \rho h_{ij}(\mathbf{x}, \mathbf{r}_{ij})\})^2 - \mu^2_{ij}\right\}.$$

Many optimization algorithms aim at computing a Karush-Kuhn-Tucker (KKT) point $(\mathbf{y}^*, \boldsymbol{\xi}^*)$ where the necessary optimality conditions for (6)-(10) are satisfied [21]. Consider a first-order primal-dual algorithm of the form

$$\mathbf{y}[k+1] = \mathcal{P}_{\mathsf{Y}}\left(\mathbf{y}[k] - \alpha_k \nabla_{\mathbf{y}}\mathcal{L}_{\rho}(\mathbf{y}[k], \boldsymbol{\xi}[k])\right) \quad (12)$$

$$\lambda^{\phi}_i[k+1] = \left[\lambda^{\phi}_i[k] + \beta_k g^{\phi}_i(a^{\phi}_i[k], \mathbf{r}^{\phi}_{i\cdot}[k], \mathbf{r}^{\phi}_{\cdot i}[k])\right]_0^{\lambda^{\phi}_{i,\max}} \quad (13)$$

$$\mu_{ij}[k+1] = \left[\mu_{ij}[k] + \beta_k h_{ij}(\mathbf{x}[k], \mathbf{r}_{ij}[k])\right]_0^{\mu_{ij,\max}}, \quad (14)$$

where $\mathcal{P}_{\mathsf{Y}}$ denotes the projection on the set Y, and $\nabla_{\mathbf{y}}$ denotes the vector of derivatives with respect to the primal variables. The prespecified stepsizes α_k, β_k can be chosen constant and sufficiently small, see [15]. We use the notation $[x]^u_l := \max\{l, \min\{u, x\}\}$ to project $x \in \mathbb{R}$ on the interval $[l, u]$. The upper bounds $\lambda^{\phi}_{i,\max}$ and $\mu_{ij,\max}$ need to be sufficiently large so that the resulting box region

$$\Xi := \{(\boldsymbol{\lambda}, \boldsymbol{\mu}) | 0 \le \lambda^{\phi}_i < \lambda^{\phi}_{i,\max}, \forall i \neq \text{dest } \phi,$$
$$0 \le \mu_{ij} < \mu_{ij,\max}, \forall i \neq j\} \quad (15)$$

contains the desired Lagrange multiplier $\boldsymbol{\xi}^*$. They can be set to $+\infty$ if no such region estimate is known, but otherwise can be used to significantly reduce oscillations in the primal variable trajectories, see [22, p. 181]. Upper bounds on Lagrange multipliers can be obtained for example from duality arguments [23, p. 6377].

A. Explicit Form and Considerations about Distributed Implementations

This section describes the nice structure given to the generic primal variable update equation (12) by the separable form of the augmented Lagrangian (11). First, for a differentiable function K and parameters λ, ρ, we have $\nabla_{\mathbf{y}}[(\max\{0, \lambda + \rho K(\mathbf{y})\})^2] = 2\rho \max\{0, \lambda + \rho K(\mathbf{y})\} \nabla_{\mathbf{y}} K(\mathbf{y})$. We can thus rewrite (12) explicitly as

$$\mathbf{x}_i[k+1] = \mathcal{P}_{\mathsf{W}}\Bigg(\mathbf{x}_i[k] + \mathrm{sat}_{\mathbf{v}}\Bigg[-\alpha_k\Bigg(\frac{\partial G}{\partial \mathbf{x}_i}(\mathbf{x}[k]) - \quad (16)$$
$$\sum_{l,m\neq l} \max\{0, \mu_{lm}[k] + \rho h_{lm}(\mathbf{x}[k], \mathbf{r}_{lm}[k])\} \frac{\partial c_{lm}}{\partial \mathbf{x}_i}(\mathbf{x}[k])\Bigg)\Bigg]\Bigg),$$

$$a_i^\phi[k+1] = \Bigg[a_i^\phi[k] - \alpha_k\Bigg(-\frac{dU_i^\phi}{da_i^\phi}(a_i^\phi[k]) + \max\{0, \lambda_i^\phi[k]$$
$$+ \rho g_i^\phi(a_i^\phi[k], \mathbf{r}_{i\cdot}^\phi[k], \mathbf{r}_{\cdot i}^\phi[k])\}\Bigg)\Bigg]_{a_{i,\min}^\phi}^{a_{i,\max}^\phi}, \forall \phi, i \neq \mathrm{dest}(\phi), \quad (17)$$

$$r_{ij}^\phi[k+1] = \Bigg[r_{ij}^\phi[k] - \alpha_k\Bigg(\frac{dV_{ij}^\phi}{dr_{ij}^\phi}(r_{ij}^\phi[k])$$
$$+ \max\{0, \mu_{ij}[k] + \rho h_{ij}(\mathbf{x}[k], \mathbf{r}_{ij}[k])\}$$
$$- \max\{0, \lambda_i^\phi[k] + \rho g_i^\phi(a_i^\phi[k], \mathbf{r}_{i\cdot}^\phi[k], \mathbf{r}_{\cdot i}^\phi[k])\}$$
$$+ \max\{0, \lambda_j^\phi[k] + \rho g_j^\phi(a_j^\phi[k], \mathbf{r}_{j\cdot}^\phi[k], \mathbf{r}_{\cdot j}^\phi[k])\}\Bigg)\Bigg]_{r_{ij,\min}^\phi}^{r_{ij,\max}^\phi},$$
$$\forall \phi, \forall i \neq \mathrm{dest}(\phi), \forall j \neq i. \quad (18)$$

In large robotic networks, distributed algorithms are preferable for their scalability and tolerance to faults. Assume that robot $i \in [n]$ is responsible for updating its position vector $\mathbf{x}_i[k]$, its traffic admission rates $\mathbf{a}_i[k] := \{a_i^\phi[k]\}_\phi$ and outgoing link rates $\mathbf{r}_{i\cdot}[k]$, and the dual variables $\boldsymbol{\lambda}_i[k] := \{\lambda_i^\phi[k]\}_\phi$ and $\boldsymbol{\mu}_{i\cdot}[k] := \{\mu_{ij}[k]\}_j$. Using its own local information, it can immediately update the dual variables $\boldsymbol{\mu}_{i\cdot}$ according to (14). By also keeping track of the rates $\mathbf{r}_{\cdot i}[k]$ from its one-hop neighbors, it can also immediately update the variables $\mathbf{a}_i$, $\boldsymbol{\lambda}_i$ according to (17) and (13). Finally, to update its link rates $\mathbf{r}_{ij}$ according to (18) for some neighbor $j \in [n]$, it also needs access to the rates $\mathbf{r}_{j\cdot}[k]$ and $\mathbf{r}_{\cdot j}[k]$ of information sent to and from node j. In other words, all the mentioned variables so far can be updated with at most two-hop information.

The complexity of the robot position update (16) depends on the separability properties of the gradient of the task potential G and on the number of links (l, m) with which robot i might interfere, i.e., for which $\partial c_{lm}/\partial \mathbf{x}_i$ is not zero. If interferences can be neglected, e.g., with a large processing gain S in (5) or an appropriate time and frequency allocation scheme, then the capacity of link (l, m) depends only on the positions of robots l and m, so $\partial c_{lm}/\partial \mathbf{x}_i$ is non-zero only for $l = i$ or $m = i$ and the sum in (16) can be computed from the information collected from the one-hop neighbors.

IV. Sampling-Based Algorithm

The algorithm presented in Section III requires a model of the average channel capacities $c_{ij}(\mathbf{x})$, but in practice such models are notably difficult to devise, and only provide rough approximations [1, chapter 1]. Practical systems must rely on measuring the channel gains (4) and estimating the resulting link capacities. Since UVS can be deployed in a priori unknown environments for which channel strength maps are not available, deployment algorithms should take the channel sampling requirement into account. In this section, we show that this is possible by only modestly increasing the complexity of the primal-dual algorithm.

A. Two-time-scale SPSA Algorithm

We assume for simplicity that we can measure the instantaneous and random channel capacities $c_{ij}[k; \mathbf{x}[k]]$ introduced in Section II-B. In practice, only the SINR (5) or channel gains (4) might be measurable, and the capacities should then be evaluated based on the chosen communication scheme. Recall from Section II-B that $c_{ij}(\mathbf{x})$ is the ergodic limit of the random variables $c_{ij}[k; \mathbf{x}]$, for $\mathbf{x}$ fixed. To implement the iterations (14), (16) and (18) of the algorithm, estimates of the link capacities $c_{ij}(\mathbf{x}[k])$ and of their gradients $\partial c_{lm}(\mathbf{x}[k])/\partial \mathbf{x}_i$ are required, and can only be obtained from the measurements $c_{ij}[k; \mathbf{x}[k]]$. We adopt a two-time scale approach [14, Section 8.6], where the estimates are computed simultaneously with the primal and dual variable updates, albeit with larger stepsizes. Intuitively this device allows us to replace the estimators by their steady-state values in the convergence analysis of the sampling-based algorithm.

First, consider the following recursive estimators for the link capacities

$$\hat{c}_{lm}[k+1] \quad = \hat{c}_{lm}[k] + \gamma_k(c_{lm}[k; \mathbf{x}[k]] - \hat{c}_{lm}[k]), \quad (19)$$

where $\hat{c}_{lm}[0] = 0$, and $\gamma_k \in (0, 1)$ are prespecified stepsizes that can be taken as constant and sufficiently small, see [15]. Next, the capacity gradients are estimated using a form of finite difference approximation. Standard finite-difference methods are not practical in multi-robot systems however, requiring too much inter-robot coordination. The issue is that to estimate the partial derivatives of the link capacities with respect to the position of a single robot, this robot must move while the other robots remain fixed. This results in unreasonably long update times to compute just one iteration of (16).

This issue can be resolved using a stochastic sampling strategy known as Simultaneous Perturbation Stochastic Approximation (SPSA), see [13]. We divide period k into two subperiods. In the first subperiod, the robots are in the configuration $\mathbf{x}[k]$, and each robot measures the link capacities $c_{lm}[k; \mathbf{x}[k]]$ that its presence influences. It also generates for that period, independently of the other robots, a random d-dimensional vector, say $\boldsymbol{\Delta}_i[k]$ for robot i, with independent and identically distributed (iid) entries in $\{+1,-1\}$ such that $P(\Delta_i^l = 1) = P(\Delta_i^l = -1) = 1/2$, $l = 1,\dots,d$. Let $\mathbf{\Delta}[k] = [\mathbf{\Delta}_1[k]^T,\dots,\mathbf{\Delta}_n[k]^T]^T$ denote the aggregate nd dimensional random vector. In the second subperiod, denoted k^+, all robots move simultaneously and randomly, with robot i moving by an amount $\delta\mathbf{\Delta}_i[k]$ from $\mathbf{x}_i[k]$, where δ is a small constant. Then, each robot again measures the relevant link capacities $c_{lm}[k^+;\mathbf{x}[k]+\delta\mathbf{\Delta}[k]]$. They can now update the following sequences

$$\hat{\mathbf{d}}_{lm,i}[k+1] = \hat{\mathbf{d}}_{lm,i}[k] + \gamma_k\big(\mathbf{\Delta}_i[k](c_{lm}[k^+;\mathbf{x}[k]+\delta\mathbf{\Delta}[k]] - c_{lm}[k;\mathbf{x}[k]]) - \hat{\mathbf{d}}_{lm,i}[k]\big), \quad (20)$$

where the γ_k are the same as in (19), and $\hat{\mathbf{d}}_{lm,i}[0] = \mathbf{0}$.

The final sampling-based algorithm followed by the robots is the same as the primal-dual algorithm (16)-(18), (13)-(14), except that in (16), (14) and (18) the capacities $c_{ij}(\mathbf{x}[k])$ in the expression $h_{ij}(\mathbf{x}[k],\mathbf{r}_{ij}[k])$ are replaced by the estimates $\hat{c}_{ij}[k]$ from (19) and in (16) the terms $\partial c_{lm}(\mathbf{x}[k])/\partial\mathbf{x}_i$ are replaced by the estimates $\hat{\mathbf{d}}_{lm,i}[k]/\delta$ based on (20). Perhaps surprisingly, numerical experiments [24] have shown that for many problems the total number of iterations to reach convergence with this 2-sample randomized approximation scheme is often of the same order as the one required with the standard central difference scheme. Hence SPSA drastically simplifies and accelerates the gradient estimation procedure by allowing all robots to sample the channels simultaneously, and potentially involves no significant loss of performance per iteration overall.

To understand the motivation behind (20), for appropriate small stepsizes γ_k and α_k, the estimates $\hat{\mathbf{d}}_{lm,i}[k]/\delta$ used in (16) in place of $\partial c_{lm}(\mathbf{x}[k])/\partial\mathbf{x}_i$ approximately contribute the following term to the dynamics of $\mathbf{x}[k]$

$$\mathbb{E}\left[\mathbf{\Delta}_i[k]\frac{c_{lm}(\mathbf{x}[k]+\delta\mathbf{\Delta}[k]) - c_{lm}(\mathbf{x}[k])}{\delta}\Big|\mathbf{x}[k]\right], \quad (21)$$

which by a Taylor expansion of $c_{lm}(\mathbf{x})$ is equal to

$$\mathbb{E}\Big[\mathbf{\Delta}_i[k]\mathbf{\Delta}[k]^T\nabla c_{lm}(\mathbf{x}[k]) + \frac{1}{2}\delta\mathbf{\Delta}_i[k]\mathbf{\Delta}[k]^T\nabla^2 c_{lm}(\mathbf{x}[k]+s\delta\mathbf{\Delta}[k])\mathbf{\Delta}[k]\Big],\ s\in[0,1]$$
$$= \partial c_{lm}(\mathbf{x}[k])/\partial\mathbf{x}_i + O(\sup_{\mathbf{z}}\delta\|\nabla^2 c_{lm}(\mathbf{z})\|), \quad (22)$$

since $\mathbb{E}\big[\mathbf{\Delta}_i[k]\mathbf{\Delta}[k]^T\big] = [0\dots I_d\dots 0]$.

V. Simulations

We briefly illustrate the practical behavior of the primal-dual algorithm for the deployment scenario depicted on Fig. 1. One robot must approach a given waypoint at a known position q^*, which is achived with the task potential (2), while transmitting back some information relayed to the base station by the other robots. The robots must also establish a communication flow between this base station and another distant one. The instantaneous channel capacities are stochastic and simulated as

$$c_{ij}[k;\mathbf{x}] = 4\,\eta_{ij}[k]\ln\left(1+\frac{1}{0.1+0.25\|\mathbf{x}_i-\mathbf{x}_j\|^2}\right), \quad (23)$$

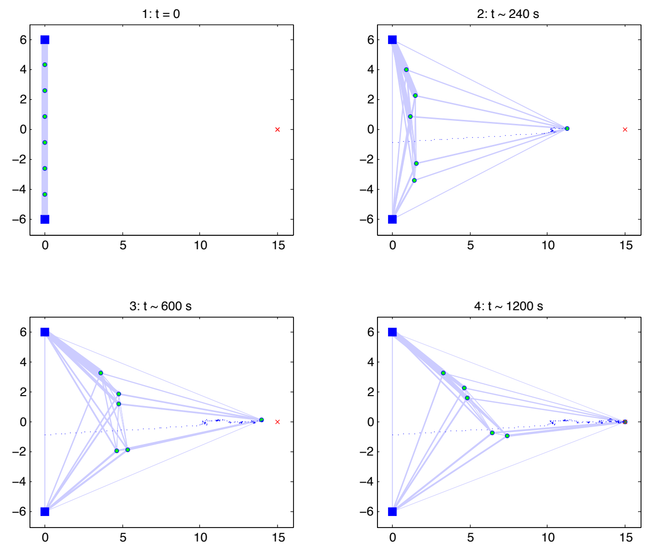


Fig. 1. One designated robot must approach the target denoted by a cross, placed at $x = 1.5$ km (the dotted curve shows the robot trajectory). It generates information for the top Base Station (BS) at unit rate. The bottom BS also transmits information to the top BS at unit rate, as part of the same and unique flow. The other robots spread to establish the necessary links. The primal and dual variables are updated every 50 ms, using the measurement-based algorithm and no knowledge of the link capacity model. The maximum velocity of the vehicles is 10 m/s (36 km/hr), as could be appropriate for ground robots.

with $\{\eta_{ij}[k]\}_k$ an iid sequence of log-normal random variables with distribution $\ln\mathcal{N}(0,0.2)$, see Fig. 2. The robots do not know the model (23) and can only rely on the fluctuating measurements $c_{ij}[k;\mathbf{x}]$ taken during deployment to update their positions and the communication rates. They implement the two-time-scale SPSA algorithm described in Section IV-A. As can be seen from Fig. 3(a), with properly chosen parameters, the instantaneous routing and capacity constraints can be essentially satisfied during the whole deployment, resulting in short communication delays and small queues at the terminals. With a variance here of about 0.27 for the η_{ij} variables, the algorithm exhibits a satisfying converging behavior. More generally the performance of such a finite difference scheme can be sensitive to the level of measured noise, and additional averaging over several capacity measurements before moving might be necessary. Variance reduction in the estimates (19), (20) can also be achieved by reducing the values of α,β,γ.

VI. Conclusions

A communication constrained robot deployment algorithm is presented in this paper to jointly optimize the steady-state locations of a group of unmanned vehicles and the wireless communication network necessary to support their mission. An optimization-based approach is proposed, where a general augmented Lagrangian primal-dual algorithm can drive the robotic network to a locally optimal final configuration. Importantly, this approach can be used as a measurement based feedback strategy, when no sufficiently reliable prior model of the communication channels is available.

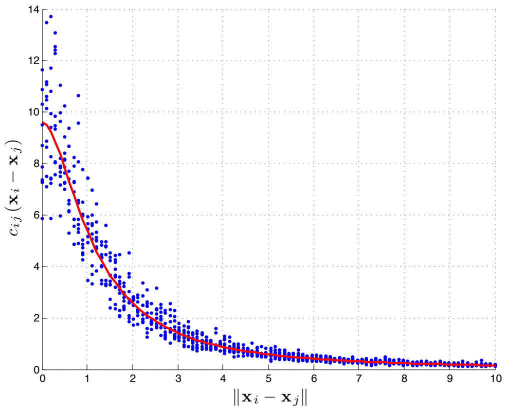
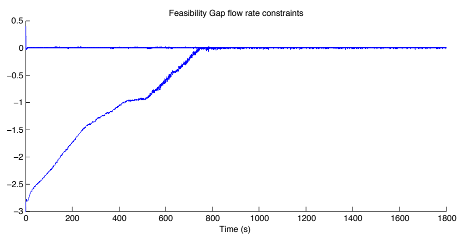
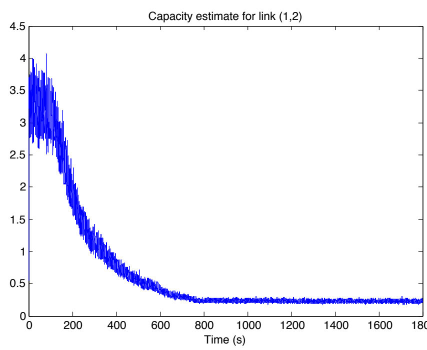


Fig. 2. Average link capacity function (solid curve) with random samples for the simulated scenario, see (23).



(a)



(b)

Fig. 3. (a) Trajectories of the routing constraint values $g_i^{\phi}(a_i^{\phi}[k], \mathbf{r}_{i\cdot}^{\phi}[k], \mathbf{r}_{\cdot i}^{\phi}[k])$. Note that the communication constraints are essentially always satisfied during deployment. (b) Sample trajectory of the capacity estimates (19) for the link $(1, 2)$, with $\gamma = 0.2$.

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