Cams, Gears, Belts & Chains
Design Game

SAAST Robotics 2007

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Cams

Transform one motion into another
Cam – curved/grooved surface
Rotational motion of cam → Follower oscillation and/or translation
Designed for motion, path or function generation

Many types of cams:

Plate/disk cam; wedge cam - translating roller followers
Cams: Types of Cams

Types of cams:

Cylindrical cam; conical cam - translating followers

Face cam; globoidal cam – oscillating follower
Cams: Types of Followers

Types of Followers:

- Translating flat-faced follower
- Translating roller follower
- Translating point follower
- Oscillating flat-face follower
- Oscillating roller-follower
Cams: Types of Followers

Types of Followers:

- Translating positive-return follower
- Oscillating spherical-face follower
- Translating double-roller follower and double-lobed cam
Cams: Applications

Design cam geometry for follower displacement according to graph

Requirements for tracer point:
- Rise off prime circle by L
- Remain for a while (dwell) @ L
- Return to prime circle
- Remain at rest in 2\textsuperscript{nd} dwell
- Repeat

Figure 6.4  Disk cam and radial roller follower with appropriate nomenclature. Distance c-d is the rise of the follower in position 7.

Figure 6.5  Follower displacement profile corresponding to Fig. 6.4. Distance c-d is the rise of the follower in position 7. Maximum follower travel L represents movement from point a on the prime circle to point b at stations 5 and 6.
Cams: Applications

Stamping Mechanism
(1) Stamping platen
(2) Flexures
(3) Stop
(4) Springs
(5) Anvil

Goal: (1) to be cyclically depressed against (5) according to time-displacement curve (6)
Gears

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Gears

- Transmit motion by means of successively engaging teeth
  - Most Common: Rotating shaft to another rotating shaft
  - Rotating shaft to a rack – translates motion to straight line

- Why gear up/down a motor?
  - Increase speed – decrease torque
  - Decrease speed – increase torque

- Types of Gears
  - Spur gears – transmit rotary motion btw/ parallel shafts
Gears: Types of Gears

- Types of Gears
  - Internal gear and pinion

- Rack and pinion – rack is spur gear with infinitely large pitch diameter
Gears: Types of Gears

- Types of Gears
  - Helical gears – transmit motion btw parallel or non-parallel shafts
Gears: Types of Gears

- Types of Gears
  - Bevel gears – transmit motion between intersecting shafts
Gears: Types of Gears

- **Types of Gears**
  - Bevel gears – transmit motion between intersecting shafts
    - Skew bevel gears – connect shafts whose axes don’t intersect – straight teeth
  - Hypoid gears – transmit motion between shafts whose axes don’t intersect
    - Curved teeth
Gears: Types of Gears

- Types of Gears
  - Worm gears – transmit motion between nonparallel non-intersecting shafts
  - Noncircular gears – non constant angular velocity ratios between input and output
Gears: Gear Tooth Nomenclature

*Pitch circles* of mating gears are tangent to each other.

*Circular pitch (CP)* – distance along arc of pitch circle between neighboring teeth.

*Diametral pitch (P)* - # of teeth on gear/inch of pitch diameter.

Backlash – difference between tooth space and thickness of engaging tooth at pitch circles.

\[ P = \frac{N}{D} \]

\[ CP = \frac{\pi D}{N} \]

- \( N \) = # of teeth
- \( D \) = pitch diameter (in)
Gears: Gear Trains

Desire: $\omega_3 = \omega_{out} = 2700$ rpm, counterclockwise input to machine
Standard motor output: $\omega_1 = \omega_2 = \omega_{in} = -1800$ rpm, clockwise
Solution: Simple Gear train – spur gears

- Pitch velocities of mating gears are equal and functions of pitch radii and angular velocity
  $$V_{p2} = r_2 \omega_2 = V_{p3} = -r_3 \omega_3 \rightarrow \frac{\omega_3}{\omega_2} = -\frac{r_2}{r_3} = -\frac{30}{20}$$
  $$\omega_3 = -\frac{3}{2} \omega_2 = -\frac{3}{2} (-1800) = 2700 \text{ rpm ccw}$$
- Angular velocities inversely proportional to pitch radii $\rightarrow$ pitch diameter $\rightarrow$ # of teeth
  $$P = \frac{N}{D} \quad \omega_{in} / \omega_{out} = r_{out} / r_{in} = \frac{N_{out}}{N_{in}}$$

$$D = \frac{N}{P}$$
$$2 \times r = \frac{N}{P}$$
$$r \approx N$$
Gears: Gear Trains

• What if want larger angular velocity ratio, like 60:1? \( N_2 = 60 \, N_1 \)
• Space limitations prevent one set of gears w/ diameter ratio of 60:1
• Solution: Compound gear train

\[
\frac{\omega_2}{\omega_3} = \frac{-N_3}{N_2}, \quad \frac{\omega_3}{\omega_4} = 1, \quad \frac{\omega_4}{\omega_5} = \frac{-N_5}{N_4}, \quad \frac{\omega_5}{\omega_6} = 1, \quad \frac{\omega_6}{\omega_7} = \frac{-N_7}{N_6}
\]

\[
\frac{\omega_2}{\omega_7} = \frac{\omega_2}{\omega_3} \frac{\omega_3}{\omega_4} \frac{\omega_4}{\omega_5} \frac{\omega_5}{\omega_6} \frac{\omega_6}{\omega_7} = \frac{N_2 \, N_5 \, N_7}{N_2 \, N_4 \, N_6}
\]

\[
\frac{\omega_2}{\omega_7} = -\left( \frac{N_2 \, N_5 \, N_7}{N_2 \, N_4 \, N_6} \right)
\]
Choose: $N_3/N_2 = 3$, $N_5/N_4 = 4$, $N_7/N_8 = 5 \Rightarrow \frac{\omega_2}{\omega_7} = -\left(\frac{N_3}{N_2} \cdot \frac{N_5}{N_4} \cdot \frac{N_7}{N_8}\right) = 60$

General Rule:

$$\text{Gear Ratio} = \left| \frac{\omega_{\text{driver}}}{\omega_{\text{driven}}} \right| = \frac{\text{product of numbers of teeth on driven gears}}{\text{product of numbers of teeth on driver gears}}$$
Gears: Gear Trains

- Planetary gear train - allow some gear axes to rotate about others
  - Sun gear, planet carrier (or arm), planet gears
  - 2 DOF’s → 2 inputs

\[ e = \frac{\omega_F - \omega_A}{\omega_L - \omega_A} \]

- \( e \) = gear ratio
- \( \omega_F \) = rpm of first gear in planetary gear train
- \( \omega_L \) = rpm of last gear in planetary gear train
- \( \omega_A \) = rpm of arm

\[ e = \frac{\omega_2 - \omega_3}{\omega_5 - \omega_3} \]
**Gears: Gear Trains**

**Planetary Gear Train Example**

In Fig. 13-28 the sun gear is the input, and it is driven clockwise at 100 rev/min. The ring gear is held stationary by being fastened to the frame. Find the rev/min and direction of rotation of the arm.

\[
e = \frac{\omega_F - \omega_A}{\omega_L - \omega_A}
\]

\[
e = \frac{\omega_{\text{driver}}}{\omega_{\text{driven}}} = \frac{\text{product } N_{\text{driven}} \text{ gears}}{\text{product } N_{\text{driver}} \text{ gears}}
\]

+ = same direction

- = opposite direction

\[
\omega_F = \omega_2 = -100 \text{ rpm}
\]

\[
\omega_L = \omega_5 = 0
\]

Unlock gear 5, and hold arm stationary →

\[
e = \frac{\omega_2}{\omega_3} = -\frac{N_4 \cdot N_5}{N_2 \cdot N_4} = -\frac{30 \cdot 80}{20 \cdot 30} = -4
\]

\[
-4 = \frac{-100 - \omega_A}{0 - \omega_A} \Rightarrow \omega_A = -20 \text{ rpm}
\]
7.1. Figure P7.1 shows a simple gear train. If the input is provided by gear A, and the output taken off the shaft of gear E, what is the ratio and the sign of $\omega_{\text{out}}/\omega_{\text{in}}$ in terms of the number of teeth on the gears?
Gears: Homework Problem 2

This gear train consists of miter gears (same-size bevel gears) having 16 teeth each, a 4-tooth right hand worm, and a 40-tooth worm gear. The speed of gear 2 is given as +200 rpm, corresponding to counterclockwise rotation about y-axis. What is the speed and direction of the worm gear (5)?
Belts & Chains

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Belts & Chains

- Used in conveying systems and for transmitting power over long distances
- Replacements for gears, shafts, bearings
  - Simplifies design, reduces cost
  - Elastic, flexible nature is good for absorbing shock loads and vibrations
  - Minimum distance between pulley axes required for proper operation

<table>
<thead>
<tr>
<th>BELT TYPE</th>
<th>FIGURE</th>
<th>JOINT</th>
<th>SIZE RANGE</th>
<th>CENTER DISTANCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flat</td>
<td></td>
<td>Yes</td>
<td>( r = \begin{cases} 0.03 \text{ to } 0.20 \text{ in} \ 0.75 \text{ to } 5 \text{ mm} \end{cases} )</td>
<td>No upper limit</td>
</tr>
<tr>
<td>Round</td>
<td></td>
<td>Yes</td>
<td>( d = \frac{1}{2} \text{ to } \frac{3}{4} \text{ in} )</td>
<td>No upper limit</td>
</tr>
<tr>
<td>( V )</td>
<td></td>
<td>None</td>
<td>( b = \begin{cases} 0.31 \text{ to } 0.91 \text{ in} \ 8 \text{ to } 19 \text{ mm} \end{cases} )</td>
<td>Limited</td>
</tr>
<tr>
<td>Timing</td>
<td></td>
<td>None</td>
<td>( p = 2 \text{ mm and up} )</td>
<td>Limited</td>
</tr>
</tbody>
</table>

Crowned pulleys
Grooved pulleys / sheaves
Grooved pulleys / sheaves
Toothed wheels / sprockets
Belts & Chains

• Belt characteristics:
  • Used for long center distances
  • Except for timing belts, slip and creep present so velocity ratios between driver and driven shafts not constant or equal to ratio of pulley diameters
  • Use of idler or tension pulley to avoid adjusting center distances – needed with old or new belts

Open-belt drive – slack side should be on top

Reversing drives - open

Reversing drives - crossed

Both sides of belt contact pulleys → these drives cannot be used with V or timing belts
Belts & Chains

Quarter-twist belt drive

Variable speed belt drive – flat belts

Variable speed belt drive – flat, V and round belts
Belts & Chains: Flat belt calculations

\[ D = \text{diameter of large pulley} \]
\[ d = \text{diameter of small pulley} \]
\[ C = \text{center distance} \]
\[ \theta = \text{angle of contact} \]
\[ L = \text{Length of belt - sum of two arc lengths with twice the distance between the beginning and end of contact:} \]
\[ L = \left[ 4 \cdot C - (D - d)^2 \right]^{1/2} + \frac{1}{2} (D \cdot \theta_D + d \cdot \theta_d) \]

\[ \theta_d = \pi - 2 \cdot \sin^{-1}\left( \frac{D - d}{2 \cdot C} \right) \]
\[ \theta_D = \pi + 2 \cdot \sin^{-1}\left( \frac{D - d}{2 \cdot C} \right) \]
Belts & Chains: Flat belt calculations

D = diameter of large pulley

d = diameter of small pulley

C = center distance

θ = angle of contact (same for each pulley)

L = Length of belt - sum of two arc lengths with twice the distance between the beginning and end of contact:

\[ L = \left[ 4 \cdot C - (D - d)^2 \right]^{1/2} + \frac{\theta}{2} (D + d) \]
Bels & Chains: Timing Belts

Timing Belts

- Do not slip, transmit power at constant angular-velocity ratio
- No initial tension needed, 97-99% efficient
- No lubrication needed, quiet
- 5 standard inch-series pitches available
  - Pitch lengths available in lengths of 6” to 180”
  - Pulleys: pitch diameters 0.60” to 35.8”, groove numbers from 10 to 120
Belts & Chains: Roller Chains

Roller Chains

• No slippage or creep
• Constant angular velocity ratio
• Requires lubrication, can be noisy
• Ability to drive multiple shafts from single source of power
• Single, double, triple, and quad strands available

Double strand roller chain
Belts & Chains: Roller Chain Calculations

\( \gamma/2 = \text{angle of articulation} \)

- Rotation of link through this causes impact between roller and sprocket teeth, wear in chain joint
- Design system to reduce this angle as much as possible

\( p = \text{chain pitch} \)
\( \gamma = \text{pitch angle} \)
\( D = \text{pitch diameter of sprocket} \)
\( N = \text{number of sprocket teeth} \)
\( L = \text{chain length} \)
\( C = \text{center distance} \)
\( N_1 = \# \text{ of teeth on small sprocket} \)
\( N_2 = \# \text{ of teeth on large sprocket} \)

\[
\sin \left( \frac{\gamma}{2} \right) = \frac{p/2}{p/2} \quad \text{or} \quad D = \frac{p}{\sin \left( \frac{\gamma}{2} \right)} \\

\text{Since } \gamma = \frac{360^\circ}{N} \quad \Rightarrow \quad D = \frac{p}{\sin \left( \frac{180^\circ}{N} \right)}
\]

\[
\frac{L}{p} = \frac{2 \cdot C}{p} + \frac{N_1 + N_2}{2} + \frac{(N_2 - N_1)^2}{4 \cdot \pi^2 \cdot \left( \frac{C}{p} \right)}
\]