Territory Partitioning for Gossiping Agents

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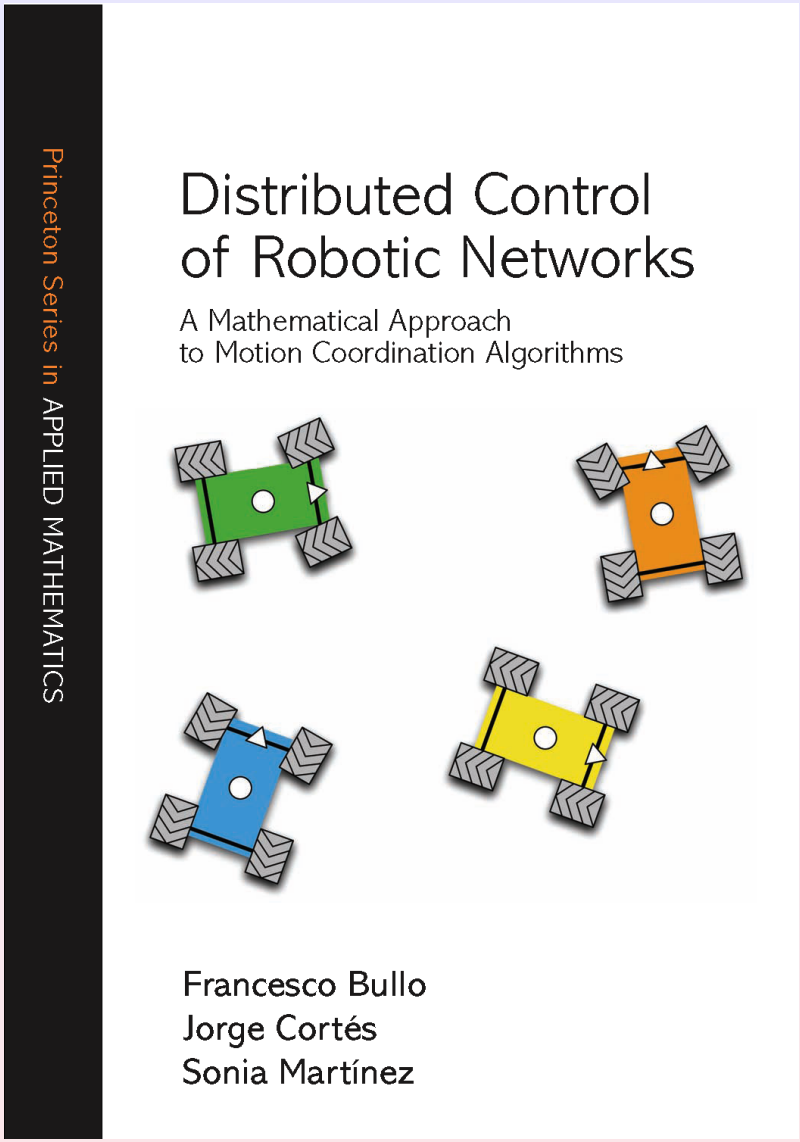
Distributed Control of Robotic Networks



1. intro to distributed algorithms (graph theory, synchronous networks, and averaging algos)
2. geometric models and geometric optimization problems
3. model for robotic, relative sensing networks, and complexity
4. algorithms for rendezvous, deployment, boundary estimation

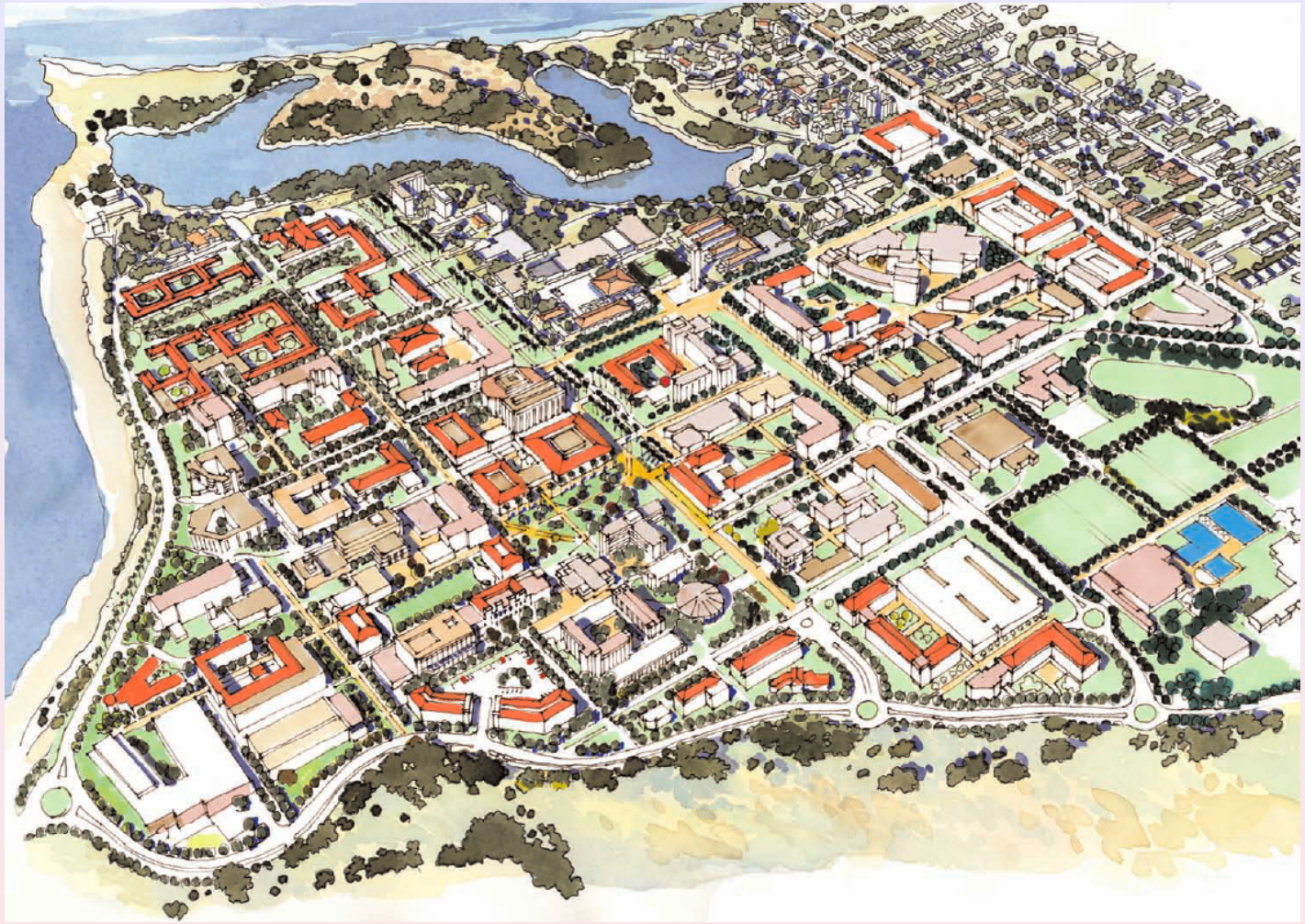
Status: Freely downloadable at http://coordinationbook.info with tutorial slides and (ongoing) software libraries.

F. Bullo, J. Cortés, and S. Martínez. *Distributed Control of Robotic Networks*. Applied Mathematics Series. Princeton Univ Press, 2009. ISBN 978-0-691-14195-4

Territory partitioning is ... art

Ocean Park Paintings, by Richard Diebenkorn (1922-1993)

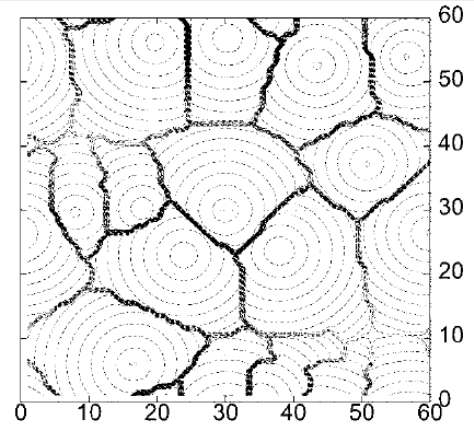
Territory partitioning is ... centralized space allocation



UCSB Campus Development Plan, 2008

Territory partitioning is ... animal territory dynamics

Tilapia mossambica, "Hexagonal Territories," Barlow et al, '74

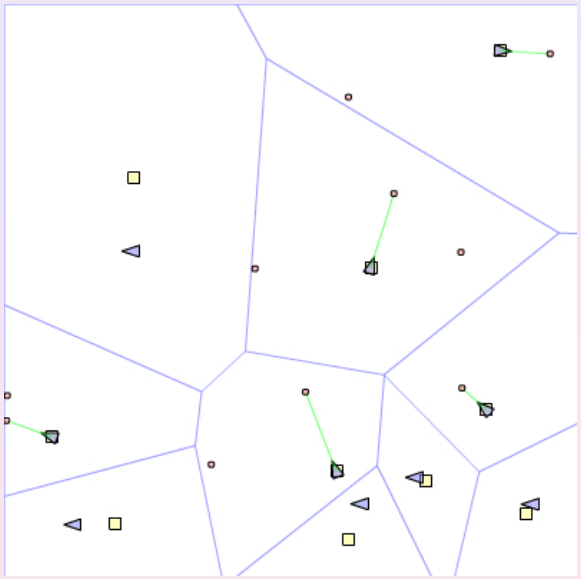


Red harvester ants, "Optimization, Conflict, and Nonoverlapping Foraging Ranges," Adler & Gordon, '03

- (exploitation + conflict) predict nonoverlapping foraging regions for distinct colonies
- "Deviations between model predictions and data indicate that colonies might allocate a larger than optimal number of foragers to areas near boundaries between foraging ranges."

Territory partitioning is ... robotic load balancing

- targets/customers appear randomly space/time
- robotic network knows locations and provides service
- goal: minimize wait time



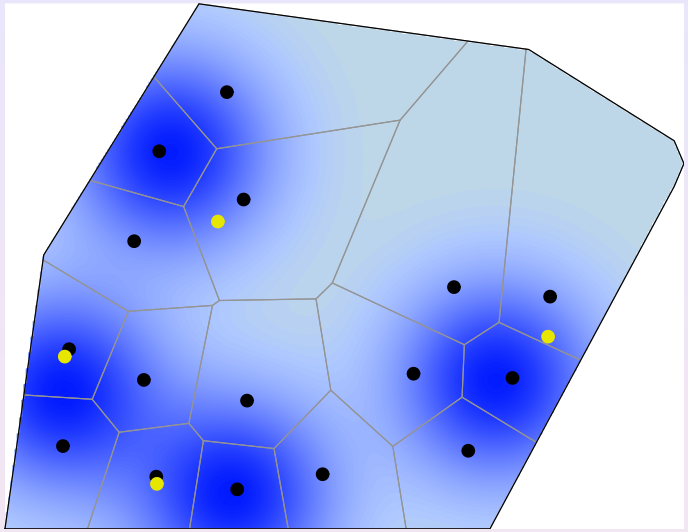
M. Pavone, E. Frazzoli, and F. Bullo. Decentralized algorithms for stochastic and dynamic vehicle routing with general target distribution. In *Proc CDC*, pages 4869–4874, New Orleans, LA, December 2007

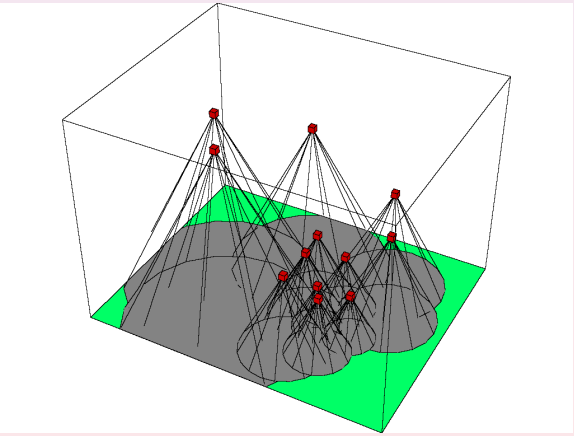
Distributed partitioning+centering algorithm

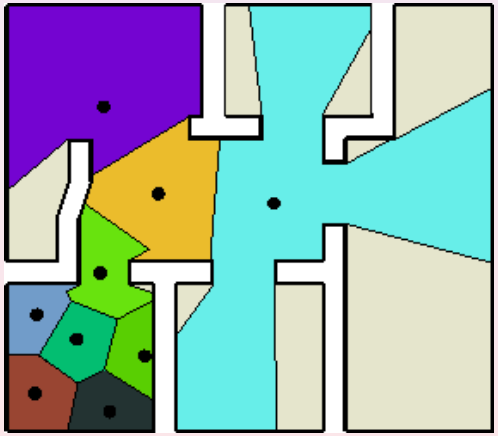
Partitioning+centering law

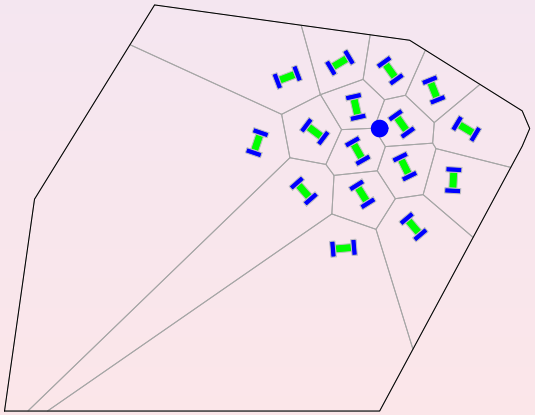
At each comm round:

1: acquire neighbors' positions
2: compute own dominance region
3: move towards centroid of own dominance region









Multi-center optimization

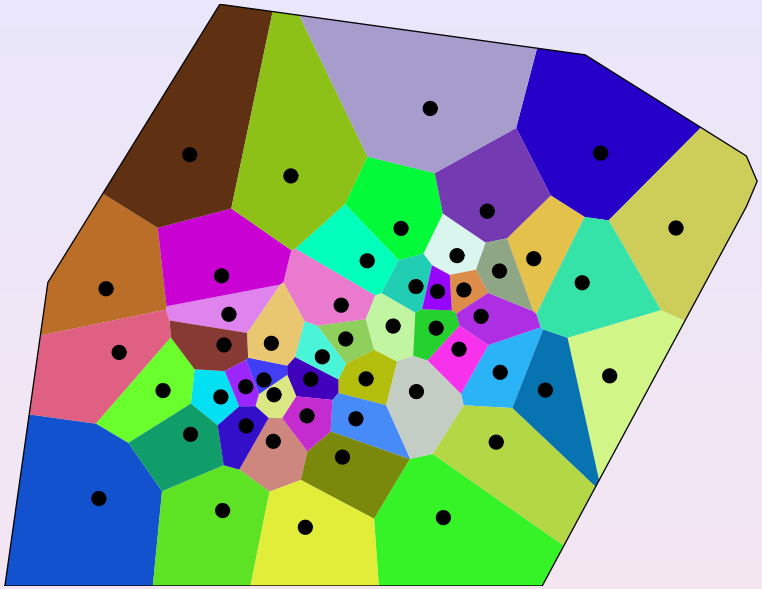
- place n robots at $p = \{p_1, \ldots, p_n\}$
- partition environment into $v = \{v_1, \ldots, v_n\}$
- define expected deviation

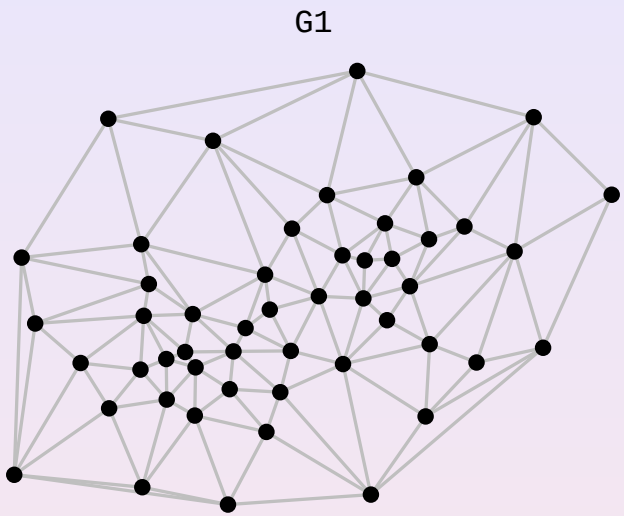
$$H(v, p) = \int_{v_1} \|q - p_1\| dq + \cdots + \int_{v_n} \|q - p_n\| dq$$

Theorem (Lloyd '57 "least-square quantization")

1. *at fixed partition, optimal positions are centroids*
2. *at fixed positions, optimal partition is Voronoi*
3. *alternate p-v optimization leads to local optimum*

Today: What are minimal communication requirements?





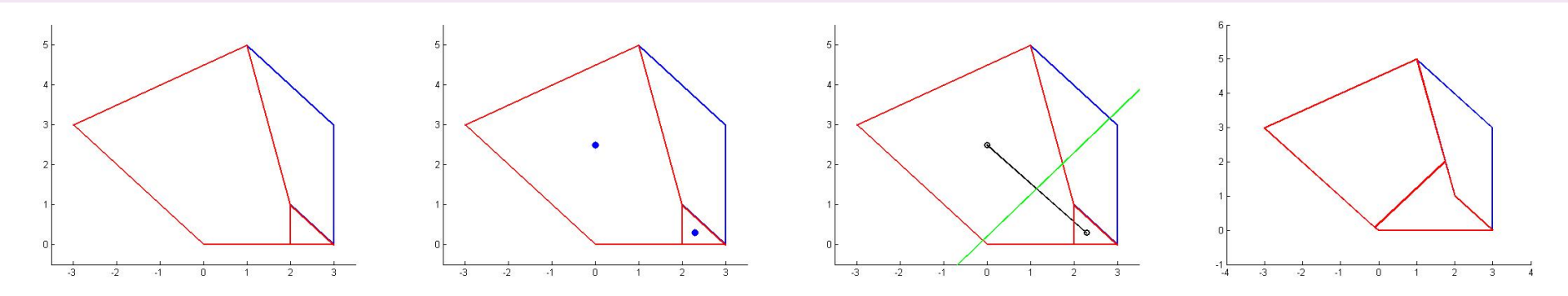
Lloyd partitioning+centering law requires:

1. synchronous & reliable communication
2. communication along edges of "adjacent regions graph"

what are minimal interaction requirements?

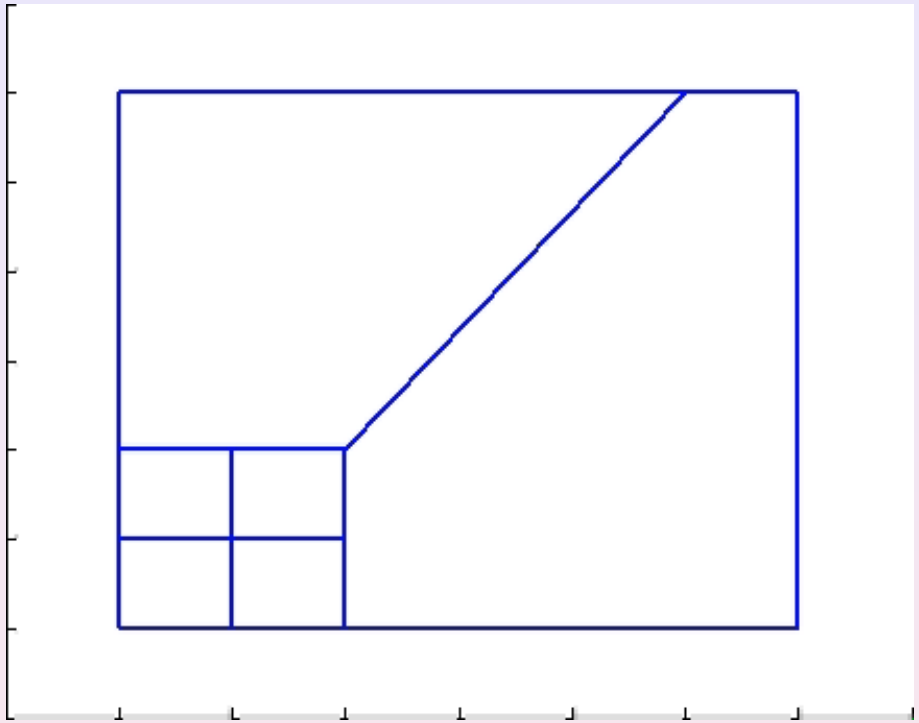
Algo #1: Gossip partitioning policy

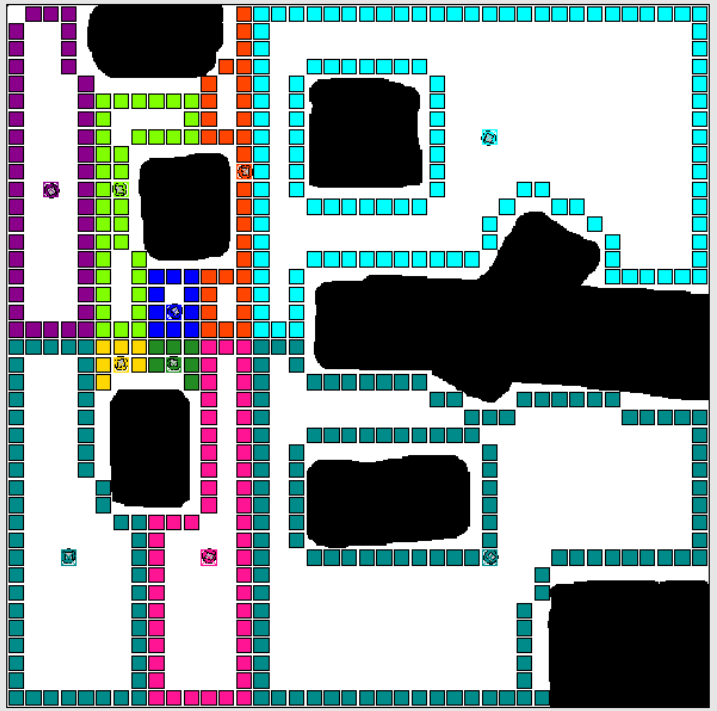
1. Random communication between two regions
2. Compute two centers
3. Compute bisector of centers
4. Partition two regions by bisector



P. Frasca, R. Carli, and F. Bullo. Multiagent coverage algorithms with gossip communication: control systems on the space of partitions. In *Proc ACC*, St. Louis, MO, June 2009. To appear

Simulation results





Implementation in player/stage

Algo #2: Randomized appointment protocols

```
Asynchronously each agent:
1: waits random time at random location near boundary;
   if neighbors appears, then region recomputation
2: performs vehicle routing for random time
```

1. bio-inspired boundary loitering
   "... larger than optimal number of foragers to areas near boundaries between foraging ranges"
2. bio-inspired region recomputation between gossiping agents

Theoretical contributions

state space:

- not finite-dimensional because a-priori arbitrary number of vertices
- non-convex disconnected polygons
- standard topology not ok

#1 topology of partitions

Space of (relatively closed) partitions is metric and compact with respect to symmetric difference

gossip map:

- ill-posed
- discontinuities
- not deterministic

#2 map modification

well-posed and continuous

#3 Lasalle invariance thm

1. persistent stochastic switching
2. common Lyapunov function
3. convergence to intersection set of fixed points

#4 Randomized appointments

... are persistent stochastic

Conclusions

Summary

1. novel gossip partitioning algorithm
2. novel randomized appointment protocol
3. space of partitions + stochastic LaSalle

Thanks to:

ARO SWARMS Award

Paolo Frasca

Ruggero Carli